- Retarded density correlation func.:

$$iD^R(r_1,t_1,r_2,t_2) = \Theta(t_1-t_2)\langle[\tilde{n}(r_1,t_1),\tilde{n}(r_2,t_2)]\rangle$$

$$D^R(r_1,t_1,r_2,t_2) = -i\Theta(t_1-t_2)\sum_{m,n} w_n A^D_{mn}(r_1,r_2)[1-e^{-\beta(K_m-K_n)}]e^{-\frac{i}{\hbar}(K_n-K_m)(t_1-t_2)}$$

$$D^R(r_1,r_2,\omega) = \int_{-\infty}^{\infty}dt\, D^R(r_1,t,r_2,0)e^{i(\omega+i\varepsilon)t}$$

$$\sim \left[\frac{e^{i[(\omega+i\varepsilon)-\frac{K_m-K_n}{\hbar}]t}}{i[\omega+i\varepsilon-\frac{K_m-K_n}{\hbar}]}\right]_0^{\infty} =$$

for $-\infty$ it's not a problem, because of $\Theta(t)$, but for ∞ we have to ensure that it will be convergent. $\rightarrow i\varepsilon$

$$= +i\sum_{mn} w_n A^D_{mn}(r_1,r_2)[1-e^{-\beta(K_m-K_n)}]\frac{1}{i[\omega+i\varepsilon-\frac{K_n-K_m}{\hbar}]}$$

$$D^R(r_1,r_2,\omega) = \sum_{mn}\frac{w_n A^D_{mn}(r_1,r_2)[1-e^{-\beta(K_m-K_n)}]}{\omega-\frac{K_n-K_m}{\hbar}+i\varepsilon}$$

$\downarrow$

$$D^R(q,\omega) = \sum_{mn}\frac{w_n A^D_{mn}(q)[1-e^{-\beta(K_m-K_n)}]}{\omega-\frac{K_n-K_m}{\hbar}+i\varepsilon}$$

- Density fluctuation op

$$D(r_1,\tau_1,r_2,\tau_2) = -\langle T_\tau(\tilde{n}(r_1,\tau_1)\cdot\tilde{n}(r_2,\tau_2))\rangle$$

$$D^R(q,\omega) = \int\frac{d\omega'}{2\pi}\frac{\rho^D(q,\omega')}{\omega-\omega'+i\varepsilon} =$$

$$= \int\frac{d\omega'}{\cancel{2\pi}}\cancel{2\pi}\sum_{m,n}\frac{w_n(1-e^{-\beta(K_m-K_n)})A^D_{mn}(q)\cdot\delta(\omega'-\frac{K_m-K_n}{\hbar})}{\omega-\omega'+i\varepsilon} =$$

$$\frac{1}{x+i\varepsilon} = \mathbb{P}\left(\frac{1}{x}\right) - i\pi\delta(x)$$

$$= \sum_{mn}\frac{w_n(1-e^{-\beta(K_m-K_n)})A^D_{mn}(q)}{\omega-\frac{K_m-K_n}{\hbar}+i\varepsilon}$$

$$D^R(l,\omega) = \sum_{m,n} \left(\frac{\omega_m A_{mn}(l)}{\omega - \left(\frac{K_m - K_n}{\hbar}\right) + i\epsilon} + \frac{\omega_n A_{mn}(l)}{\omega + \left(\frac{K_m + K_n}{\hbar}\right) + i\epsilon} \right) -$$

β

mean in the 2nd term

$$- \frac{\omega_m e^{-\beta(K_n - K_m)} A_{nm}(l)}{\omega - \frac{K_n - K_m}{\hbar} + i\epsilon}$$

$$\mathrm{Im}\left(D^R(q,\omega)\right) = -\frac{1}{2}\rho^0(q,\omega)$$

$$\int \frac{d\omega'}{2\pi}\, \frac{\rho^0(q,\omega')}{i\omega_n - \omega'} = D(q, i\omega_n)$$ → gives the correct Lehman-representation.

can be calculated by the Temp. dependent Matsubara-formalism.

then with analytic continuation D^R can be obtained...

$$A_{nm}(q) = A^*_{m,n}(-q)$$

$$n^+(q) = n(-q)$$ comes from here

In isotropic system: there is $|q|$-dependance!

⇒ $|-q| = q$ ...

we can also find out $A^*_{m,n} \in \mathbb{R}$

$A_{nm}(q) = A_{mn}(q)$ symmetric-matrix

// See page 13. //

Old facts concerning the bubble:

$$D(\vec{q}, i\omega_n) = \frac{\Pi(\vec{q}, i\omega_n)}{\mathcal{E}(\vec{q}, i\omega_n)} \qquad \mathcal{E}(\vec{q}, i\omega_n) = 1 - V(q)\Pi(\vec{q}, i\omega_n)$$

approx.: $\Pi(\vec{q}, i\omega_n) \approx$  (Random-phase-approx)

→ where are the poles?

$$D^R(\vec{q},\omega) = \frac{\Pi^R(\vec{q},\omega)}{\mathcal{E}(\vec{q},\omega)}$$

$i\omega_n \to \omega + i\epsilon$ analytic continuation

$$\Pi(\vec{q}, i\omega_n \to \omega + i\varepsilon) = \Pi^R(\vec{q}, \omega)$$

1.) Π^R has a pole $\to$ no pole in D^R

2.) $\boxed{\varepsilon^R \text{ has } \emptyset}$

- ε contains Π^R
- they "cancel out"

$$\varepsilon^R(\vec{q}, \omega) = \emptyset$$

$\downarrow$

from the location of the pole

$$\omega = \Omega + i\Gamma \to \text{lifetime}$$

dispersion relation : $\Omega(q)$

Jelly - model

- In RPA:

$$\Pi^{(0)}(\vec{q}, i\omega_n) = -\frac{m\, q_F}{\pi^2 \hbar^2} R(\zeta) \quad \text{where} \quad \zeta = \frac{\omega_n m}{\hbar\, q_F\, q}$$

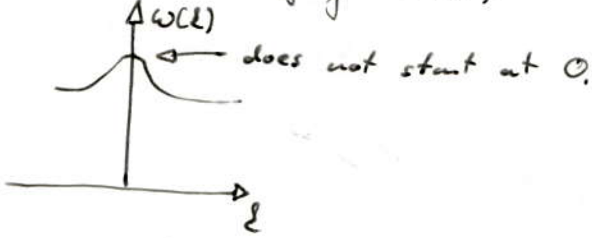
when $q \to 0$

- in huge systems long wavelength dominates.
- where $R(\zeta) = 1 - \zeta \operatorname{arctg}\left(\frac{1}{\zeta}\right)$

- analytic continuation: $\zeta' = \frac{(\omega + i\varepsilon) m}{i \hbar\, q_F\, q}$ $\rightsquigarrow$ purely imaginary (we forget ε...)

if $q \to 0$

$|\zeta'| \to \infty$



- taylor-expansion : $\operatorname{arctg}(x) = x - \frac{x^3}{3} + (\ldots)$

$$R(\zeta') \approx 1 - \zeta'\left(\frac{1}{\zeta'} - \frac{1}{3\zeta'^3}\right) \approx \frac{1}{3\zeta'^2}$$

$$\Pi(q,\omega) \cong -\frac{m q_F}{\pi^2 \hbar^2}\,\frac{1}{3\,\xi'^2} = -\frac{m q_F}{\pi^2\hbar^2}\,\frac{1}{3\left(\frac{\omega m}{i\hbar q_F q}\right)^2} = \frac{q^2 q_F^3}{3\omega^2\pi^2 m}$$

$$0 = \varepsilon^R(q,\omega) = 1 - \underbrace{e_0^2\, 4\pi \frac{1}{q^2}}_{v(q)}\,\frac{q^2 q_F^3}{3\omega^2\pi^2 m}$$

→ the dispersion relation really starts at finite ω

$$\omega^2_{Plasmon} = \frac{4}{3}\,\frac{e_0^2 q_F^3}{m\pi}$$ and $\boxed{q_F^3 = 3\pi^2 n}$

where n is the density

(from "$N = \sum_n F_{FD}$"; $\frac{\hbar q_F^2}{2m} = \mu$ at $T=0$)

$$\omega_{Plasmon} = \sqrt{\frac{4\pi e_0^2 n}{m}}$$

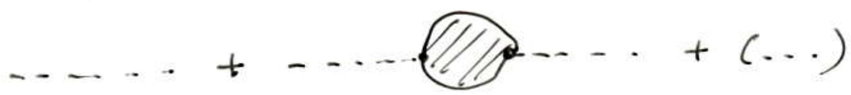
→ no imaginary part. ⇝ plasmons are long lived

• with further expansion, there will be imag. part...

$$\omega(q) = \omega_{Plasmon}\left[1 + \frac{9}{10}\left(\frac{q}{q_F}\right)^2\right.$$

excitations of $1e^-$ from fermi sphere. } disp. location meets this collective excitations decay to 1 part excitations. → Landau-dampening

Superfluidity, etc.



$$V_{eff} = \frac{4\pi e_0^2}{q^2 \varepsilon_e} \qquad e_0^2 = \frac{e^2}{4\pi\varepsilon_0}\,,\quad a_0 = \frac{\hbar^2}{m e_0^2}$$ (1st Bohr-radius)

$\varepsilon_e(0,\omega) = 1 - \frac{\omega_p^2}{\omega^2}$ ω_p is plasma freq.

$\varepsilon_e(q,0) = 1 + \frac{q_{TF}^2}{q^2}$

$q_{TF}^2 = \frac{4}{\pi} \frac{1}{q_F a_0} q_F^2$

Thomas - Fermi

$q_F^3 = 3\pi^2 n$

$v_F = \frac{\hbar q_F}{m}$

for electrons

$\omega \ll q v_F$

we can take $\varepsilon_e(q,0)$

for ions

$q v_{ion} \ll \omega$

ion velocity

electrons

Ions move from now on ...

$\varepsilon_{ion} = 1 - \frac{\Omega_p^2}{\omega^2}$

different plasma freq. (different mass, charge)
