

$$\Sigma(p) = \sum_{\text{sources}} \left(\text{diagram} \right)$$

QED II

$$-\frac{\partial}{\partial p^\mu} = \Sigma$$

$$\Sigma(p) = A + B(p-m) + \Sigma^*(p)$$

$$\underline{\Sigma_1 = \Sigma_2} \quad e_e = \underline{B} \quad \text{figyelni mindenkor}$$

Univerzialis töltésre vonnunk, kiegészít az IR divergenciók

ora

Electron formfactor

$$f_\mu^\mu \rightarrow \gamma_\mu \rightarrow \gamma_\mu + \Gamma_\mu^R(p, p')$$

$$\Gamma_\mu^R = \gamma_\mu F_1(q^2) + \frac{i}{2m} \sigma_{\mu\nu} q^\nu F_2(q^2)$$

$$F_1(q^2) \approx \frac{d}{3\pi} \frac{q^2}{m^2} \left(\log \frac{m}{\mu} - \frac{3}{8} \right)$$

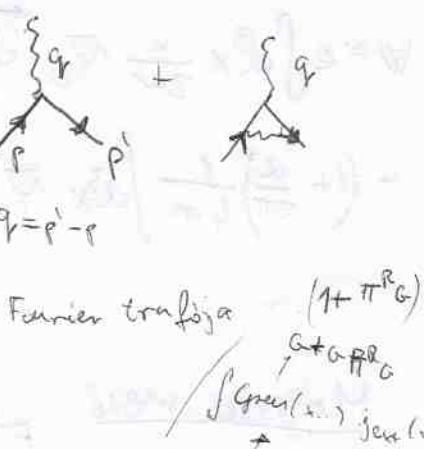
$$F_2(0) = \frac{d}{2\pi}$$

$$W = \int d^3x \bar{\psi}_\mu A_\mu^R = e \int d^3x \bar{\psi}_p \left(\gamma_\mu + \Gamma_p^R(p, p') + i \Gamma_{\mu\nu}^{R\alpha} \not{A}_\alpha \right) \psi_p A_\mu^R$$

$$\text{töltés eloszlás} \quad \text{Fourier transz.} \quad (1 + \pi^R G)$$

$$\approx e \int d^3x \bar{\psi}_p \left[1 + \frac{d}{3\pi} \frac{q^2}{m^2} \left(\log \frac{m}{\mu} - \frac{3}{8} - \frac{1}{5} \right) + \frac{d}{2\pi} \frac{i}{2m} \sigma_{\mu\nu} q^\nu \right] \psi_p A_\mu^R$$

$$A_\mu^R \sim e^{-iqx}$$



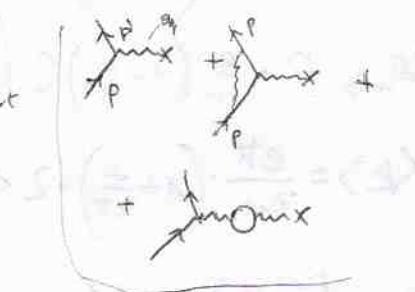
Gordon azonosság

$$\bar{\psi}(p) \gamma_\mu \psi(p) = \frac{1}{2m} \bar{\psi}(p) \left\{ (p+p')_\mu + i \sigma_{\mu\nu} (p-p')^\nu \right\} \psi(p)$$

$$\bar{\psi}(p) \gamma_\mu (p-m) \psi(p) = 0$$

$$\bar{\psi}(p) (p'-m) \gamma_\mu \psi(p) = 0$$

$$\frac{1}{2m} \bar{\psi}(p) \gamma^\mu \psi(p) = \bar{\psi}(p) \left(\gamma_\mu \not{p} + \not{p} \gamma_\mu \right) \psi(p)$$



$$\gamma_\mu \gamma_\nu = \frac{1}{2} [(\gamma_\mu, \gamma_\nu) + (\gamma_\nu, \gamma_\mu)]$$

$$2\gamma_{\mu\nu} = -2i\sigma_{\mu\nu}$$

(13)

$$\% = e^{\int d^3x \bar{\Psi}_p \left\{ \frac{1}{2m} (\vec{p} + \vec{p})_\mu \left[1 + \frac{d}{2\pi} \frac{q^2}{m^2} (\log \frac{m}{\Lambda} - \frac{3}{8} - \frac{1}{5}) \right] + \left(1 + \frac{d}{2\pi} \right) \frac{i}{2m} \sigma_{\mu\nu} q^2 \right\} A_p^\mu A_{ext}^\mu}$$

$$q \approx 0 \quad \vec{p} = p$$

$$\bar{\Psi}_p \frac{p^\mu}{m} \psi_p \rightarrow \frac{p^\mu}{E} = (1, \vec{v})$$

↑
Saját mágneses
nyomaték

$$\bar{u}_{p=0} u_{p=0} = 1$$

$$u_p = \sqrt{\frac{m}{E}} D(L(p)) u_{p=0}$$

$$\bar{u}_p u_p = \frac{m}{E}$$

$$\bar{u}_p = \sqrt{\frac{m}{E}} \cdot \bar{u}_{p=0} D(L(p))^{-1}$$

$$q_\mu \rightarrow +i\partial_\mu \quad A_{ext} - \tau e$$

$$p_r \quad \psi_{-n} + i\partial_\mu$$

$$p_r' \quad \psi'_+ - i\overleftrightarrow{\partial}_\mu$$

$$W \approx e \int d^3x \frac{i}{2m} \bar{\Psi}_p \overleftrightarrow{\partial}_\mu \psi(x) \left\{ 1 - \frac{d}{2\pi} \frac{1}{m^2} (\log(\frac{m}{\Lambda}) - \frac{3}{8} - \frac{1}{5}) D \right\} A_{ext}^\mu$$

$$- \left(1 + \frac{d}{2\pi} \right) \frac{1}{4m} \int d^3x \bar{\Psi}_p(x) \sigma_{\mu\nu} \psi_p(x) F^{\mu\nu}$$

$$\sigma_{\mu\nu} q_\nu A_{ext}^\mu \rightarrow \frac{1}{2} \sigma_{\mu\nu} (q_\nu A_{ext}^\mu - q_\mu A_{ext}^\nu)$$

Mágneses mező

$$F^{12} = -B_3^3 = -F^{21}$$

$$\sigma_{12} = \sum_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} = -\sigma_3$$

$$\sum = \begin{pmatrix} \sigma_{23} & \sigma_{31} & \sigma_{12} \\ \sigma_{12} & \sigma_{13} & \sigma_{21} \\ \sigma_{31} & \sigma_{21} & \sigma_{13} \end{pmatrix}$$

$$W_{min} \approx -\frac{e}{4m} \left(1 - \frac{d}{2\pi} \right) 2 \int d^3x \bar{\Psi}(x) \sum \psi(x) \cdot \underline{B} = -\langle \underline{\mu} \rangle \underline{B}$$

$$\langle \underline{\mu} \rangle = \frac{e\hbar}{2m} \left(1 + \frac{d}{2\pi} \right) \cdot 2 \langle \underline{s} \rangle = g\mu_B \langle \underline{s} \rangle \quad S = \frac{1}{2} \sum$$

$$g = 2 \left(1 + \frac{d}{2\pi} \right) \approx 2 \left(1 + 0.00116141 \dots \right)$$

$$g_{exp} = 2 \cdot \left(1 + 0.001159652193 \right)$$

$$g_{theor} = 2 \left(1 + 0.001159652140 \pm \frac{0.000000000028}{0.000000000028} \right)$$

\pm hibák / feltételek (0.00)

$$\frac{g_{exp} - 2}{2} = 0.001165917 \pm 0.0000000007$$

$$a = \frac{g_{\text{ther}}^{\mu} - 2}{c^2} = 0.0011659202 \pm 0.0000000015$$

QED II

$(\Delta a_p)_{\text{new}} = (691 \pm 7) \cdot 10^{-10}$ genauer ist nun

$$(\Delta a_p)_{\text{strong}} = (15.1 \pm 0.4) \cdot 10^{-10}$$

Sugawara Konstruktion

① Flusskontraktion

② Brachstrahlung

Klassisches Szindrom



mug's

$t=0$

$x=0$

$\tau \rightarrow f(\tau)$ inv

$$j^{\mu}(x) = e \int d\tau \frac{dy^{\mu}}{d\tau} \delta^{(4)}(x^{\mu} - y^{\mu}(\tau))$$

$y^{\mu}(\tau) = \tau$

fröhlicher welliger ebben

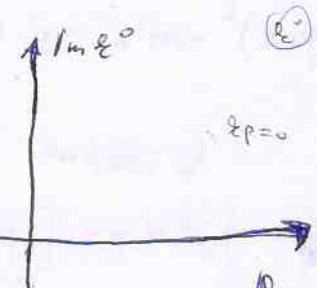
$$y^{\mu}(\tau) = \begin{cases} \frac{p^{\mu}}{m} \tau & \tau < 0 \\ \frac{p^{\mu}}{m} \tau & \tau > 0 \end{cases} \quad \text{eig. parametereinzahl}$$

$$\hat{j}^{\mu}(\epsilon) = \int d^4x e^{i\epsilon x} j^{\mu}(x) = ie \left(\frac{p^{\mu}}{\epsilon p^1 + i\epsilon} - \frac{p^{\mu}}{\epsilon p^1 - i\epsilon} \right) e^{-i\epsilon t}$$

Lorentz-invarianten

$$\tilde{A}^{\mu}(\epsilon) = -\frac{1}{\epsilon^2} \hat{j}^{\mu}(x) \rightarrow A^{\mu}(x) = \int \frac{d^4\epsilon}{(2\pi)^4} e^{-i\epsilon x} \tilde{A}_{\mu}(\epsilon)$$

$$\frac{1}{\epsilon^2 + i\epsilon(\text{sign} x)}$$



$$\sim E(x), B(x) = \text{rot} A$$

$$= -\nabla A - \dot{A}$$

leisungsrechte Energia = $\int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \frac{e^2}{2} \left| \sum_{\lambda=1,2}^{(x)} (\epsilon) \left(\frac{p^1}{\epsilon - p^1} - \frac{p^1}{\epsilon + p^1} \right) \right|^2 =$

$$= \frac{e^2}{\pi^2} \int d\omega J(\omega, \omega) \int \frac{d^3k}{(2\pi)^3}$$

$\sum_{\lambda=1,2} \rightarrow \sum_{\lambda=1}^{(x)}$ $E^{(0)}(\epsilon) + E^{(1)}(\lambda) \sim \delta_{\epsilon} \left(\frac{i}{\epsilon - p^1} - \frac{i}{\epsilon + p^1} \right) = 0$

$$\sum_{\lambda=1}^3 \epsilon_{\mu}^{(\lambda)}(x) \epsilon_{\nu}^{(\lambda)*}(x) = -g_{\mu\nu}$$

$$P = E(x, y) \quad P' = E'(x, y')$$

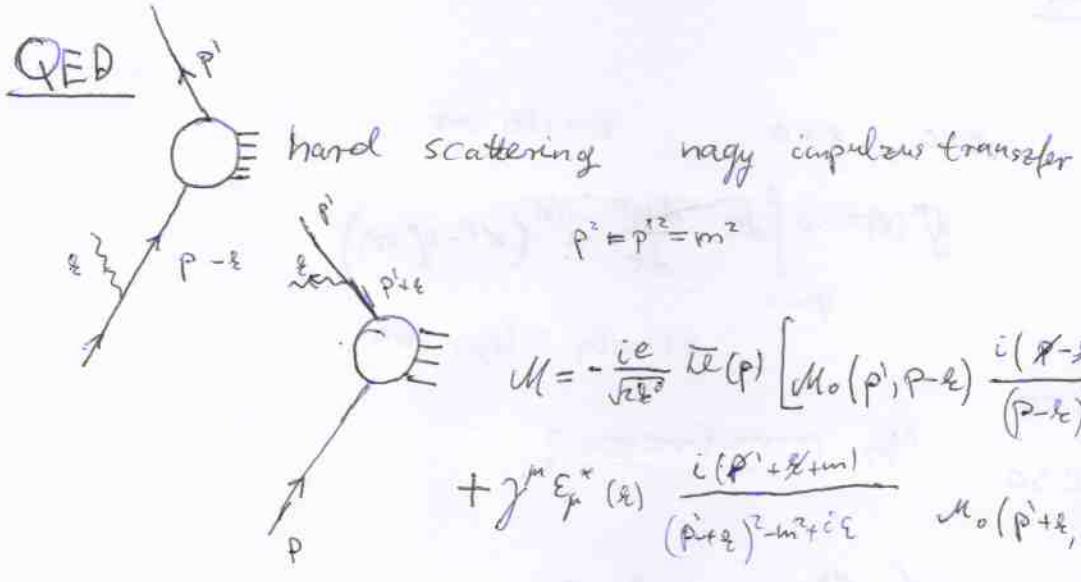
$$I(y, y') = \int \frac{dS \cdot k}{4\pi} \left[\frac{2(1-y-y')}{(1-\frac{k}{E} \cdot y)(1-\frac{k}{E} \cdot y')} - \frac{m^2/E'^2}{(1-\frac{k}{E} \cdot y')^2} - \frac{m^2/E^2}{(1-\frac{k}{E} \cdot y)^2} \right]$$

$$N_{\gamma} = \frac{1}{\pi} \int_{E_-}^{E_+} \frac{dw}{w} I(y, y') = \frac{1}{\pi} \log \frac{E_+}{E_-}$$

ω fixieren

$E_+ \approx$ bei Abschall
 $E_+ \approx 1$

Selection feld unter



Soft foton

$$|q| \ll |q'| = |\vec{p}' - \vec{p}|$$

$$(p - q)^2 - m^2 = -2p \cdot q$$

$$q = \omega(n, \frac{q}{n})$$

$$(p + q)^2 - m^2 = 2p \cdot q$$

frei zulässiger $M_0(p', p - q) = M_0(p', p)$

Stabilität von f -> erlaubt (6)

$$(p - m) \gamma^\mu u(p) = \underbrace{[2p^\mu + \gamma^\mu(-p + m)]}_{\bar{u}(p)} u(p)$$

$$\bar{u}(p) \gamma^\mu (p' + m) = \bar{u}(p) 2p'^\mu$$

$$M = \bar{u}(p) M_0(p', p) u(p) \left(\frac{p \cdot \epsilon^*}{p \cdot q} - \frac{p' \cdot \epsilon^*}{p' \cdot q} \right) \frac{-ie}{\sqrt{2w}}$$

$$d\sigma = d\sigma(p \rightarrow p) \frac{d\omega}{2w} \sum_{\lambda=1,2} \left| \epsilon_\lambda \left(\frac{p'}{p \cdot q} - \frac{p}{p \cdot q} \right) \right|^2$$

$$d(\text{prob}) = \frac{d^3 k}{(2\pi)^3} \cdot \frac{e^2}{zw} \sum_{\lambda=1,2} |$$

QED II

[Peschkin-Schrodinger-Gauß
Gauß]

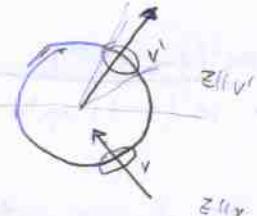
IR problem

$$J(v, v') = \int \frac{d\omega_k}{4\pi} \left\{ \frac{2(1-vv')}{(1-\frac{E}{E'}v)(1-\frac{E}{E'}v')} - \frac{m^2/E^2}{(1-\frac{E}{E'}v)^2} + \frac{m^2/E^2}{(1-\frac{E}{E'}v')^2} \right\}$$

ultrad. hataréset:

$E, E' \gg m$

$$|v|, |v'| \sim 1$$



$$J(v, v') \approx \int d(\cos\theta) \frac{1-v \cdot v'}{(1-v \cos\theta)(1-v' \cos\theta)}$$

$$\underline{v} \cdot v = vv$$

$$\begin{aligned} & \text{ordning upphöra att} \\ & \text{fört känslan att försöker} \end{aligned} \quad \left. \begin{aligned} & + \int d\cos\theta \frac{1-v \cdot v'}{(1-v \cdot v')(1-v' \cos\theta)} \end{aligned} \right\} \quad \begin{aligned} & \underline{v} \cdot v = vv \\ & \underline{v} \cdot v' = v'v \end{aligned}$$

$$\underline{v} \cdot v = 1 - x(1-v \cdot v) \quad x > 0 \quad x \text{ ve leggen så att nägdeln in i}$$

$$\Rightarrow J(v, v') = \log \left(\frac{1-v \cdot v}{1-vv} \right) + \log \left(\frac{1-v' \cdot v}{1-v'v} \right) = \log \frac{(E'E - p \cdot p)^2}{E'E(E-p)(E'-p)} =$$

$$p = |\underline{p}|$$

$$E = p + \frac{m^2}{2p} + O(\frac{1}{p}) + \dots$$

$$E(E-p) \approx \frac{E+p}{2} E - p = \frac{m^2}{2}$$

$$= 2 \log \frac{p \cdot p}{m^2/2} = 2 \log \left(-\frac{q^2}{m^2} \right)$$

$$q^2 = p^2 - p'^2$$

$$q^2 = p^2 + p'^2 - 2p \cdot p'$$

(R div.

$$d\sigma(p \rightarrow p' + \gamma(k)) = d\sigma(p \rightarrow p') \frac{1}{\pi} \int_{\mu}^{q^2} \frac{dw}{w} \log \left(-\frac{q^2}{m^2} \right) \stackrel{q^2 \rightarrow \infty}{\simeq} d\sigma(p \rightarrow p') \underbrace{\frac{1}{\pi} \log \left(\frac{q^2}{m^2} \right) \log \left(-\frac{q^2}{m^2} \right)}_{\text{Sudakov dupla logaritmer}}$$

Otra

IR Divergenz an vertex hängen

$$\gamma^\mu + \Gamma^M(p, p) \quad q = p - p' \quad \text{wegen} \\ \gamma^\mu F_1(q^2) + \frac{i}{2m} \sigma_{\mu\nu} q^\nu F_2(q^2) \\ \hookrightarrow F_2(0) = \frac{1}{2\pi}$$

$$F_1^R = F_1^R(q^2) - F_1(0)$$

oder
nicht renormiert

$$F_1(q^2) = \frac{1}{2\pi} \int dx dy dz \delta(x+y+z-1) \cdot \left\{ \log \frac{m^2(1-z)^2}{m^2(1-z)^2 - q^2 xy} + \frac{m^2(1-4z+z^2) + q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2 xy + p^2 z} \right. \\ \left. - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + p^2 z} \right\}$$

$\mu \rightarrow 0$ Divergenz bei Null!

Divergenz: $\mathbb{R} \ni 1$ kritisch $\Rightarrow x, y \approx 0$ $x = 1-y-z$

$$\simeq \frac{1}{2\pi} \int_0^1 dz \int_0^{1-z} dy \left[\frac{-2m^2 + q^2}{m^2(1-z)^2 - q^2 y(1-y-z) + \mu^2} - \frac{-2m^2}{m^2(1-z)^2 + \mu^2} \right] =$$

$$y = (1-z) \quad w = 1-z$$

$$= \frac{1}{2\pi} \int_0^1 ds \frac{1}{2} \int_0^1 dw \left[\frac{-2m^2 + q^2}{[m^2 - q^2 s(1-s)] w^2 + \mu^2} - \frac{-2m^2}{m^2 w^2 + \mu^2} \right] =$$

$$= \frac{1}{4\pi} \int_0^1 ds \left[\frac{-2m^2 + q^2}{m^2 - q^2 s(1-s)} \log \frac{m^2 - q^2 s(1-s)}{\mu^2} + 2 \log \frac{m^2}{\mu^2} \right] =$$

$$F_1(q^2) = -\frac{1}{2\pi} f_{IR}(q^2) \log \frac{(m^2 - q^2)}{\mu^2} \quad \text{nur unphysikalische Punkte, sonst}$$

$$f_{IR}(q^2) = \int_0^1 \frac{m^2 - q^2/2}{m^2 - q^2 s(1-s)} ds - 1$$

pl.

$$\gamma_\mu \rightarrow \gamma_\mu \left(1 + F_1(q^2) \right) + \frac{i}{2m} \sigma_{\mu\nu} q^\nu F_2(q^2)$$

hierbei
scheitern

(most ist intrinsisch fraglich)

$$\frac{d\sigma}{d\Omega} \rightarrow \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{1}{\pi} f_{IR}(q^2) \log \left(\frac{-q^2 \sqrt{q^2 + m^2}}{\mu^2} \right) + O(\alpha) \right]$$

Legen

$$-q^2 \rightarrow \infty$$

$$q^2 \gg m^2$$

→ nella tavola + 1 e 0 mil

$$\frac{1}{\sqrt{1-\xi}}$$

$$f_{IR}(\eta^2) = \log \frac{-\eta^2}{m^2}$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{e \rightarrow e} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{\alpha}{\pi} \log \left(\frac{\eta^2}{m^2} \right) \log \left(\frac{-\eta^2}{\mu^2} \right) \right]$$

Valigiden nur es a mind
hochstesatzmetrop

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{rest}} = \left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e)} + \underbrace{\left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e+\gamma)}}_{E < E_{\text{kin}}} + \underbrace{\left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e+\gamma)}}_{\gamma}$$

$$\left(\frac{d\sigma}{d\Omega} \right)_0 \left(\frac{\alpha}{\pi} \log \left(\frac{-\eta^2}{m^2} \right) \log \left(\frac{-\eta^2}{\mu^2} \right) \right)$$

(R div liesst

durchfor folgt nicht
nur traten abgelenkt,
sritt-e sicht folgen in

A Habis her. fügs a dichten Reibungshypothese

$$\left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e+\gamma(E < E_i))} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left(\frac{\alpha}{\pi} \log \left(\frac{-\eta^2}{m^2} \right) \log \left(\frac{E_i^2}{\mu^2} \right) \right)$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{rest}} = \left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e)} + \left(\frac{d\sigma}{d\Omega} \right)_{(e \rightarrow e+\gamma(E < E_i))} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left(1 - \frac{\alpha}{\pi} \log \left(\frac{\eta^2}{m^2} \right) \log \left(\frac{-\eta^2}{E_i^2} \right) \right)$$

ha $-\eta^2 \gg m^2$

probabilität: mindig jö

elektrischen: und negativ lenkt, ha E_F müssen leicht, elektronen magnetisch werden hell

stimmt

Block-Nordsieck → ^{Restein-Schröder}
Lösungstechnik

All: aktor ist liesst or R div, ha $-\eta^2 \gg m^2$ nem feloszt

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{rest}} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{\alpha}{\pi} f_{IR}(\eta^2) \log \left(\frac{-\eta^2 \text{ vacuum}}{\mu^2} \right) + \frac{1}{2\pi} J(v, v') \log \frac{E_f^2}{\mu^2} + O(\lambda^2) \right]$$

$$\text{ha } -\eta^2 \gg m^2 \quad \underbrace{f_{IR}(\eta^2)}_{\text{es ignabil mindig log}} = J(v, v') \pi \log \left(-\frac{\eta^2}{m^2} \right)$$

es ignabil mindig log

$$J(v, v') = \int \frac{d\Omega_{v'}}{4\pi} \left(\frac{2\eta F'}{(v'_z - v_z)(v'_\perp - v_\perp)} - \frac{m^2}{(v'_z - v_z)^2} - \frac{m^2}{(v'_\perp - v_\perp)^2} \right)$$

↑ ↑
 les Szerezték

2. integral

$$\int \frac{d\Omega}{4\pi} \frac{1}{(\hat{q} \cdot \hat{p})^2} = \frac{1}{2} \int_{-1}^1 d(\cos\theta) \frac{1}{(p^0 + p \cos\theta)^2} = \frac{1}{(p^0)^2 + (p^1)^2} = \frac{1}{m^2}$$

$$\hat{q} = (1, \vec{q}) \quad \int \frac{d\Omega}{4\pi} \frac{1}{(\hat{q} \cdot \hat{p})(\hat{q} \cdot \hat{p})} = \int_0^1 d\beta \int \frac{d\Omega}{4\pi} \frac{1}{[(p^0 + (1-\beta)\vec{q} \cdot \vec{p})^2]^2}$$

$$\frac{1}{AB} = \int_0^1 d\beta \frac{1}{[p^0 + (1-\beta)B]^2}$$

$$\frac{1}{(p^0 + (1-\beta)B)^2}$$

$$\int d\beta \frac{1}{(p^0 + (1-\beta)B)^2} = \int_0^1 d\beta \frac{1}{m^2 - \beta(1-\beta)q^2} \quad q = B - p^0$$

$$2pq = 2m^2 - q^2$$

$$J(v, v) = \int_0^1 \frac{2m^2 - q^2}{m^2 - q^2(1-\beta)q^2} d\beta = 2$$

$$f_{IR}(q^2) = \int_0^1 d\beta \frac{m^2 - q^2/2}{m^2 - q^2(1-\beta)q^2} - 1 \quad J(v, v) = 2f_{IR}$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{next} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{1}{\pi} f_{IR}(q^2) \log \left(-\frac{q^2}{E_i^2} \right) + O(\alpha^2) \right]$$

Block ~ Nordström \neq rendis felix sbezogene \Rightarrow IR seines trügt

$$\left(\frac{d\sigma}{d\Omega} \right)_{next} = \left(\frac{d\sigma}{d\Omega} \right)_0 \underbrace{\left[1 - \frac{1}{\pi} f_{IR}(q^2) \log \left(-\frac{q^2}{E_i^2} \right) \right]}_{0 < < 1}$$

$$E_e = 0 \cdot m \quad \left(\frac{d\sigma}{d\Omega} \right)_{next} = 0 \quad \text{Rendis} \rightarrow \text{e}^- \text{ infarrescence (infraparticle)}$$

Rendis tetszölge rendis

für max
q fokalisiert

$$\mathcal{L} = -\frac{1}{4} F_0^2 + \frac{1}{2} \mu_0^2 A_0^2 - \frac{1}{2} \lambda_0 (\nabla A)^2 + i \nabla_0 (\partial - m_0) \psi - e_0 \nabla_0 A_0 \psi_0$$

regularized by cell

symmetr. part. e. - ad

$$m(m_0, \lambda_0, \mu_0)$$

$$A_0 = \sqrt{\epsilon_3} A$$

$$e(m_0, e_0, \lambda_0, \mu_0)$$

$$n_0 = \sqrt{\epsilon_3} n$$

$$\lambda(m_0, e_0, \lambda_0, \mu_0)$$

$$\psi_0 = \frac{\sqrt{\epsilon_3} \epsilon_2}{\epsilon_1} \psi = \sqrt{\epsilon_3} e$$

$$\mu_0$$

$$G_0(p_1, \dots, p_m, q_1, \dots, q_n, \mu_0, m_0, e_0, \lambda_0, \Lambda) = \sqrt{\epsilon_2} \sqrt{\epsilon_3}^{1/2} G_0(p_1, \dots, p_m, q_1, q_2, \mu_1, m_0, e_0, \lambda)$$

$\Lambda = \max \omega_{max}$

QED II

$$\mathcal{L} = -\frac{1}{4} Z_3 F^2 + \frac{1}{2} \mu^2 Z_3 A^2 - \frac{1}{2} \lambda_0 Z_3 (\partial A)^2 + Z_3 (\bar{\psi} \gamma^4 - (m - \delta m) \psi \gamma^4) - Z_1 e \bar{\psi} \gamma^4$$

$$= -\frac{1}{4} F^2 + \frac{1}{2} \mu^2 A^2 - \frac{1}{2} (\partial A)^2 + i \bar{\psi} (\gamma - m) \psi - e \bar{\psi} \gamma^4 + L.c.t.$$

elentagó
(counter term)

renormált perturbáció és szerint feltüntetésre

ezeket is e-ben
perturbálásnak nezzük

Hatvány számolás

a gravitációs belső impulzusa \rightarrow

$$I = \int d^4 q_1 \dots d^4 q_n \frac{1}{p_1^2 - m^2} \dots \frac{1}{p_n^2 - m^2} ()$$

p_1, \dots, p_n = leírhatóak q_i -el \rightarrow kijelölt vonalak
(Kirchhoff t.u.)

$$\ell \rightarrow \lambda \ell$$

$$\lambda \gg 1 \quad I \rightarrow \lambda^{w(\omega)} I$$

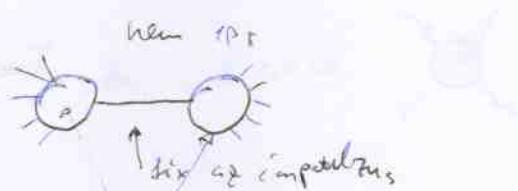
$$w(\omega) = 4L - 2I_B - I_F + \sum_{v=1}^V \delta_v$$

↓
 bázis
 prop

v vertex
 divergencia
 száma (QEDben nincs ilyen)

$$\omega(G) \begin{cases} < 0 & I \text{ univerzálisan divergens} \\ = 0 & I(\log) \text{ divergens} \\ > 0 & I \text{ divergens} \end{cases}$$

1PI: véges. összeg



primitív divergenciákkal \Rightarrow 1PI divergencia

Tétel $G \nabla \geq G$, $\nabla 1PI \quad \omega(r) < 0 \Rightarrow G$ konvergens

ezért működik a reducible renormalizálás

(Collins: renormalization)

27

QED II

$$\omega(G) = 4L + \sum_{v=1}^V \delta_v - E_F - 2E_B$$

$$L = I_B + I_F - V + 1$$

$$\omega(G) = 4 = 3I_F + 2I_B + \sum_{v=1}^V (\delta_v - 4)$$

$$2I_F + E_F = \sum_{v=1}^V f_v \quad 2I_B + E_B = \sum_{v=1}^V b_v$$

heute
berücksichtigt
seiner

$$\omega(G) = 4 = \sum_{v=1}^V \left(\frac{3}{2} f_v + b_v - \delta_v - 4 \right) - \frac{3}{2} (E_F - E_B) \quad \begin{array}{l} \text{hängt Impuls auf} \\ \text{hebt Leibnizregel} \end{array}$$

$$\Pi_{\mu\nu}(k) = \underbrace{\left(\eta^{\mu\nu} - \eta^{\mu\nu} k^2 \right)}_{\delta=2} \Pi(k^2) \quad k^2 = \pi R / 2 \pi$$

$$\omega = 2 \quad \delta = 2 \quad \omega = 0$$

$$\omega(G) = 4 = - \sum_{v=1}^V [g_v] - \frac{3}{2} (E_F - E_B) - 5$$

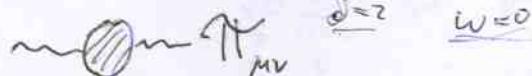
QED

$$[e_7] = 0$$

$$g_v + [g_v] = 6 \quad \omega_v - 4 = -[g_v]$$

$$\omega = 4 - \frac{3}{2} (E_F - E_B) - 5$$

$$E_B = 2, \quad E_F = 0$$



$$\delta = 2 \quad \omega = 0$$

$$E_F = 2, \quad E_B = 0$$

$$\rightarrow \text{shaded circle} \rightarrow \sum_{\mu} \epsilon_{\mu} \sum_{\rho} \delta_{\mu\rho} + \epsilon(\rho - \mu) + O((\rho - \mu)^2)$$

$$E_F = 2, \quad E_B = 1$$



$$\omega = 1 \text{ lin. d.}$$

direkt scheinbar nicht passen

$$E_F = 0, \quad E_B = 4$$



$$\omega = 0 \text{ bei } \omega_0, \text{ da } \delta = 4 \rightarrow \omega = -4$$

es kann divergieren

QED:

z_1, z_2, z_3, s_m

(λ, μ)

QED II

pl. $A \bar{v} i \gamma^\mu - B \bar{v} \gamma^\mu$

$A, B \sim O(e^2)$

1. renormalisierungsfeldtheorie

$$L = L_R + L_{ct}$$

\uparrow fiktiv freie Parameterausprägung

$$\left. \sum_e(\phi) \right|_{\phi=m} = 0$$

$$\frac{i}{\phi - m - \sum(p) - i\varepsilon}$$

$$m_R = m_{phys}$$

$$A = A_0 + A_1 + \dots$$

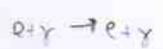
elsörendbar
nur ad
j'ndelbst

$e_2 +$
 $h \times e -$
wellen

$$\left. \frac{\partial \sum_e(\phi)}{\partial \phi} \right|_{\phi=m} = 1$$

$$\left. \Gamma^R(p, p) \right|_{\phi=m} = \gamma^R$$

(Thomson Wellstreuerzielung $\rightarrow \omega$



$$\omega = \frac{e^2}{4\pi}$$

letralne ecce pl. $\left. \sum(p) \right|_{p=0} = -m$

$$\left. \frac{\partial \sum(p)}{\partial p} \right|_{p=0} = 1$$

$$\left. \Gamma^R(p, p) \right|_{p=0} = \gamma^R$$



renormalisierbarer A-tmar el lebt
 $\rightarrow \infty$ erhalten

$$-i \sum_0(p, \lambda_1, m_1, R_1) - i(A_1, R + B_1) = -i \sum_R(p, \lambda, m_R, e_R)$$

$$\sum_0(p=m_R, m_1, R_1) = -B_1$$

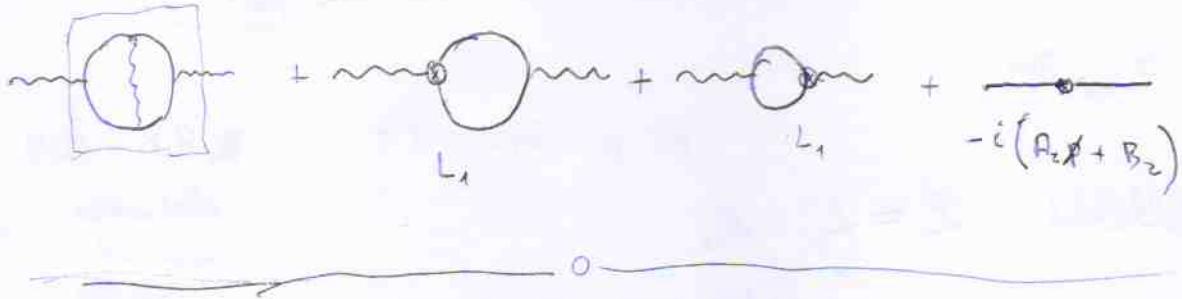


$$L \bar{v} \gamma^\mu A_\mu +$$

$$\left. \frac{\partial \sum_0}{\partial p} \right|_{p=m_R} = -A$$

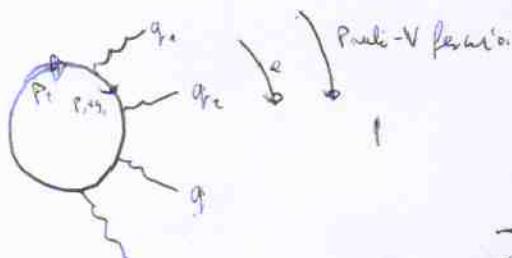
$$\Gamma^R_R(p, p) = -L \gamma^\mu$$

$$L \gamma^\mu = \left. \Gamma^R(p, p) \right|_{p=m_R}$$



Bebizonytűvel, hogy az eljárás hozzávetőleges

Pauli-Villars teljesítés rendjén ($t_{\text{zufgson}} = t_{\text{zufb}} = t$)



$$\frac{1}{p-M+i\varepsilon} \rightarrow \frac{p+M}{p^2-M^2+i\varepsilon}$$

poláromás
/
+

$$I = \sum_{j=0}^s C_j \int d^4 p \operatorname{tr} \left(\frac{1}{p-M_1+i\varepsilon} \gamma_\mu \frac{1}{p+q_1-M_1+i\varepsilon} \dots \gamma_{2p} \right) = \sum_{j=0}^s C_j \int \frac{d^4 p}{(2\pi)^4}$$

$$\frac{P_n(p^2; p, q_c, q_c^2) + M_j^2 P_{n-1}(p^2; p, q_c, q_c^2)}{Q_{2n}(p^2; p, q_c, q_c^2) + M_j^2 Q_{2n-1}(p^2; p, q_c, q_c^2)}$$

$$M_0 = m$$

$$C_0 = 1 \quad \text{a fölbbi feltétel}$$

racionális
+

$$\frac{P_n}{Q_{2n}} + M_1^2 \left(\frac{P_{n-1}}{Q_{2n}} - \frac{P_n Q_{2n-1}}{Q_m^2} \right) +$$

teljes

$$\sim p^{-2n} \quad p \rightarrow \infty$$

$$M_1^2 \text{ éha } \sim p^{-2n-2} \quad p \rightarrow \infty$$

$$\sum_{j=0}^s C_j = 0 \quad \sum_{j=0}^s C_j M_j^2 = 0 \quad \sim \text{előre ha, } k=2$$

legfelüli p^- -rel való

$$M_1^2 = m^2 + 2\Lambda^2 \quad C_1 = 1$$

$$M_2^2 = m^2 + \Lambda^2 \quad C_2 = -2$$

Λ leágazás, $\Lambda \rightarrow \infty$ -re viszszakapja az eredeti integrált

$$\lim_{\Lambda \rightarrow \infty}$$

Fotonra:

$$G_{S\sigma}(k) = -i \left(\frac{q_S - -q_S k_0/\mu^2}{\omega - \mu^2 + i\varepsilon} - \frac{q_S q_S/\mu^2}{\omega^2 - \mu^2/\lambda} \right)$$

$$G_{S\sigma}(k \rightarrow 0, \mu) - G_{S\sigma}(k, \mu)$$

$$[\mu, \sqrt{\lambda}]$$

$$\mathcal{L}_{\text{reg}} = \frac{1}{\mu_1^2 - \mu_2^2} \left\{ -\frac{i}{4} (\partial_S A_\alpha - \partial_\alpha A_S) (\square + \mu_1^2 + \mu_2^2) (\partial^\alpha A^\alpha - \partial^\beta A^\beta) \right. \\ \left. + \frac{\mu_1^2 \mu_2^2}{2} A^2 - \frac{1}{2} \partial A (\lambda \square + \mu_1^2 + \mu_2^2) \partial A \right\} + \sum_{j=0}^3 \bar{\Psi}_j (i \not{D} - e A - M_j) \Psi_j$$

ADA

$\partial_\mu j^{(k=0)}$ merktetn.

λ, μ_1, μ_2

$j=2,3$ "reale" elöjd a hosszabn \Rightarrow bázisfehér kell kezden

de nem báj, mert $\partial_\alpha A^\alpha \rightarrow \infty$ minden előtérben

0+α

dec. 20., Jan. 3., Jan 10., Jan 18., Jan 22.

$$\partial_x^k \langle 0 | T j^\mu(x) \psi(x_1) \bar{\psi}(y_1) \dots \psi(x_n) \bar{\psi}(y_n) | A_{\mu_1}(z_1) \dots A_{\mu_k}(z_k) | 0 \rangle =$$

$$= \sum_{i=1}^n \langle 0 | T \{ [j_\mu(x), \psi(x_i)] \delta(x^0 - x_i^0) \bar{\psi}(y_i) + \psi(x_i) [j_\mu(x), \bar{\psi}(y_i)] \} \delta(x^0 - y_i^0) \}$$

ψ ↗ ite van valdene

$$\partial_x^k \langle 0 | T j_\mu(x) \psi(x_1) \bar{\psi}(y_1) \dots A_{\mu_1}(z_1) | 0 \rangle = e \langle 0 | T \bar{\psi}(x_1) \psi(y_1) \dots A_{\mu_1}(z_1) | 0 \rangle \left(\sum_{i=1}^n \delta^{(4)}(x - y_i) \right)$$

$$- \sum_{i=1}^n \delta^{(4)}(x - z_i)$$

PV regularizálva $L \rightarrow \infty$ szimultánul!

PV elavolítása ($\Lambda \rightarrow \infty$)

Feltességek, hogy $L \rightarrow \infty$ mindenhol rendig minden OK.

→ csak overall divergenciákkal kell foglalkoznom

$N_B = 2, N_F = 0$

$\omega = 2$

$$\text{Diagram} = \text{main} + \text{loop}$$

(QED II)

$$G_{\beta\sigma}(x) = G_{\beta\sigma}^{(0)}(x) - i e \int d^4 x' G_{\beta\sigma}^{(0)}(x-x') \langle 0 | T j^\sigma(x') A_\sigma(0) | 0 \rangle$$

$$\partial_x^3 G_{\beta\sigma}(x) = \partial_x^3 G_{\beta\sigma}^{(0)}(x) + \text{Wand-Termschubel}$$

propagator divergenz neu normieren

$$\partial_x^3 G_{\beta\sigma}(x) = \underbrace{\partial_x^3 G_{\beta\sigma}^{(0)}(x)}_{-i \left(\frac{\eta_{\beta\sigma} - \eta_s \eta_\sigma / \mu^2}{\mu^2 - \mu^2/\lambda} + \frac{\eta_s \eta_\sigma / \mu^2}{\mu^2 - \mu^2/\lambda} \right)} = (\mu \rightarrow \mu_1) \frac{\mu^2/\lambda}{\mu^2 - \mu^2/\lambda}$$

$$G_{\beta\sigma}^{-1} = -i \Gamma_{\beta\sigma} \quad \text{Paralle + Verteil.}$$

$$\Gamma_{\beta\sigma} = \frac{-\mu^2/\lambda}{\mu^2(\mu^2 - \mu^2/\lambda)} \partial_x^\sigma \Gamma_{\beta\sigma}(\epsilon) \quad \begin{matrix} \nearrow \eta_{\beta\sigma} \eta_\sigma \\ \nearrow \eta_s \eta_\sigma \end{matrix} \quad \log -\frac{\eta_s \eta_\sigma}{\epsilon^2}$$

$$\Gamma_{\beta\sigma}(\epsilon) = A(\epsilon^2) \left(\eta_{\beta\sigma} \epsilon^2 - \eta_s \eta_\sigma \right) + B(\epsilon) \eta_s \eta_\sigma$$

$$B(\epsilon^2) = \lambda \frac{\epsilon^2 - \mu^2/\lambda}{\epsilon^2} \quad \text{mehrheitl. infre reg. von kap. konstant}$$

$$\downarrow \epsilon \approx \eta_{\beta\sigma} \approx \eta_{\text{ext}}$$

$$A(\epsilon^2) = 1 + \mathcal{T}(\epsilon^2)$$

$$\Rightarrow \delta m_p = 0$$

$\bar{\epsilon}_3$ bzg. divergenz

$$\delta = 2$$

$$\bar{\epsilon} = \mu$$

$$\sum(p) \leftrightarrow \Gamma_\mu \quad \underline{z_1 = z_2} \quad \text{Wand ansonsten}$$

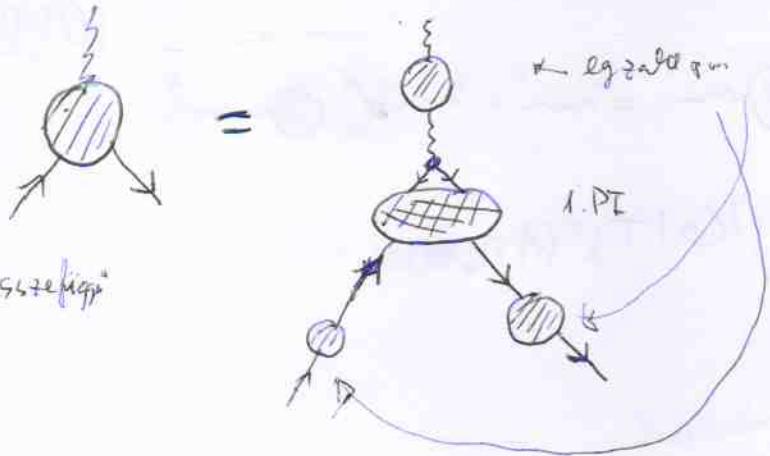
segmentat. bzg. erg. mas. bzgl. reiz amplitud

$$n=1, l=0$$

$$-ie(2\pi)^4 \delta^{(4)}(p-p-q) V_3(p,p) = \int d^4 x \, d^4 x_1 \, d^4 y_1 \, e^{i(p \cdot x_1 - p \cdot y_1 - q \cdot x)} \langle 0 | \bar{\psi} \psi_{\mu}(x) \psi_{(x_1)} \bar{\psi}_{(y_1)} | 0 \rangle$$

o-ad. und 6n

$$= 0$$



$$V_3(p', p) = G_{gg}^s(q) S(p) \Lambda^s(p', p) S(p)$$

$$\% = -i G_{gg}^{(0)}(q) \int d^4x d^4x_1 d^4y_1 e^{i(p' \cdot x_1 - q' \cdot y_1 - px)} \langle 0 | T j^0(x) \psi(x_1) \bar{\psi}(y_1) | 0 \rangle$$

$$e(2\pi)^4 \delta^{(4)}(p-p-q) \underbrace{q^s G_{gg}^{(0)}(q)}_{G^{(0)}} S(p) \Lambda^s(p', p) S(p) =$$

elb "zusammenf"

* Derivationsregeln für Vertexfunktionen

$$= -i \underbrace{q^s G_{gg}^{(0)}(q)}_{Lq^s} \int d^4x \quad = i \cancel{x} \int d^4x d^4x_1 d^4y_1 e^{i(p' \cdot x_1 - p \cdot y_1 - q \cdot x)} \langle 0 | T j^0(x) \psi(x_1) \bar{\psi}(y_1) | 0 \rangle$$

$$e(2\pi)^4 \delta^{(4)}(p-p-q) S(p) q^s \Lambda_g(p', p) S(p) = ie \int d^4x d^4x_1 d^4y_1 e^{i(p' \cdot x_1 - p \cdot y_1 - q \cdot x)} \langle 0 | T \bar{\psi}_1(x_1) \psi_1(y_1) | 0 \rangle$$

$$\bullet \left(\delta^{(4)}(x-y_1) - \delta^{(4)}(x-x_1) \right)$$

$$S(p) q^s \Lambda_g(p', p) S(p) = S(p) - S(p')$$

$$q^s \Lambda_g(p', p) = S^{-1}(p') - S^{-1}(p)$$

$\Lambda \rightarrow S$ renormalisiert

QED II

$$S' = \bar{x} - m - \sum(p)$$

beobachtet

reduziert somit $e^2 = e$

1PI

$$\text{q}^* \Gamma_3(p, p) = \sum(p) - \sum(p)$$

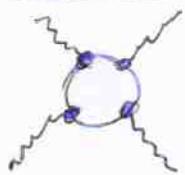
$$\Gamma \rightarrow \Gamma$$

$\gamma -$

$$\boxed{\Gamma_3(p, p) = -\frac{\partial}{\partial p^3} \sum(p)}$$

$$\Rightarrow Z_1 = Z_2$$

$\gamma - \gamma$ Stoß's



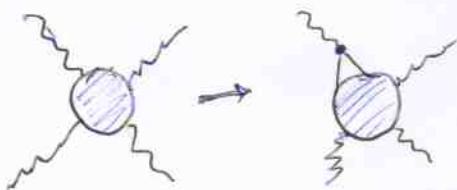
$$\sim \int \frac{d^4 k}{k^4}$$

$\sim \log \lambda^2 / \sim A_\mu^4$ abhängig? (nur ex. f. g. Regionen)

• Vakuum Hellstroms

+ Verzögern ex. & groß werden

$\gamma - \gamma$ Stoß's



$$\frac{\Gamma_{g_1 g_2 g_3 g_4}(k_1, k_2, k_3, k_4)}{\Gamma_{g_1 g_2 g_3 g_4}(k_1, k_2, k_3, k_4)} \sim G_{gg}(k_1) \langle 0 | T j^\mu(k_1) A_{g_2}(k_2) A_{g_3}(k_3) A_{g_4}(k_4) | 0 \rangle$$

$$k_1 G_{gg} = k_1 G_{gg}^{(0)} \sim k_1^5$$

$$\frac{\Gamma_{g_1 g_2 g_3 g_4}}{\Gamma_{g_1 g_2 g_3 g_4}}(k_1, k_2, k_3, k_4) = 0$$

$\delta = 1$

$$\Gamma_{g_1 g_2 g_3 g_4} = (k_1)^5 \Gamma_{g_1 g_2 g_3 g_4} = (k_1)^5 (k_2)^5 \Gamma_{g_1 g_2 g_3 g_4}$$

ambiguität

$$k_1 \rightarrow k_1$$

ist von
ex interw. S

$$\Gamma \rightarrow \lambda \Gamma$$

$(\omega = 0)$ stat. Lösungen

$$\Gamma_4 \rightarrow \lambda^{-4} \Gamma_4$$

$$F^a \rightarrow F^a$$

Egyenlőtlen konvergencia → mérhetetlenség.

$$F_{\mu\nu}(x) \approx g_{\mu\nu} A_\nu(x) - g_{\nu\mu} A_\mu(x)$$

0 -

összefoglalás:

L-hoz rendig minden elterjedés mérhetővannak

+ PV regularizáció mérhető.

↓

L+1-hoz rendig egrifelv. Ward-Talek

↳ az α_2 ki elterjedésre is teljesül

\Rightarrow L+1 rendig L_c is tudja \rightarrow L+1-ed rendig is finomabb

renormálás után a WT szembenességi \Rightarrow QED o. Pkt. stat. + rendigben renormálható!

QED renormálhatósága

a PT & renormálhatósága alkja elektrom., amik

① fizikai & divergenciament.

② megrétege & mérhetőinvariancia (WT) \rightarrow Graphok Bleuer működés
renormálható elterjedésre

\Rightarrow a renormálás után az
elterjedés megrétege poz. definit,
megmaradó valószínűségi részük vételez
S-matrix

→ nemholmi "szörzés alk. séprese!"

& elne'let alkotható $\Lambda \approx 0$ mellett elne'le!

I. alk. (renormált) paraméterek, amelyekkel leírjuk a fizikai

mennyiségeket ($E \ll 1$, struktúra) eredményeit a hibás ("nincs")

nem renormálhatóban: Pontos számításba feltehetetünk a Λ hivatalos
fizikaiért!