Name: 　　　　　　　　　**Solid State Physics Problems, A**　　　　　**Solutions:**

Rules:
The test has a time limit of 60 minutes. You may use writing and drawing tools.
Scoring:
correct answer: 1 point;　　incorrect answer: 0 point;　　empty cell: 0 point
Evaluation:
0 – 4 point: try it again　　5 – 10 point: pass the test

1. Consider a one-dimensional chain of lattice constant a in the empty-lattice approximation. Calculate the energy of the **third**-lowest energy eigenstate for the wavenumber $k = \frac{\pi}{2a}$. The mass of the electron is denoted by m.

A) $\frac{3}{4}\frac{\hbar^2\pi^2}{ma^2}$　**B)** $\frac{9}{4}\frac{\hbar^2\pi^2}{ma^2}$　**C)** $\frac{3}{8}\frac{\hbar^2\pi^2}{ma^2}$　**D)** $\frac{9}{8}\frac{\hbar^2\pi^2}{ma^2}$　**E)** $\frac{25}{8}\frac{\hbar^2\pi^2}{ma^2}$

2. a and $a^\dagger$ are bosonic annihilation and creation operators. Simplify the following commutator $\left[a^\dagger a, a\right]$.

A) $-a$　**B)** $-a^\dagger$　**C)** 1　**D)** $a^\dagger$　**E)** a

3. Consider a cube lattice with lattice parameter a in the empty-lattice approximation. The number of the conduction electrons is $\frac{3}{2}N$, where N is the number of the primitive cell in the sample. Calculate the radius k_F of the Fermi sphere.

A) $\sqrt[3]{9\pi^2}\frac{1}{a}$　**B)** $\sqrt[3]{\frac{9\pi^2}{2}}\frac{1}{a}$　**C)** $\sqrt[3]{\frac{9\pi^2}{4}}\frac{1}{a}$　**D)** $\sqrt[3]{\frac{9\pi^2}{8}}\frac{1}{a}$　**E)** $\sqrt[3]{\frac{9\pi^2}{16}}\frac{1}{a}$

4. a_j and $a_j^\dagger$ are annihilation and creation operators on the lattice point j. We also define the creation and annihilation operators in the k-space by Fourier transformation:

$$a_{\mathbf{k}} = \frac{1}{\sqrt{N}}\sum_j a_j e^{i\mathbf{k}\mathbf{R}_j},$$

where N is the number of primitive cells, the sum is over every lattice points, furthermore, $\mathbf{R}_j$ is the position vector of lattice point j and the wavenumber $\mathbf{k}$ is the element of the Brillouin zone. Express the following sum $\sum_j a_j^\dagger a_j$ by the operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$.

A) $-\sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$　**B)** $\sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$　**C)** $\sum_{\mathbf{k}} a_{-\mathbf{k}}^\dagger a_{\mathbf{k}}$　**D)** $\sum_{\mathbf{k}} a_{-\mathbf{k}} a_{\mathbf{k}}^\dagger$　**E)** $-\sum_{\mathbf{k}} a_{-\mathbf{k}}^\dagger a_{\mathbf{k}}$

5. Investigate the in-plane phonon modes of the graphene. How many optical and acoustic modes are there?

A) 2 acoustic, 2 optical　**B)** 2 acoustic, 0 optical　**C)** 2 acoustic, 4 optical
D) 1 acoustic, 5 optical　**E)** 3 acoustic, 3 optical

6. Elementary excitations of ferromagnets at low temperature can be described by the Hamiltonians $H = E_0 + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$, where E_0 is the energy-offset, $a_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}$ are bosonic operators. The dispersion relation of spin waves is a quadratic function, i.e. $\hbar\omega_{\mathbf{k}}$ is proportional to $|\mathbf{k}|^2$. The number of thermally excited magnons at low temperature T, i.e. $\sum_{\mathbf{k}}\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}}\rangle$, is proportional to

A) $\sqrt{T}$.　**B)** T.　**C)** $\sqrt{T^3}$.　**D)** T^2.　**E)** $\sqrt{T^5}$.

7. Electrons in a two-dimensional system can be described by the following Hamiltonian:

$$H(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m} + U(y).$$

Consider a homogeneous magnetic field with magnitude B in the z-direction. Which vector potential was chosen if the Peierls substitution Hamiltonian $H(\mathbf{p} - e\mathbf{A})$ commutes with the operator p_x?

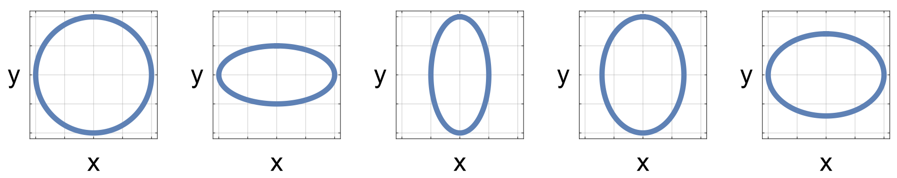
A) $B(-y, 0, 0)$　**B)** $\frac{B}{2}(-y, x, 0)$　**C)** $B(0, x, 0)$　**D)** $B(0, y, 0)$　**E)** $\frac{B}{2}(y, x, 0)$

8. Electrons can be described in a solid state by a quadratic Hamiltonian $H = \frac{\hbar^2}{2}\mathbf{k}\mathbf{M}^{-1}\mathbf{k}$, where the inverse mass tensor in the x, y, z basis has the form

$$\mathbf{M}^{-1} = \begin{pmatrix} 1/m & 0 & 0 \\ 0 & 1/(2m) & 0 \\ 0 & 0 & 1/m \end{pmatrix}.$$

Which sketch shows the classical orbital of an electron in the **real** space if we apply homogeneous weak magnetic field $\mathbf{B} = (0, 0, B)$? We assume that $m > 0$.

A)　**B)**　**C)**　**D)**　**E)**



9. Find the cyclotron mass m_c for the previous problem. The cyclotron mass is defined by $m_c = \frac{\hbar^2}{2\pi}\frac{\partial\mathcal{A}}{\partial\epsilon}$, where $\mathcal{A}$ is the k-space area enclosed by the loop of the motion. The energy is denoted by ϵ.

A) m　**B)** $2m$　**C)** $\frac{3m}{2}$　**D)** $\frac{3m}{\sqrt{2}}$　**E)** $\sqrt{2}m$

10. Consider a linear chain of lattice constant a with atoms of mass M at the lattice points. We take into account only the interaction between the nearest neighbors atoms, which interaction is modeled by an unstrained classical spring with spring constant K. Calculate the energy of the phonon mode with wavenumber $k = \frac{\pi}{3a}$.

A) $\sqrt{3}\sqrt{\frac{K}{M}}$　**B)** $\sqrt{\frac{K}{M}}$　**C)** $\sqrt{\frac{3}{2}}\sqrt{\frac{K}{M}}$　**D)** $\sqrt{6}\sqrt{\frac{K}{M}}$　**E)** $\sqrt{2}\sqrt{\frac{K}{M}}$

Solutions: EABBA CAEEB