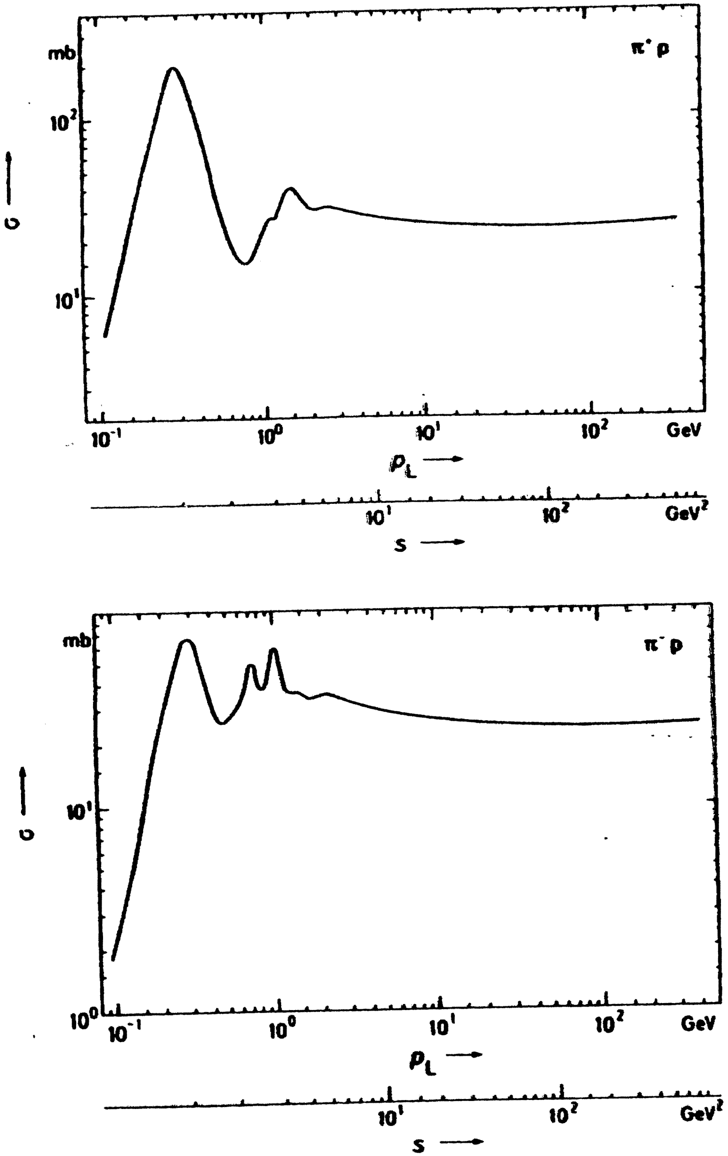

Figure 16.1 The total $\pi^+ p$ and $\pi^- p$ cross-sections (after Particle Data Group 1984).

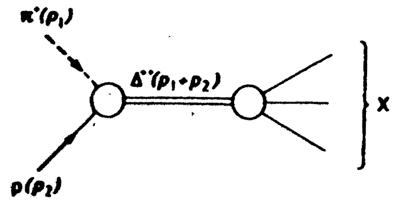

Figure 16.2 Diagram for the production and decay of the Δ^{++} resonance in $\pi^+ p$ scattering. The final state X consists to nearly 100% of $\pi^+ p$ again.

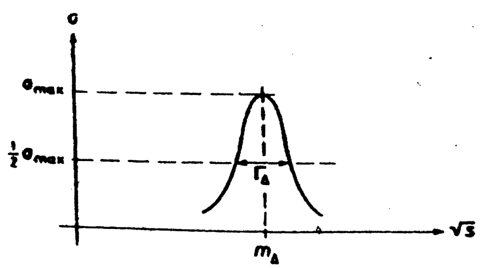

Figure 16.3 Resonance curve and width Γ_Δ corresponding to (16.7) (schematic).

29.2.6. *The group SU(3)*

The fundamental representation of the group SU(3) is given by the matrices

$$U = \exp(\tfrac{1}{2}\lambda_i\omega_i), \qquad i = 1,2,\ldots,8,$$

where λ_i are the Gell-Mann matrices, and ω_i are eight real parameters. Usually the matrices λ_i are chosen in the form:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The matrices λ satisfy the following relations:

$$\mathrm{Tr}\,\lambda_i\lambda_j = 2\delta_{ij},$$

$$[\lambda_i\lambda_j] = 2if_{ijk}\lambda_k,$$

$$[\lambda_i\lambda_j]_+ = \tfrac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k, \qquad \text{where } i,\,j,\,k = 1,2,\ldots,8.$$

Here f_{ijk} are structure constants of the group SU(3), d_{ijk} are symmetrical and f_{ijk} are antisymmetrical with respect to permutations of any pair of indices. Direct calculations easily give 54 non-zero constants f_{ijk} and 58 non-zero constants d_{ijk}:

ijk	f_{ijk}	ijk	d_{ijk}	ijk	d_{ijk}
123	1	118	$1/\sqrt{3}$	355	$\frac{1}{2}$
147	$\frac{1}{2}$	146	$\frac{1}{2}$	366	$-\frac{1}{2}$
156	$-\frac{1}{2}$	157	$\frac{1}{2}$	377	$-\frac{1}{2}$
246	$\frac{1}{2}$	228	$1/\sqrt{3}$	448	$-\frac{1}{2}\sqrt{3}$
257	$\frac{1}{2}$	247	$-\frac{1}{2}$	558	$-\frac{1}{2}\sqrt{3}$
345	$\frac{1}{2}$	256	$\frac{1}{2}$	668	$-\frac{1}{2}\sqrt{3}$
367	$-\frac{1}{2}$	338	$1/\sqrt{3}$	778	$-\frac{1}{2}\sqrt{3}$
458	$\frac{1}{2}\sqrt{3}$	344	$\frac{1}{2}$	888	$-1/\sqrt{3}$
678	$\frac{1}{2}\sqrt{3}$				

($54 = 9 \times 6$ where 6 is the number of permutations of indices $i \neq j \neq k$, and $58 = 4 \times 6 + 11 \times 3 + 1$). Note that $d_{ijk} = 0$ if the number of indices 2, 5, 7 is odd. On the other hand, $f_{ijk} = 0$ if the number of these indices is even. These indices, 2, 5, 7, are special because the corresponding matrices λ are antisymmetric.

29.2.7. *Fierz identities for λ matrices*

Using the completeness of the nine matrices δ^α_β, λ^α_β, we can write:

$$\delta^\alpha_\beta\delta^\gamma_\delta = A\delta^\alpha_\delta\delta^\gamma_\beta + B\lambda^\alpha_\delta\lambda^\gamma_\beta,$$

$$\lambda^\alpha_\beta\lambda^\gamma_\delta = C\delta^\alpha_\delta\delta^\gamma_\beta + D\lambda^\alpha_\delta\lambda^\gamma_\beta,$$

where A, B, C and D are coefficients to be determined and where

$$\lambda\cdot\lambda = \lambda_i\lambda_i, \qquad i = 1,2,\ldots,8.$$

Multiplication of these two equalities by $\delta^\delta_\alpha\delta^\beta_\gamma$ yields

$$3 = 9A, \qquad 16 = 9C,$$

and multiplication by $\delta^\beta_\alpha\delta^\delta_\gamma$ yields

$$9 = 3A + 16B, \qquad 0 = 3C + 16,$$

whence

$$\delta^\alpha_\beta\delta^\gamma_\delta = \tfrac{1}{3}\delta^\alpha_\delta\delta^\gamma_\beta + \tfrac{1}{2}\lambda^\alpha_\delta\cdot\lambda^\gamma_\beta,$$

$$\lambda^\alpha_\beta\cdot\lambda^\gamma_\delta = \tfrac{16}{9}\delta^\alpha_\delta\delta^\gamma_\beta - \tfrac{1}{3}\lambda^\alpha_\delta\cdot\lambda^\gamma_\beta.$$

Now it is not difficult to show that

$$8\delta^\alpha_\beta\delta^\gamma_\delta + 3\lambda^\alpha_\beta\cdot\lambda^\gamma_\delta = +\left(8\delta^\alpha_\delta\delta^\gamma_\beta + 3\lambda^\alpha_\delta\cdot\lambda^\gamma_\beta\right),$$

$$4\delta^\alpha_\beta\delta^\gamma_\delta - 3\lambda^\alpha_\beta\cdot\lambda^\gamma_\delta = -\left(4\delta^\alpha_\delta\delta^\gamma_\beta - 3\lambda^\alpha_\delta\cdot\lambda^\gamma_\beta\right).$$

Applied to the product of two triplet spinors, the first of these expressions selects the state 6, and the second one selects the state $\bar{3}$ (recall that $3 \times 3 = 6 + \bar{3}$).

29.2.8. *SU(3) multiplets*

A contravariant three-component spinor t^α is transformed by the matrices $U = \exp(\frac{1}{2}i\omega_i\lambda_i)$; it is denoted by 3. A covariant spinor t_α is transformed by complex conjugate matrices $U^* = \exp(-\frac{1}{2}i\omega_i\lambda_i^*)$; it will be denoted by $\bar{3}$. Representations of higher dimensions can be constructed out of 3 and $\bar{3}$ by making use of the invariant tensors δ^α_β, $\varepsilon_{\alpha\beta\gamma}$, and $\varepsilon^{\alpha\beta\gamma}$:

$$3\times\bar{3} = 8+1: \quad \text{singlet}, \quad 1 \sim t^\alpha t_\beta \delta^\beta_\alpha;$$
$$\text{octet}, \quad 8 \sim T^\alpha_\beta = t^\alpha t_\beta - \tfrac{1}{3}\delta^\alpha_\beta (t^\gamma t_\gamma).$$

$$3\times 3 = 6+\bar{3}: \quad \text{antitriplet}, \quad \bar{3} \sim T_\gamma = t^\alpha t^\beta \varepsilon_{\alpha\beta\gamma};$$
$$\text{sextet}, \quad 6 \sim T^{\alpha\beta} = t^\alpha t^\beta + t^\beta t^\alpha.$$

$$3\times 6 = 8+10: \quad 8 \sim T^\gamma_\delta = t^\alpha t^{\beta\gamma}\varepsilon_{\alpha\beta\delta};$$
$$\text{decuplet}, \quad 10 \sim T^{\alpha\beta\gamma}.$$

$$\bar{3}\times 6 = 3+15: \quad 3 \sim T^\gamma = t_\alpha T^{\alpha\gamma};$$
$$15 \sim T^{\beta\gamma}_\alpha.$$

$$8\times 8 = 1+8+8+10+\overline{10}+27: \quad \overline{10} \sim T_{\alpha\beta\gamma}; \quad 27 \sim T^{\gamma\delta}_{\alpha\beta}.$$

An arbitrary tensor can be written in the form

$$T^q_p = T^{\beta_1\beta_2\ldots\beta_q}_{\alpha_1\alpha_2\ldots\alpha_p},$$

where symmetrization is carried out separately over all upper and lower indices, and the trace for any pair $\alpha_i\beta_k$ is zero. The total number of components of the multiplet T^q_p is found easily:

$$N = \tfrac{1}{2}(p+1)(q+1)(p+q+2).$$

Examples of physical SU(3) multiplets:

$$q^\alpha = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \text{quark triplet},$$

$$\bar{q}_\alpha = (\bar{u}, \bar{d}, \bar{s}) \quad \text{antiquark (anti)triplet},$$

$$P^\alpha_\beta = \begin{pmatrix} \sqrt{\frac{1}{6}}\,\eta^0 + \sqrt{\frac{1}{2}}\,\pi^0 & \pi^+ & K^+ \\ \pi^- & \sqrt{\frac{1}{6}}\,\eta^0 - \sqrt{\frac{1}{2}}\,\pi^0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix} \quad \text{octet of pseudo-scalar mesons},$$

$$B^\alpha_\beta = \begin{pmatrix} \sqrt{\frac{1}{6}}\,\Lambda^0 + \sqrt{\frac{1}{2}}\,\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \sqrt{\frac{1}{6}}\,\Lambda^0 - \sqrt{\frac{1}{2}}\,\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{1}{6}}\,2\Lambda^0 \end{pmatrix} \quad \text{octet of baryons}.$$

When the isotopic subgroup SU(2) of group SU(3) is singled out, it is convenient to plot the particles of the multiplet on the so-called T_3Y diagrams. Examples are given in figs. 29.1, 2, 3.

By combining d and s (or s and u) quarks, instead of u and d, into an SU(2) doublet we single out the U (or V) spin subgroup* of SU(3) (see fig. 29.4). Figs. 29.1–4 demonstrate that particles within one U-multiplet have identical charges. The composition of U-multiplets is obvious in these figures, with the exception of the central particles on the T_3Y diagram for the octet. The point is that the Σ^0 and Λ^0 states possess a definite T-spin but no definite U-spin. It is their linear superpositions

$$\Sigma^0_U = -\tfrac{1}{2}\Sigma^0 + \tfrac{1}{2}\sqrt{3}\,\Lambda^0, \; \Lambda^0_U = -\tfrac{1}{2}\sqrt{3}\,\Sigma^0 - \tfrac{1}{2}\Lambda^0,$$

that possess definite U-spin: unity for the first and zero for the second.

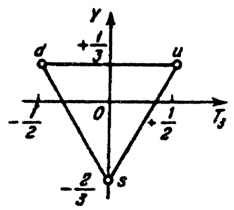

Fig. 29.1

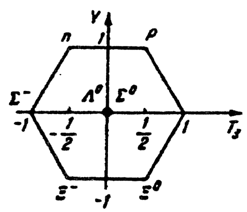

Fig. 29.2

*Sometimes the minus sign is assigned to some of the particles of the SU(3) multiplet in order to make positive the matrix elements of the ladder operators of a given SU(2) subgroup (see J. J. de Swart, *Rev. Mod. Phys.* 35 (1963) 916).