**Particle physics - written test n. 1**

**17th December 2019**

Solve the following three problems (including the extra questions only if desired). The test is passed with a score of at least 15 points.

**Problem 1 (5 points)**

Under which conditions will the 4-momentum of a system of 2 photons correspond to zero mass?

**Problem 2 (10 points)**

Determine which of the following processes can take place and which ones cannot:

$$\pi^+ \to e^+ \, e^- \, e^+ \, ; \qquad \pi^0 \to \gamma \, \gamma \, ; \qquad \mu^- \to e^- \, \gamma \, ;$$
$$K^+ \to \mu^+ \, \nu_\mu \, ; \qquad K^- \, n \to \Sigma^- \, \pi^0 \, .$$

For the forbidden processes, explain why they are forbidden; for the allowed ones, write down which is the relevant interaction.

**(Extra question, +6 points)**

Are the following processes allowed or forbidden? If allowed, write down what is the relevant interaction, if they are forbidden explain why:

$$\pi^+ \to \pi^0 \, e^+ \, \bar{\nu}_e \, ; \qquad \pi^+ \to \pi^0 \, e^+ \, \nu_e \, ; \qquad \pi^0 \to \pi^+ \, e^- \, \bar{\nu}_e \, .$$

**Problem 3 (5 points)**

Let $b_1^\dagger$ and $b_2^\dagger$ be the creation operators of the ground state $|G\rangle$ and the excited state $|X\rangle$ of an atom,

$$|G\rangle = b_1^\dagger|0\rangle \, , \qquad |X\rangle = b_2^\dagger|0\rangle \, , \qquad b_1|0\rangle = b_2|0\rangle = 0 \, ,$$

with $|0\rangle$ the vacuum state, and $[b_i, b_j^\dagger] = \delta_{ij}$, $[b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0$, $i, j = 1, 2$. Show that the following holds for the operators $\Pi = b_1^\dagger b_2$ and $\Pi^\dagger = b_2^\dagger b_1$:

$$\Pi^\dagger|G\rangle = |X\rangle \, , \qquad \Pi|X\rangle = |G\rangle \, , \qquad \Pi^\dagger|X\rangle = \Pi|G\rangle = 0 \, .$$

**(Extra question, +4 points)**

Show that the operators $I_1 = \frac{1}{2}(\Pi^\dagger + \Pi)$, $I_2 = \frac{1}{2i}(\Pi^\dagger - \Pi)$ and $I_3 = \frac{1}{2}(b_2^\dagger b_2 - b_1^\dagger b_1)$ obey the SU(2) commutation relations,

$$[I_a, I_b] = i\varepsilon_{abc} I_c \, .$$