2019.05.09. [1st.]

- there are several other terms in H $\rightsquigarrow$ very different graphs (new)

$$G_{\alpha\beta}(\vec{k},\tau) = -\langle T_\tau b_{k\alpha}(\tau)\, b^+_{k\beta}(0)\rangle$$

$$b_{k\alpha} = \begin{cases} b_k & \text{if } \alpha = 1 \\ b^+_{-k} & \text{if } \alpha = 2 \end{cases}$$

$\rightsquigarrow$ to treat the bose-condensed phase we need a 2×2 mx. gf.

$$\underline{\underline{G}} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

$$\hat{O}(\tau) = e^{\frac{\hat{K}\tau}{\hbar}}\, \hat{O}_s\, e^{-\frac{\hat{K}\tau}{\hbar}}$$

$$\langle \hat{O} \rangle = \mathrm{Tr}(\hat{\rho}\,\hat{O})$$

$$\hat{\rho} = \frac{e^{-\beta K}}{Z_G}$$

$$Z_G = \langle e^{-\beta K}\rangle$$

$$G_{11}(k,\tau) = -\langle T_\tau b_k(\tau)\, b^+_k(0)\rangle$$

$$G_{22}(k,\tau) = -\langle T_\tau b^+_{-k}(\tau)\, b_{-k}(0)\rangle$$

- order of op.-s can be changed freely (bosonic b-s...)

$$= -\langle T_\tau b_{-k}(0)\, b^+_{-k}(\tau)\rangle = G_{11}(-k,-\tau)$$

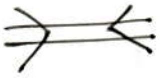
- the 2 elements are not independent!

$$G_{22}(k,\tau) = G_{11}(-k,-\tau) \longrightarrow \boxed{G_{22}(k, i\omega_n) = G_{11}(-k,-i\omega_n)}$$

$$G_{21}(k,\tau) = G_{12}(-k,-\tau) \longrightarrow \boxed{G_{21}(k, i\omega_n) = G_{12}(-k,-i\omega_n)}$$

$$\underset{\text{normal system}}{\hat{K}} = \hat{K}_0 + \underbrace{\hat{K}_1' + \hat{K}_0'}_{\text{1 particle}} + \underbrace{\hat{K}_{I4} + \hat{K}_{I3} + \hat{K}_{I2} + \hat{K}_{I1} + \hat{K}_{I0}}_{\text{interaction part.}}$$

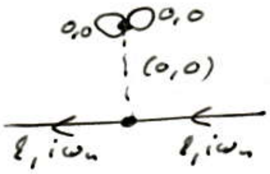
• some notations:

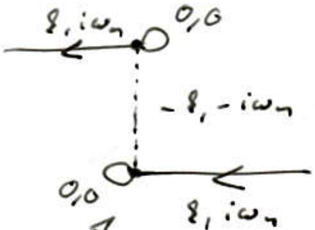
 $= -G_{11}(\ell, i\omega_n)$  $= -G_{21}(\ell, i\omega_n)$

 $= -G_{12}(\ell, i\omega_n)$  $= -G_{22}(\ell, i\omega_n)$

 =  +  +  +

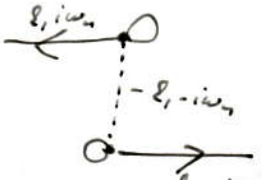
$-G_{11}(\ell, i\omega_n)$ $-G_0(\ell, i\omega_n)$ $\left(-G_{11}^{(0)}(\ell, i\omega_n)\right)$ (Hartree-term) (Fock-term)

+ 

+ 

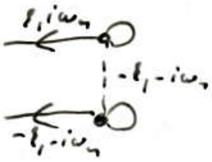
+ ... (higher order terms)

these are not present above T_c. (since there is no condensate)

 = 

$-G_{12}(\ell, i\omega_n)$

does not exist above T_c (anomalous gf.)

(same structure: 

)

- Analytical forms:

$$-G_{11}(\ell, i\omega_n) = -G_{11}^{(0)}(\ell, i\omega_n) - \frac{1}{\hbar} V(0)\, G_{11}^{(0)\,2}(\ell, i\omega_n) \int \frac{d^3q}{(2\pi)^3} \frac{1}{\beta\hbar} \sum_m \Big[-G_0(q, i\omega_m)\, e^{i\omega_m \eta} \Big] +$$

$$- \frac{1}{\hbar} V(0)\, G_0^{\,2}(\ell, i\omega_n) \int \frac{d^3q}{(2\pi)^3} \frac{1}{\beta\hbar} \sum_m \left(-G_0(q, i\omega_m)\, e^{i\omega_m \eta} \right) +$$

$$- \frac{1}{\hbar} V(0)\, G_0^{\,2}(\ell, i\omega_n) \frac{N_0}{V} +$$

$$- \frac{1}{\hbar} V(-\ell)\, G_0^{\,2}(\ell, i\omega_n) \frac{N_0}{V} + (\text{higher order terms} \ldots)$$

$$-G_{12}(\ell, i\omega_n) = -\frac{1}{\hbar} V(-\ell)\, G_0(\ell, i\omega_n)\, G_0(-\ell, -i\omega_n) \frac{N_0}{V} + (\ldots)$$

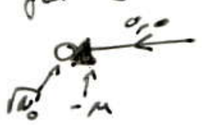
$$0 \sim \sqrt{N_0}$$

- but how to calculate N_0?

$$\hat{b}_\ell = \hat{a}_\ell - \sqrt{N_0}\, \delta_{\ell 0}$$

$$\langle \hat{b}_\ell \rangle = 0 \quad \forall \vec{\ell}$$

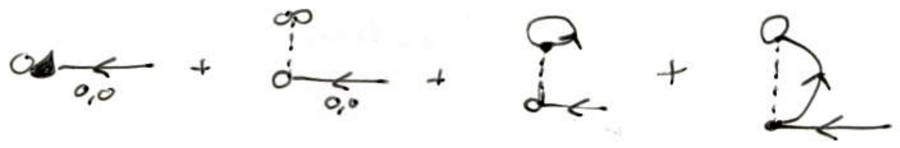
for $\ell = 0$



but for non-0 ℓs:

something has to be on the end!

$0 = \langle b_0 \rangle =$



$\rightsquigarrow$ 1 incoming arrow, but no outgoing!

$$\equiv \boxed{\Sigma_{10}} \xleftarrow[(0,0)]{}$$

$0\Delta\!\leftarrow\; = (-\mu)\sqrt{N_0}\, b_0$

$\frac{N_0^{3/2}}{2V} V(0)\, b_0$ } they both have to have the same dimension!

$$0 = \left(-\frac{1}{\hbar}\right)\left[\sqrt{N_0}(-\mu) + \left(\frac{N_0^{3/2}V(0)}{V} + \frac{\sqrt{N_0}}{V}\sum_q (V(0)+V(q))\underbrace{\left(\frac{1}{\beta\hbar}\right)\sum_m G_0(q,i\omega_m)}_{n_q' = \frac{1}{e^{\beta(e_q-\mu)}-1}} \ldots\right.\right.$$

solving for μ

$$\mu = \frac{N_0}{V}V(0) + \frac{1}{V}\sum_q (V(0)+V(q))\, n_q' + \ldots$$

~ thermal occupation of the normal atoms

↓

for small q this can be < 0!

- we now have a scalar-eq for N_0, if μ is a parameter of the model.

- let's suppose $(e_q - \mu) > 0$

↳ what is $T=0$ limit?

↓

$n_q' = 0$!

↓

Bogoliubov-approx: (same as in Qgases)

Gross-Pitajevski-eq.

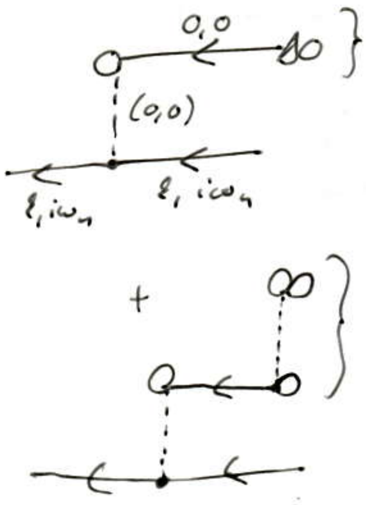
$$\boxed{\mu = \frac{N_0}{V}V(0)}$$

$\left\{\begin{array}{l} V(0) = g = \frac{4\pi\hbar a}{m} \\ \mu = gn \end{array}\right.$ for homogenius system

$\Sigma_{10} = $ o▲ + (self energy)

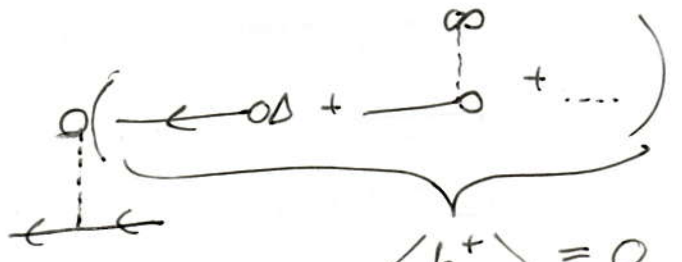
(• up to 2nd order magic has to be done)

- let's say



} some higher order terms with zero momentum

$\rightsquigarrow$



$= 0$

$\langle b_0^+ \rangle = 0$

by definition

- what happens if $\mu > 0$? $\rightarrow n_q' < 0 \rightarrow$ has to be fixed
- in Bogoliubov approx: $\mu = \frac{N_0}{V} V(0)$

  $\rightsquigarrow$ if s-wave scattering length $< 0 \rightarrow$ dynamical instability

  $\rightsquigarrow$ needs trapping potential

$\rightarrow \mu > 0$ in B. approx at $T=0$ for weakly int. Bose-gas.

$$\hat{K}_0 = \sum_k (e_k - \mu)\hat{b}_k^+ \hat{b}_k = \underbrace{\sum_k (e_k - \mu_0)\hat{b}_k^+ \hat{b}_k}_{\hat{K}_0'} + \underbrace{\sum_k (\mu_0 - \mu)\hat{b}_k^+ \hat{b}_k}_{\text{compensation}}$$

$\rightarrow$ redefine the non-int. part

compensation $\downarrow$ this goes to the perturbation as a new term

- we can say $\mu_0 = 0$, $\blacktriangle = -\mu$

  $\rightsquigarrow$ it is okay now...
- with this:

$$G_{11}^{(0)} = \frac{1}{i\omega_n - \frac{e_k}{\hbar}} \qquad G_{22}^{(0)} = \frac{1}{-i\omega_n - \frac{e_k}{\hbar}}$$

$\downarrow$ Matsubara sum

$$\frac{1}{e^{\beta e_k} - 1}$$

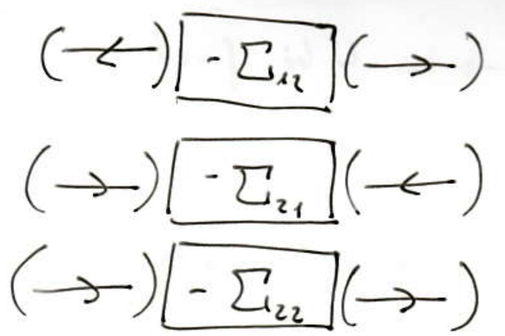
$$G_{12}^{(0)} = G_{21}^{(0)} = 0$$

but we have $K_2' = \sum_k (-\mu)\hat{b}_k^+ \hat{b}_k \rightarrow \leftarrow\blacktriangle\leftarrow$

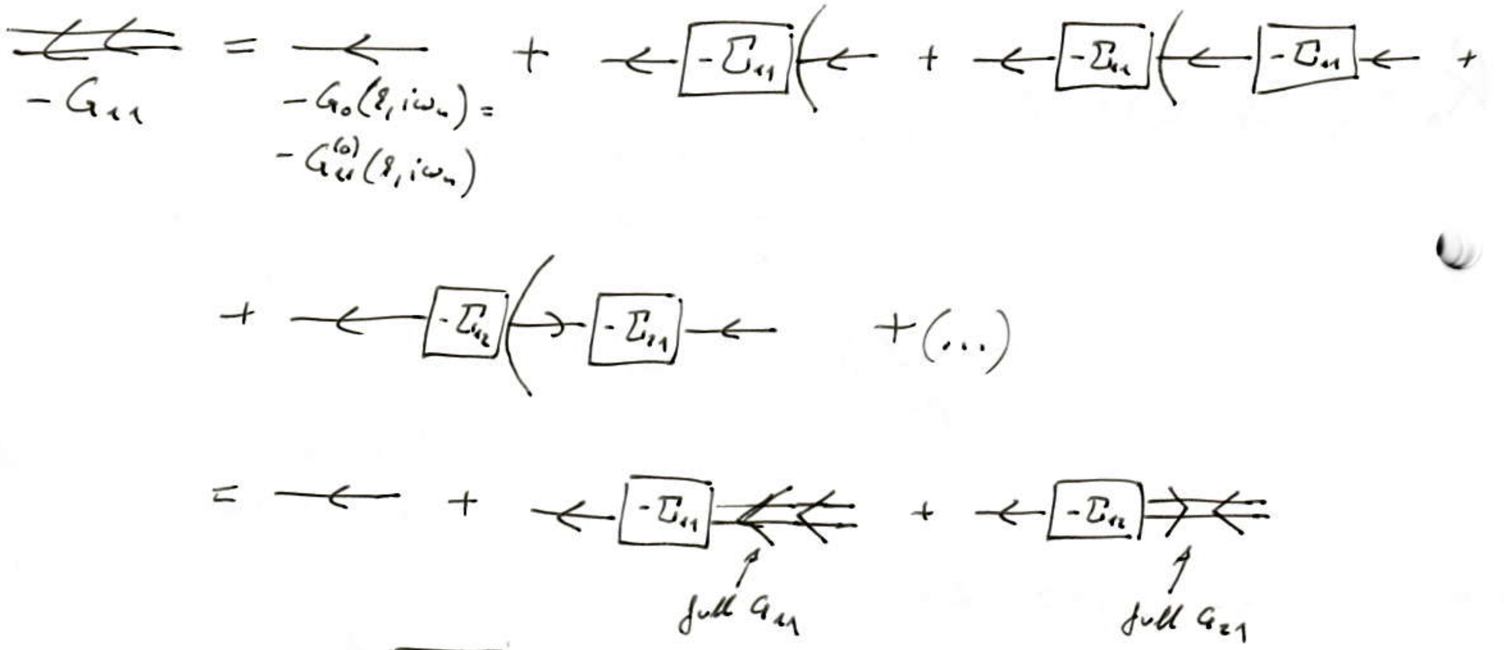
<u>Generalizing the Dyson-eq.</u>

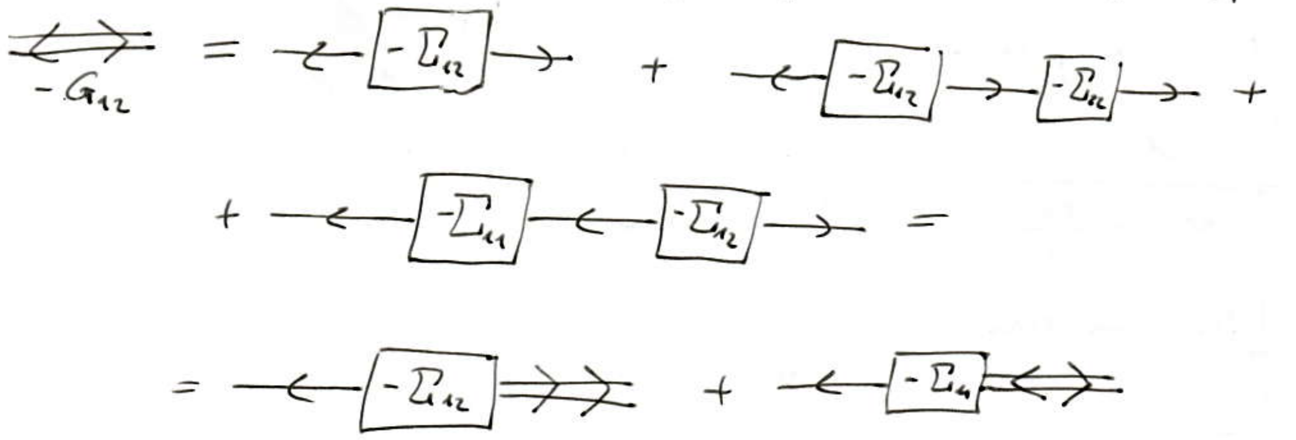
$$(\leftarrow)\boxed{-\Sigma_{11}}(\leftarrow)$$

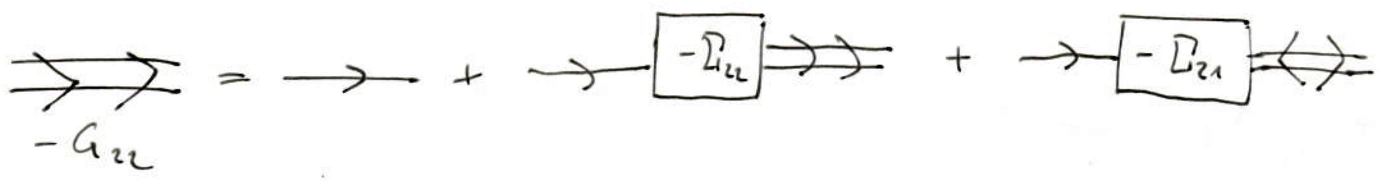
- this is known from the scalar case



- we have 4 different self-energies, that can be ordered in a 2×2 mx.







- Beliaev - Dyson - equation:

$$\boxed{G_{\alpha\beta}(\vec{q}, i\omega_n) = G^{(0)}_{\alpha\beta}(\vec{q}, i\omega_n) + \sum_{\gamma,\delta} G^{(0)}_{\alpha\gamma}(\vec{q}, i\omega_n)\,\Sigma_{\gamma\delta}(\vec{q}, i\omega_n)\,G_{\delta\beta}(\vec{q}, i\omega_n)}$$

$$G^{(0)}_{\alpha\beta}(\vec{q}, i\omega_n) = \begin{pmatrix} \frac{1}{i\omega_n - \frac{e_q}{\hbar}} & 0 \\ 0 & \frac{1}{-i\omega_n - \frac{e_q}{\hbar}} \end{pmatrix}$$

$$\Sigma_{\alpha\beta}(\vec{q}, i\omega_n) = \begin{pmatrix} \Sigma_{11}(\vec{q}, i\omega_n) & \Sigma_{12}(\vec{q}, i\omega_n) \\ \Sigma_{21}(\vec{q}, i\omega_n) & \Sigma_{22}(\vec{q}, i\omega_n) \end{pmatrix}$$
