Particle physics problem sheet

1. A pion with mass M is moving by velocity β in the lab frame and decays to two photons. Determine the relationship between the energies of the photons and their angles (relative to the motion of the pion)!
2. A photon collides with a proton of mass M at rest in the lab frame. What is the necessary minimal energy for the photon such that in the final states we will have a neutral pion of mass M_0 in addition to the original proton?
3. Under which conditions will the 4-momentum of a system of 2 photons correspond to zero mass?
4. Consider the process $e^+e^- \to Z\gamma$. Find the processes related to this one by the crossing relations and also the kinematical bounds on the Mandelstam variables s, t and u!
5. It is often useful to use the I_3 and step operators $I_\pm = I_1 \pm iI_2$ instead of the original I_j $j = 1, 2, 3$ isospin components. Show that the commutation relations are

$$[I_+, I_-] = 2I_3, \qquad [I_3, I_\pm] = \pm I_\pm$$

6. The irreducible representation of $SU(2)$ labelled by j is spanned by the basis vectors $|j, m\rangle$ ($m \in -j, -j+1, \ldots, j$). The action of $I_\pm = I_1 \pm iI_2$ and I_3 on these is given by

$$I_\pm|j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|j, m \pm 1\rangle, \quad I_3|j, m\rangle = m|j, m\rangle$$

Construct the 3×3 representation matrices for I_1, I_2 and I_3 for the $j = 1$ representation!

7. The Casimir operator of $SU(2)$ is given by $I = I_1^2 + I_2^2 + I_3^2$. Express I by I_3 and $I_\pm$ and using the expressions from the previous exercise show that

$$I|j, m\rangle = j(j+1)|j, m\rangle$$

8. Plot the pseudo-scalar meson octet consisting of a quark and anti-quark and the baryon octet consisting of 3 quarks on the I_3, Y plane and label each particle by its quark content (e.g. $\pi^+ = (u, \bar{d})$, $n = (d, d, u)$)!
9. Consider the flavour $SU(3)$ decuplet in the I_3, Y plane. For each particle calculate $I(I+1) - Y^2/4$ where I is the total isospin and show that it is a linear function of Y and find the coefficients α, β in $I(I+1) - Y^2/4 = \alpha Y + \beta$!
10. Plot the states corresponding to the $\bar{3}$ representation of flavour $SU(3)$ in the I_3, Y plane and label each with its quark content! These states correspond to $\bar{q}$ anti-quarks.

11. • Plot the states corresponding to the 6 representation of flavour $SU(3)$ in the I_3, Y plane and label each with its quark content! The 6 representation is the 2-index-symmetric representation hence these states correspond to qq-states or diquarks.

12. • Determine which processes can take place and which ones can not:

$$\pi^+ \to e^+ e^- e^+ \qquad \pi^0 \to \gamma\gamma \qquad \mu^- \to e^- \gamma$$

$$K^+ \to \mu^+ \nu_\mu \qquad K^- + n \to \Sigma^- + \pi^0$$

13. • Let $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ be the creation operators of free, 0 spin, mass m particles corresponding to a complex field. Show that the following states are eigenstates of the energy, momentum and charge operators and find the eigenvalues as well:

$$|ab\rangle = a^\dagger(\mathbf{p}) b^\dagger(-\mathbf{p})|0\rangle \qquad \text{és} \qquad |aa\rangle = a^\dagger(\mathbf{p}) a^\dagger(\mathbf{p})|0\rangle$$

14. • Let $b_1^\dagger$ and $b_2^\dagger$ be the creation operators of the ground state and excited state of an atom

$$|G\rangle = b_1^\dagger|0\rangle, \quad |X\rangle = b_2^\dagger|0\rangle, \quad b_1|0\rangle = 0 = b_2|0\rangle$$

fulfilling the commutation relations $[b_i, b_j^\dagger] = \delta_{ij}$ $i, j = 1, 2$. Show that the following holds for $\Pi = b_1^\dagger b_2$ and $\Pi^\dagger = b_2^\dagger b_1$:

$$\Pi^\dagger|G\rangle = |X\rangle, \quad \Pi|X\rangle = |G\rangle, \quad \Pi^\dagger|X\rangle = 0, \quad \Pi|G\rangle = 0 \; .$$

15. • Let ϕ be a real scalar field whose self-interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + g\phi^3$$

Determine the following
(a) mass dimension of g,
(b) the expression for the vertex in Feynman diagrams
(c) lowest order Feynman diagrams for the $2 \to 2$ and the $2 \to 3$ scatterings

16. • Let $\phi(x)$ be a real scalar field and $\psi(x)$ a Dirac spinor field and assume that their interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \bar{\psi} i \gamma^\mu \partial_\mu \psi + M\bar{\psi}\psi + g\phi\bar{\psi}\psi$$

(The last term is often called Yukawa interaction.) Determine the following
(a) mass dimension of g
(b) the expression for the vertex in Feynman diagrams

17. • Show that the time ordered product of spin 1/2 fields satisfies the equation

$$(i\gamma^\mu \partial_\mu^x - m)_{\alpha\beta} T(\psi_\beta(x)\bar{\psi}_\gamma(y)) = i\delta_{\alpha\gamma}\delta^{(4)}(x-y)$$

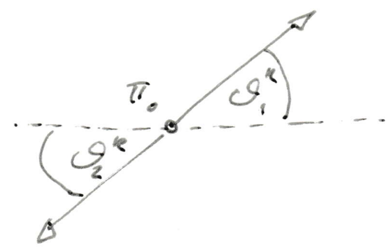
Particle physics exam problems

① A pion with mass M is moving by velocity β in the lab frame and decays to two photons. Determine the relationship between the energies of the photons and their angles (relative to the motion of the pion)!

In the rest frame of π_0:

$$p_i = \begin{pmatrix} M \\ 0 \end{pmatrix} \quad \text{and} \quad p_i = p_f$$

$$p_f = \begin{pmatrix} \omega_1^R + \omega_2^R \\ \underline{\xi}_1^R + \underline{\xi}_2^R \end{pmatrix}$$



$$\longrightarrow \underline{\xi}_1^R = -\underline{\xi}_2^R \ ; \ \omega_1^R = \omega_2^R := \omega^R \qquad \longrightarrow \vartheta_1^R = \vartheta_2^R = \vartheta^R$$

$$|\underline{\xi}| = \omega^R = \frac{M}{2}$$

The lab-frame moves with β:

$$\underline{\underline{\Lambda}} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{-\beta}{\sqrt{1-\beta^2}} & & \\ \frac{-\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

using this we get the photon-energies in lab frame:

$$\omega^L = \frac{1}{\sqrt{1-\beta^2}} \underbrace{\omega^R}_{\frac{M}{2}} + \frac{-\beta}{\sqrt{1-\beta^2}} \underbrace{\xi_x^R}_{\frac{M}{2}\cos(\vartheta^R)}$$

$$\omega^L = \underbrace{\frac{M}{2\sqrt{1-\beta^2}}}_{E^L_{\pi_0}/2} \left(1 - \beta\cos(\vartheta^R)\right)$$

$$\vartheta^R \in [-\pi, \pi] \longrightarrow \frac{E^L_{\pi_0}}{2}\left(1-\beta\cos(\vartheta^R)\right) \leq \omega^L \leq \frac{E^L_{\pi_0}}{2}\left(1-\beta\cos(\vartheta^R)\right)$$

(2.) A photon collides with a proton of mass M at rest in the Lab frame. What is the necessary minimal energy for the photon such that in the final states we will have a neutral pion of mass M_0 in addition to the original proton?

$$p_\gamma + p_p = p_p' + p_{\pi_0}$$

$$s = (p_\gamma + p_p)^2 = m_p^2 + m_\pi^2 + 2\, p_p' p_{\pi^0}$$

$$\parallel$$

$$m_p^2 + 2 m_p E_\gamma$$

best case: $\vec{p}_p = \vec{p}_{\pi_0} = 0 \rightsquigarrow s = (m_p + m_{\pi_0})^2$

$$E_\gamma^{min} = \frac{(m_p + m_{\pi_0})^2 - m_p^2}{2 m_p} = \underline{\underline{m_{\pi_0} + \frac{m_{\pi_0}^2}{2 m_p}}}$$

(3.) Under which conditions will the 4-momentum of a system of 2 photons correspond to zero mass?

$$(p_\gamma + p_\gamma')^2 = M^2 \stackrel{!}{=} 0 \qquad \left(\left(\sum_i p_i\right)^2 = M^2 \right)$$

$$\downarrow$$

$$p_\gamma p_\gamma' = 0$$

$$E_\gamma E_\gamma' - \vec{p}_\gamma \vec{p}_\gamma' = 0$$

$$E_\gamma E_\gamma' (1 - \cos\vartheta) = 0$$

Solutions:

a) $\boxed{E_\gamma = 0}$

we have no photons at all.

b) $\cos\vartheta = 1$; $\boxed{\vartheta = 0}$

colinear photons: if two γ-s move in the same direction, they look like one γ.

4. Consider the process $e^+e^- \to Z\gamma$. Find the process related to this one by the crossing relations and also the kinematical bounds on the Mandelstam variables s, t, u!

Cross processes:

$Ze^+ \to e^+\gamma$

$\gamma e^+ \to Ze^+$

$e^-Z \to e^-\gamma$

$e^-\gamma \to Ze^-$

$Z\gamma \to e^+e^-$

$Z \to \gamma e^+e^-$

$$s = (p_Z + p_\gamma)^2 \geq m_Z^2$$

$$\boxed{s \geq m_Z^2}$$

$$t = (p_e - p_\gamma)^2 = m_e^2 - 2E_\gamma^{CM}(E_e^{CM} - |\vec{p}_e|\cos(\vartheta^{CM}))$$

$$E_e^{CM} = \frac{\sqrt{s}}{2}$$

$$E_\gamma^{CM} = \frac{s - m_Z^2}{2\sqrt{s}}$$

$$t = m_e^2 - \frac{s - m_Z^2}{2}\left(1 - \underbrace{\cos\vartheta}_{\pm 1}\sqrt{1 - \frac{4m_e^2}{s}}\right)$$

$$\boxed{m_e^2 - \frac{s - m_Z^2}{2}\left(1 - \sqrt{1 - \frac{4m_e^2}{s}}\right) \leq t \leq m_e^2 - \frac{s - m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4m_e^2}{s}}\right)}$$

$$u = 2m_e^2 + m_Z^2 - s - t$$

$$\boxed{m_e^2 + m_Z^2 - s + \frac{s - m_Z^2}{2}\left(1 - \sqrt{1 - \frac{4m_e^2}{s}}\right) \leq u \leq m_e^2 + m_Z^2 - s + \frac{s - m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4m_e^2}{s}}\right)}$$

5. It is often useful to use I_3 and step operators $I_\pm = I_1 \pm i I_2$ instead of the original I_j $j = 1, 2, 3$ isospin components.
Show that the commutation relations are

$[I_+, I_-] = 2I_3$; $[I_3, I_\pm] = \pm I_\pm$; we know: $[I_i, I_j] = i\varepsilon_{ijk} I_k$

$$[I_1 + iI_2, I_1 - iI_2] = -2ii\, I_3 = 2I_3 \quad \square$$

$$[I_3, I_1 \pm i I_2] = iI_2 \pm i(-iI_1) = \pm(I_1 \pm iI_2) = \pm I_\pm \quad \square$$

6. The irreducable representation of $SU(2)$ labelled by j is spanned by the basis vectors $|jm\rangle$ ($m \in -j, -j+1, \dots j$). The action of $I_\pm = I_1 \pm i I_2$ and I_3 on these is given by

$$I_\pm |jm\rangle = \sqrt{(j \mp m)(j \pm m + 1)}\, |j\, m \pm 1\rangle \ , \quad I_3 |jm\rangle = m\, |jm\rangle$$

Construct the 3×3 representation matrices for I_1, I_2, I_3 for the $j = 1$ representation!

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\langle j m' | I_\pm | j m \rangle = \delta_{m', m \pm 1} \sqrt{(1 \mp m)(2 \pm m)}$$

$$M_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \qquad M_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$$M_1 = \frac{M_+ + M_-}{2} \quad \text{and} \quad M_2 = \frac{M_+ - M_-}{2i}$$

$$M_1 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad M_2 = -i \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

7. The Casimir operator of $SU(2)$ is given by $I^2 = I_1^2 + I_2^2 + I_3^2$. Express I^2 by I_3 and $I_\pm$ and using the expressions from the previous exercise show that

$$I\,|j\,m\rangle = j(j+1)\,|j\,m\rangle$$

$$I_+ I_- = (I_1 + i I_2)(I_1 - i I_2) = I_1^2 + I_2^2 + \underbrace{i\,(I_2 I_1 - I_1 I_2)}_{-i[I_1, I_2] = I_3}$$

$$I_+ I_- = I_1^2 + I_2^2 + I_3$$

$$\Longrightarrow \boxed{I^2 = I_+ I_- - I_3 + I_3^2}$$

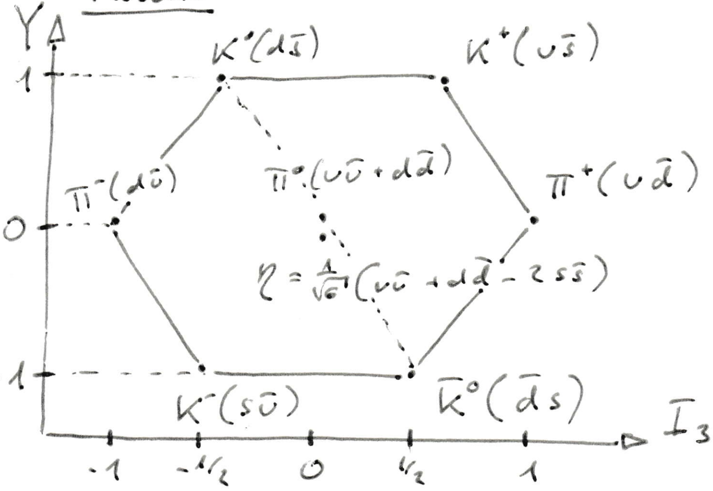
$$I^2 |j\,m\rangle = (I_+ I_- - I_3 + I_3^2)\,|j\,m\rangle = (\sqrt{\dots}\sqrt{\dots} - m + m^2)\,|j\,m\rangle =$$

$$= \left(\sqrt{j(j+1) - m(m-1)}\,\sqrt{j(j+1) - (m-1)(m-1+1)} - m + m^2\right)|j\,m\rangle =$$

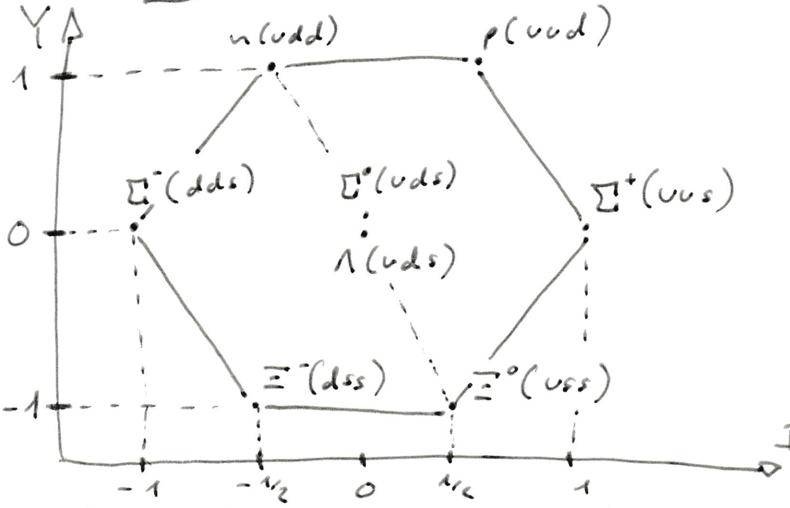
$$= \left(j(j+1) - \cancel{m^2} + \cancel{m} - \cancel{m} + \cancel{m^2}\right)|j\,m\rangle \quad \square$$

8. Plot the pseudo-scalar meson octet consisting of a quark and anti-quark and the barion octet consisting of 3 quarks on the I_3, Y plane and label each particle by its quark content (e.g. $\pi^+ = (u\bar{d})$, $n = (d, d, u)$)!

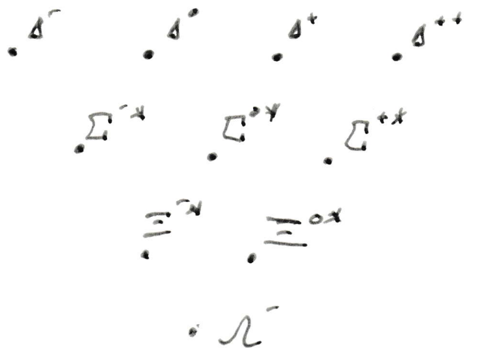
Mesons



Barions



9. Consider the flavour $SU(3)$ decuplet in the I_3, Y plane. For each particle calculate $I(I+1) - \frac{Y^2}{4}$ where I is the total isospin and show, that it is a linear function of Y and find the coefficients α, β in $I(I+1) - \frac{Y^2}{4} = \alpha Y + \beta$! 6.



$I = \frac{3}{2} \quad Y = 1$

$I = 1 \quad Y = 0$

$I = \frac{1}{2} \quad Y = -1$

$I = 0 \quad Y = -2$

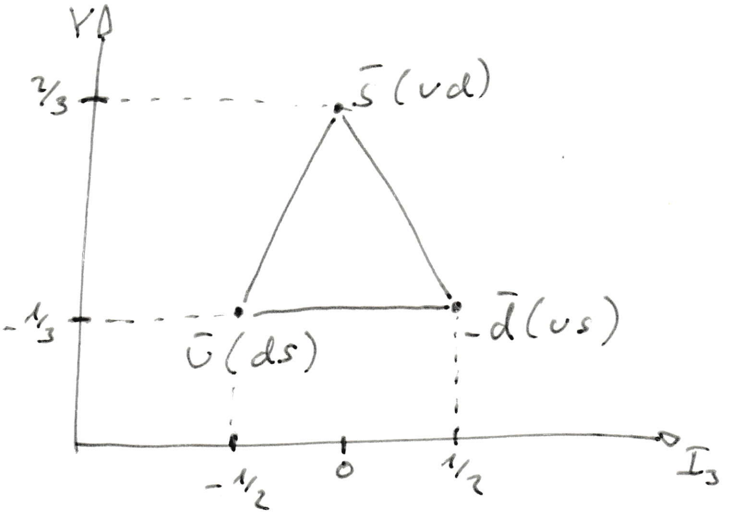
$$I(I+1) - \frac{Y^2}{4} = \frac{7}{2} = \alpha \cdot 1 + \beta \rightarrow \boxed{\alpha = 3/2}$$

$$= 2 = \alpha \cdot 0 + \beta \rightarrow \boxed{\beta = 2}$$

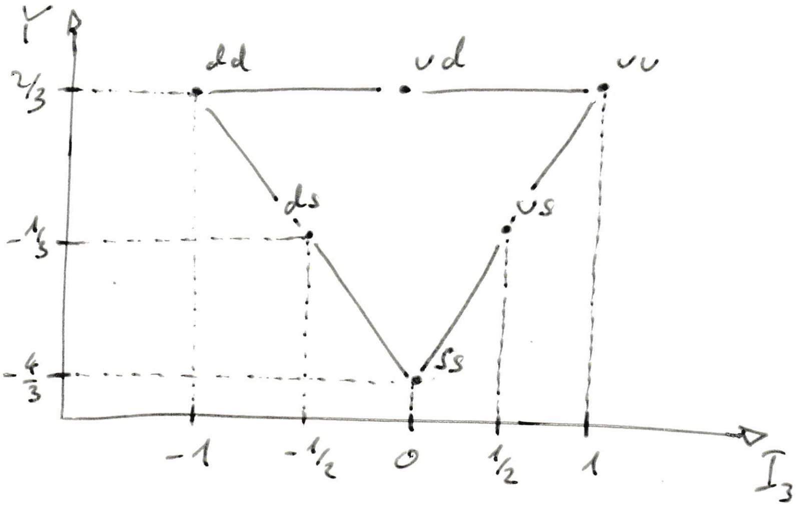
$$= \frac{1}{2} = \alpha \cdot (-1) + \beta \ \checkmark$$

$$= -1 = \alpha(-2) + \beta \ \checkmark$$

10. Plot the states corresponding to the $\bar{3}$ representation of flavour $SU(3)$ in the I_3, Y plane and label each with its quark content! These states correspond to $\bar{q}$ anti-quarks.



11.) Plot the states corresponding to the 6 representation of flavor $SU(3)$ in the I_3, Y plane and label each with its quark content! The 6 representation is the 2-index-symmetric representation hence these states correspond to qq-states or diquarks.



12.) Determine which processes can take place and which ones can not:

$\pi^+ \to e^+ e^- e^+$ ✗ lepton number!

$\pi^0 \to \gamma\gamma$ ✓

$\mu^- \to e^- \gamma$ ✗ lepton number!

$K^+ \to \mu^+ \nu_\mu$ ✓ weakly

$K^- + n \to \Sigma^- + \pi^0$ ✓ strongly

$\pi^+ \to e^+ \nu_e$ ✓

$\mu^+ \nu_\mu$ ✓

$\tau\ \nu_\tau$ ✗ Mass!

$\pi^0 \to \gamma\gamma\gamma$ ✗

$\bar{\nu}_e p \to n e^+$ ✓ "cross-β-decay"

13. Let $a^+(p)$ and $b^+(p)$ be the creation operators of free, 0 spin, mass m particles corresponding to a complex field. Show that the following states are eigenstates of energy, momentum and charge operators and find the eigenvalues aswell:

$$|ab\rangle = a^+(p)\, b^+(-p)|\emptyset\rangle \quad \text{and} \quad |aa\rangle = a^+(p)\, a^+(p)\,|\emptyset\rangle$$

$$E = \int \frac{d^3q}{(2\pi)^3 \cancel{p^0}} \cancel{p^0}\; a^+(q)\, a(q)$$

$$E|ab\rangle = \int \frac{d^3q}{\cancel{(2\pi)^3}} a^+(q) \underbrace{a(q)\, a^+(p)}_{[a(q),\, a^+(p)] = \cancel{(2\pi)^3}\, 2p^0 \delta(q-p)} b^+(-p)\,|\emptyset\rangle = 2p^0|ab\rangle$$

$$\boxed{E = 2p^0 = 2\sqrt{m^2 - p^2}}$$

$$p^\mu = \int \frac{d^3q}{(2\pi)^3 p^0}\, p^\mu\, a^+(q)\, a(q)$$

$$p^\mu|ab\rangle = \int \frac{d^3q}{(2\pi)^3 p^0}\, p^\mu\, a^+(q) \underbrace{a(q)\; a^+(p)}_{[a(q),\, a^+(p)]} b^+(-p)\,|\emptyset\rangle = \underbrace{2p^\mu}_{=0}|ab\rangle$$

$$Q = \int \frac{d^3\tilde{q}}{(2\pi)^3 p^0}\, q\left(a^+(\tilde{q})\, a(\tilde{q}) - b^+(\tilde{q})\, b(\tilde{q})\right) a^+(p)\, b^+(-p)|\emptyset\rangle =$$

$$= 2q|ab\rangle - \int \frac{d^3\tilde{q}}{(2\pi)^3 p^0}\, q\, a^+(p)\, b^+(\tilde{q}) \underbrace{b(\tilde{q})\, b^+(-p)}_{[b(\tilde{q}),\, b^+(-p)]}\,|\emptyset\rangle =$$

$$= 2q|ab\rangle - 2q|ab\rangle = \underline{0 \cdot |ab\rangle}$$

$$E|aa\rangle = 2p^0|aa\rangle = 2\sqrt{m^2 - p^2}\,|aa\rangle$$

$$p^\mu|aa\rangle = 2p^\mu|aa\rangle = 2\vec{p}\,|aa\rangle$$

$$Q\,|aa\rangle = 2q\,|aa\rangle$$

(14.) Let b_1^+ and b_2^+ be the creation operators of the grnd state and excited state of an atom.

$$|G\rangle = b_1^+ |\emptyset\rangle, \; |X\rangle = b_2^+ |\emptyset\rangle, \; b_1|\emptyset\rangle = b_2|\emptyset\rangle = 0$$

fulfilling the commutation relations $[b_i, b_j^+] = \delta_{ij}$ $i,j = 1,2$.

Show, that the following holds for $\Pi = b_1^+ b_2$ and $\Pi^+ = b_2^+ b_1$:

$$\Pi^+ |G\rangle = |X\rangle, \; \Pi|X\rangle = |G\rangle, \; \Pi^+|X\rangle = 0, \; \Pi|G\rangle = 0$$

$$\Pi^+|G\rangle = b_2^+ \underbrace{b_1 b_1^+}_{[b_1, b_1^+] = 1} |\emptyset\rangle = b_2^+|\emptyset\rangle = \underline{\underline{|X\rangle}}$$

$$\Pi|X\rangle = b_1^+ \underbrace{b_2 b_2^+}_{[b_2, b_2^+] = 1} |\emptyset\rangle = b_1^+|\emptyset\rangle = \underline{\underline{|G\rangle}}$$

$$\Pi^+|X\rangle = b_2^+ \underbrace{b_1 b_2^+}_{[b_1, b_2^+] = 0} |\emptyset\rangle = \underline{\underline{0}}$$

$$\Pi|G\rangle = b_1^+ \underbrace{b_2 b_1^+}_{[b_2, b_1^+] = 0} |\emptyset\rangle = \underline{\underline{0}}$$

(15.) Let ϕ be a real scalar field whose self-interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_m \phi)^2 - \frac{m^2}{2}\phi^2 + g\phi^3$$

Determine the following

a) mass dimension of g

$$[x] = -1 \rightarrow [\partial_x] = +1$$

$$[S] = 0 \xrightarrow[S = \int d^4x \, \mathcal{L}]{} [\mathcal{L}] = 4 \rightarrow [\phi] = 1$$

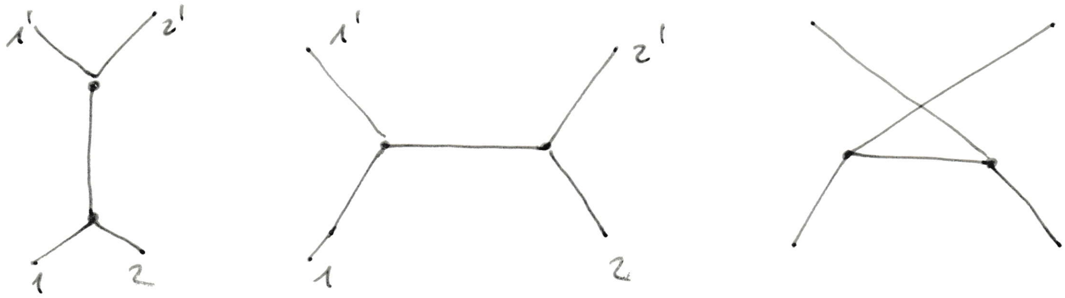
$$[g] + 3[\phi] \overset{!}{=} 4$$

$$\boxed{[g] = 1}$$

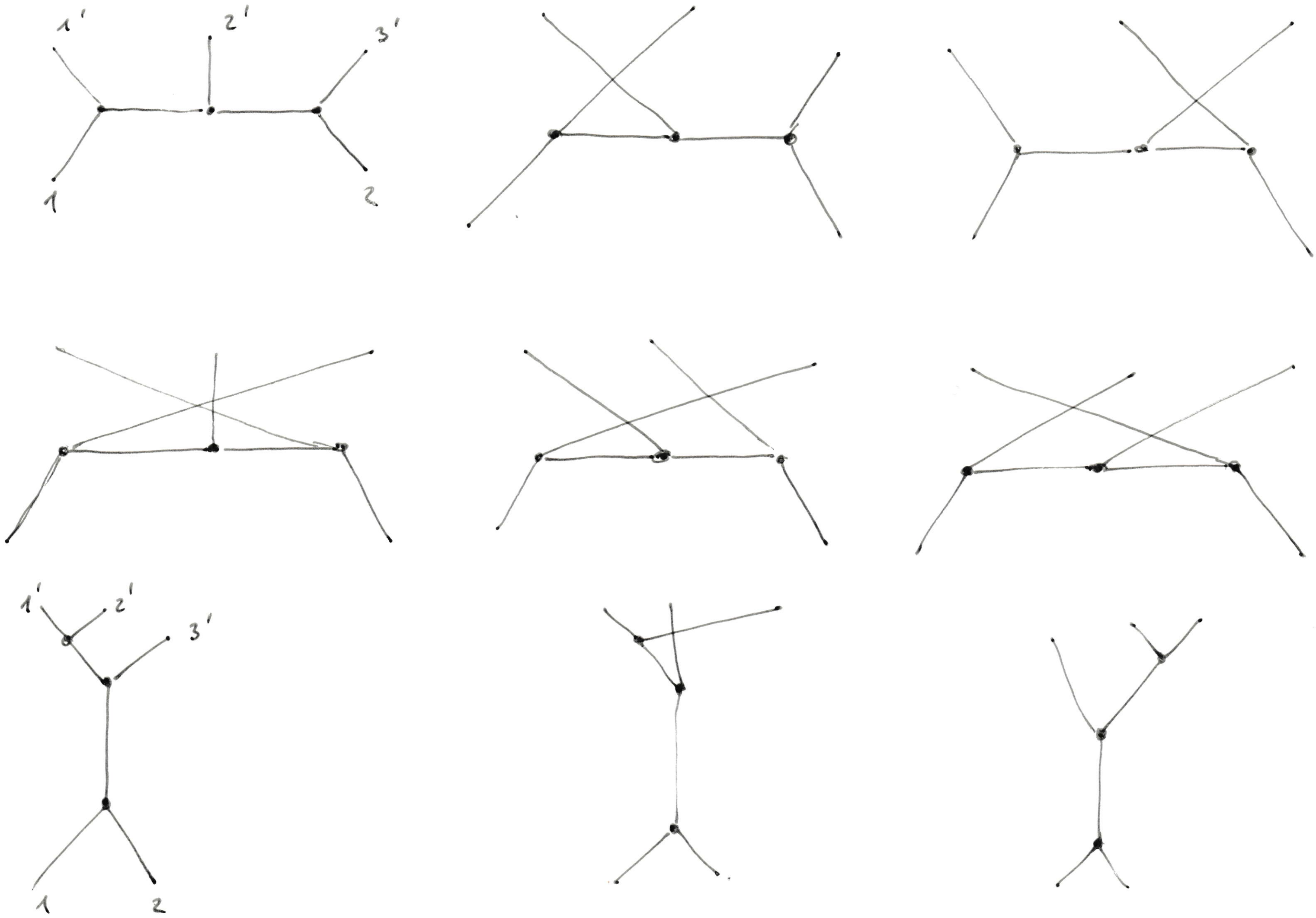
b) the expression of the vertex in Feynman diagrams

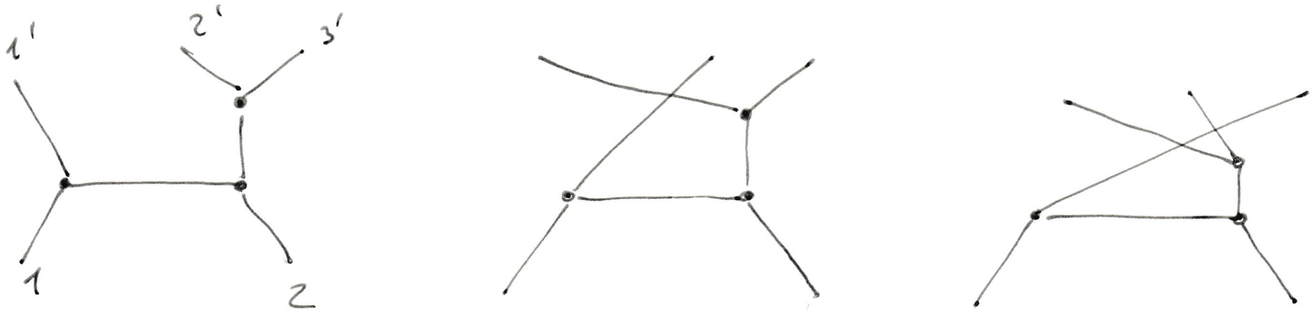
c) lowest order Feynman diagrams for $2 \to 2$ and the $2 \to 3$ scatterings

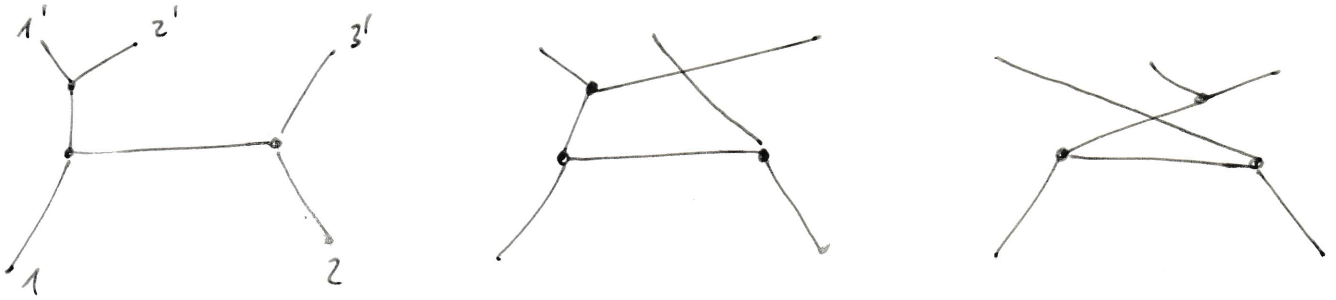
$\boxed{2 \to 2}$ (3)



$\boxed{2 \to 3}$ (15)







(16.) Let $\phi(x)$ be a real scalar field and $\psi(x)$ a Dirac spinor field and assume that their interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \bar{\psi} i \gamma^\mu \partial_\mu \psi + M \bar{\psi}\psi + g \phi \bar{\psi} \psi$$

(The last term is often called Yukawa interaction)

Determine the following

a) mass dimension of g

$$[\mathcal{L}] = 4$$

$$[\phi] = 1$$

$$[\bar{\psi} \partial \psi] = [\partial] + 2[\psi] = 1 + 2[\psi] \overset{!}{=} 4$$

$$[\psi] = \frac{3}{2}$$

$$[g] + [\phi] + 2[\psi] = [g] + 1 + 3 \overset{!}{=} 4 \rightarrow \boxed{[g] = 0}$$

b) the expression of the vertex in Feynman diagrams

(17.) Show that the time ordered product of spin $\frac{1}{2}$ fields satisfies the equation

$$(i\gamma^\mu \partial_\mu^x - m)_{\alpha\beta}\, T\left(\psi_\beta(x)\, \overline{\psi}_\gamma(y)\right) = i\,\delta_{\alpha\gamma}\,\delta^{(4)}(x-y)$$

– ectation value of

$$(i\not{\partial}^x - m)_{\alpha\beta}\, T\left(\psi_\beta(x)\,\overline{\psi}_\gamma(y)\right) =$$

$$T\left((i\vec{\nabla}\vec{\gamma} - m)_{\alpha\beta}\,\psi_\beta(x)\,\overline{\psi}_\gamma(y)\right) +$$

$$+\, i\partial_0\,\gamma^0_{\alpha\beta}\, T(\psi_\beta \overline{\psi}_\gamma) =$$

$$= T\Big(\underbrace{(i\not{\partial}^x - m)_{\alpha\beta}\,\psi_\beta}_{0}\,\overline{\psi}_\gamma\Big) +$$

$$+\, i\,\delta(x^0 - y^0)\,\underbrace{\{\gamma^0_{\alpha\beta}\psi_\beta, \overline{\psi}_\gamma\}_{ET}}_{\gamma^0_{\alpha\beta}\{\psi_\beta,\, \psi^+_\gamma\}\gamma^0_{\gamma\delta} = \delta_{\alpha\gamma}\,\delta^3(\vec{x}-\vec{y})} = i\,\delta_{\alpha\gamma}\,\delta^4(x-y)$$

$\emptyset\rangle$

$y)$

– Fourier - transform: $S(x) = \int \frac{d^4p}{(2\pi)} e^{-ipx}\, \tilde{S}(p)$

$$(\not{p} - m)_{\alpha\beta}\,\tilde{S}_{\beta\gamma} = i\,\delta_{\alpha\gamma} \qquad /\cdot(\not{p} + m)$$

– we have to invert this expression.

$$(\not{p} + m)(\not{p} - m) = \not{p}^2 - m^2$$

$$\not{p}^2 = p_\mu p_\nu \gamma^\mu \gamma^\nu = p_\mu p_\nu \frac{1}{2}\underbrace{\{\gamma^\mu, \gamma^\nu\}}_{g^{\mu\nu}} = p^2$$

$$(p^2 - m^2)\,\tilde{S} = i(\not{p} + m)$$

$$\boxed{\tilde{S}(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}}$$

Retarded Green's func. of free particle.