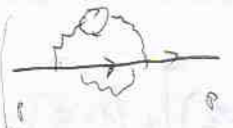
QED II

$\Sigma(p) = \sum\limits_{\substack{\uparrow \\ \text{összeg}}} \Big($  $\Big)$

$-\frac{\partial}{\partial p^\mu}$ $= \Sigma$ 

$$\Sigma(p) = A + B(\not p - m) + \Sigma^{\pi}(p)$$

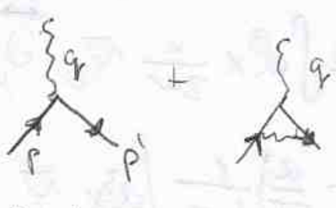
$Z_1 = Z_2$ ✓ $e_e = \sqrt{Z_3}\, e$ független mindenkire

Univerzális töltés renorm, kiegéb az IR divergencia

óra

Elektron formfaktor

$$\gamma_\mu \longrightarrow \gamma_\mu + \Gamma^R_\mu(p, p')$$



$q = p' - p$

$$\Gamma^R_\mu = \gamma_\mu F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu} q^\nu F_2(q^2)$$

$$F_1(q^2) \simeq \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\log\frac{m}{\mu} - \frac{3}{8}\right)$$ töltés eloszlás Fourier trafója

$(1+\pi^R G)$

$G + G\pi^R G$

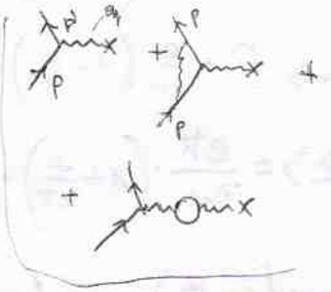
$\int \text{Green}(x\ldots)\, j_{ext}(\ldots)$

$$F_2(0) = \frac{\alpha}{2\pi}$$

$$W = \int d^3x\; j_\mu A^\mu_{ext} = e\int d^3x\; \bar\Psi_{p'}\left(\gamma_\mu + \Gamma^R_\mu(p,p') + i\pi^R_{\mu\nu} D_F^{\nu\sigma}\gamma_\sigma\right)\Psi_p A^\mu_{ext}$$

kis q

$$\simeq e\int d^3x\; \bar\Psi_{p'}\Big[\underbrace{1}_{\gamma_\mu} + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\log\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right) + \frac{\alpha}{2\pi}\frac{i}{2m}\sigma_{\mu\nu}q^\nu\Big]\Psi_p A^\mu_{ext}$$



$A^0_{ext} \sim e^{-iqx}$

Gordon azonosság

$$\bar u(p')\gamma_\mu u(p) = \frac{1}{2m}\bar u(p')\left\{(p+p')_\mu + i\sigma_{\mu\nu}(p'-p)^\nu\right\}u(p)$$

$$\bar u(p')\gamma_\mu(\not p - m)u(p) = 0$$

$$\bar u(p')(\not p' - m)\gamma_\mu u(p) = 0$$

$$\frac{1}{2m}\bar u(p')\gamma^\mu u(p) = \bar u(p')\big(\overbrace{\gamma_\mu \not p + \not p'\gamma_\mu}^{\gamma_\mu\gamma_\nu p^\nu + \gamma_\nu\gamma_\mu p'^\nu}\big)u(p)$$

$$\gamma_\mu\gamma_\nu = \frac{1}{2}\Big[\underbrace{\{\gamma_\mu,\gamma_\nu\}}_{2\eta_{\mu\nu}1} + \underbrace{[\gamma_\mu,\gamma_\nu]}_{-2i\sigma_{\mu\nu}}\Big]$$

$$\mathcal{V}_0 = e\int d^3x\, \bar\Psi_{p'} \left\{ \frac{1}{2m}(p+p')_\mu \left[1+\frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\log\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right] + \left(1+\frac{\alpha}{2\pi}\right)\frac{i}{2m}\sigma_{\mu\nu} q^\nu \right\} \Psi_p A^\mu_{ext}$$

Saját mágneses nyomaték

$q\sim 0 \quad p'=p$

$$\bar\Psi_p \frac{p^\mu}{m}\Psi_p \to \frac{p^\mu}{E} = (1, \underline{v})$$

$\bar u_{p=0} u_{p=0} = 1$ $\qquad u_p = \sqrt{\frac{m}{E}}\, D(L(p))\, u_{p=0}$

$\qquad \bar u_p = \sqrt{\frac{m}{E}}\, \bar u_{p=0}\, D(L(p))^{-1}$

$\bar u_p u_p = \frac{m}{E}$

$q_\mu \to +i\partial_\mu \quad A_{ext}$-re

$p_\mu \quad \Psi_p$-n: $+i\partial_\mu$

$p'_\mu \quad \Psi'_p$: $-i\overleftarrow{\partial}_\mu$

$$W \simeq e\int d^3x \frac{i}{2m}\bar\Psi_{p'}\overleftrightarrow{\partial}_\mu \Psi(x)\left\{1-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\log\left(\frac{m}{\mu}\right)-\frac38-\frac15\right)\Box\right\}A^\mu_{ext}$$

$$-\left(1+\frac{\alpha}{2\pi}\right)\frac{1}{4m}\int d^3x\,\bar\Psi_{p'}(x)\sigma_{\mu\nu}\Psi_p(x)F^{\mu\nu}$$

$$\sigma_{\mu\nu}q_\nu A^\mu_{ext} \to \frac12\sigma_{\mu\nu}\left(q_\nu A^\mu_{ext} - q_\mu A^\nu_{ext}\right)$$

Mágneses mező $\qquad F^{12} = -B^3 = -F^{21}$

$$\sigma_{12} = \Sigma_3 = \begin{pmatrix}\sigma_3&0\\0&\sigma_3\end{pmatrix} = -\sigma_{21} \qquad \underline\Sigma = \left(\sigma_{23},\sigma_{31},\sigma_{12}\right)$$

$$\begin{pmatrix}\sigma_1&\\&\sigma_1\end{pmatrix}\begin{pmatrix}\sigma_2&\\&\sigma_2\end{pmatrix}\begin{pmatrix}\sigma_3&\\&\sigma_3\end{pmatrix}$$

$$W_{mágn} \simeq -\frac{e}{4m}\left(1-\frac{\alpha}{2\pi}\right)2\int d^3x\,\bar\Psi(x)\underline\Sigma\,\Psi(x)\cdot\underline B = -\langle\underline\mu\rangle\underline B$$

$\underline S = \frac12\underline\Sigma$

$$\langle\underline\mu\rangle = \frac{e\hbar}{2m}\left(1+\frac{\alpha}{2\pi}\right)\cdot 2\langle\underline S\rangle = g\mu_B\langle \underline S\rangle$$

$$g = 2\left(1+\frac{\alpha}{2\pi}\right) \simeq 2\left(1+0.00116141\ldots\right)$$

$$g_{exp} = 2\cdot\left(1+0.001159652193\right)$$

α hibája fölé

$$g_{elm} = 2\left(1+0.001159652140 \underset{\text{hiba}}{\pm} 0.000000000028\right)$$

e, μ, τ kell csak (QED)

$$\frac{g^\mu_{exp}-2}{2} = 0.00116591\,77 \pm 0.0000000007$$

$a = \frac{g^{\mu}_{\text{theo}} - 2}{2} = 0.00116592\,02 \pm 0.000000015$

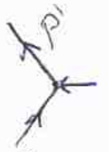
$(\Delta a_\mu)_{\text{weak}} = (691 \pm 7) \cdot 10^{-10}$ gyenge kh járulék

$(\Delta a_\mu)_{\text{strong}} = (15.1 \pm 0.4) \cdot 10^{-10}$

Sugárzási korrekciók

1. Hurok korrekció
2. Bremsstrahlung

Klasszikus számolás



rugás

$t=0 \qquad \underline{x}=0 \qquad \tau \to f(\tau)$ inv

$$j^\mu(x) = e\int d\tau \, \frac{dy^\mu}{d\tau}\, \delta^{(4)}\left(x^\mu - y^\mu(\tau)\right)$$

$y^0(\tau)=\tau$ fizikailag világos, ebben

$$y^\mu(\tau) = \begin{cases} \frac{p^\mu}{m}\tau & \tau<0 \\ \frac{p'^\mu}{m}\tau & \tau>0 \end{cases}$$

így paraméterezzük

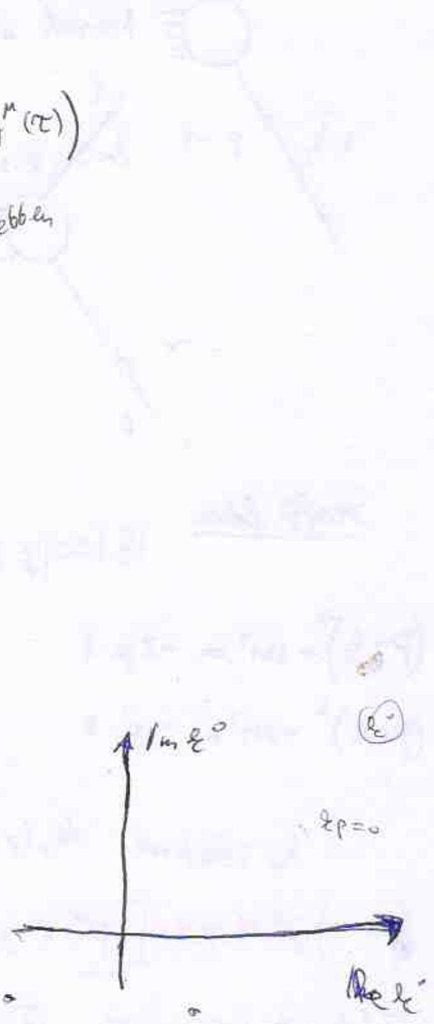
$$\tilde{j}^\mu(k) = \int d^4x\, e^{ikx} j^\mu(x) = ie\left(\frac{p'^\mu}{kp'+i\varepsilon} - \frac{p^\mu}{kp - i\varepsilon}\right)$$

$e^{-\varepsilon(\tau)}$

Lorentz-mértékben

$$\tilde{A}^\mu(k) = -\frac{1}{k^2}\tilde{j}^\mu(k) \to A^\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx}\tilde{A}_\mu(k)$$

$\frac{1}{k^2 + i\varepsilon(\operatorname{sign} k_0)}$



$\rightsquigarrow \underline{E}(x),\ \underline{B}(x) = \operatorname{rot}\underline{A}$

$= -\underline{\nabla}A - \dot{\underline{A}}$

negyes vektor lehet $\varepsilon^{(\lambda)}(k)$

$$\text{kisugárzott energia} = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \frac{e^2}{2} \left| \varepsilon^{(\lambda)}(k)\left(\frac{p'}{k\cdot p'} - \frac{p}{k\cdot p}\right)\right|^2 =$$

$\frac{1}{16\pi^3}$

$\sum_{\lambda=1,2} \to \sum_{\lambda=0}^{3}$

$\varepsilon^{(0)}(k) + \varepsilon^{(3)}(k) \sim k\left(\frac{p'}{k\cdot p'} - \frac{p}{kp}\right) = 0$

$$= \frac{\alpha}{\pi}\int d\omega\, \mathcal{J}(\underline{v}, \underline{v}')$$

$\int \frac{d\Omega_k}{4\pi}$

$$\sum_{\lambda=1}^{3} \varepsilon_\mu^{(\lambda)}(k)\, \varepsilon_\nu^{(\lambda)}(k)^* = -\eta_{\mu\nu}$$

$$p = E(1, \underline{v}) \qquad p' = E'(1, \underline{v}')$$

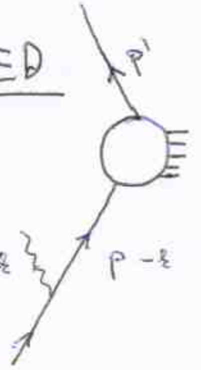
$$I(\underline{v}, \underline{v}') = \int \frac{d\Omega_k}{4\pi} \left[\frac{2(1 - \underline{v}\cdot\underline{v}')}{(1-\hat{\underline{k}}\cdot\underline{v})(1-\hat{\underline{k}}\cdot\underline{v}')} - \frac{m^2/E'^2}{(1-\hat{\underline{k}}\cdot\underline{v}')^2} - \frac{m^2/E^2}{(1-\hat{\underline{k}}\,\underline{v})^2} \right]$$

$$N_\gamma = \frac{\alpha}{\pi} \int_{E_-}^{E_+} \frac{d\omega}{\omega} \underbrace{I(\underline{v}, \underline{v}')}_{\omega \text{ független}} = \frac{\alpha}{\pi} \log \frac{E_+}{E_-}$$

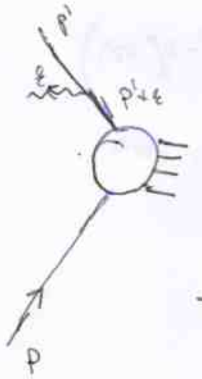
$E_+ \tau_s \sim 1$ — kísérl. elrendezés

detektor felbontás

QED



hard scattering — nagy impulzus transzfer



$$p^2 = p'^2 = m^2$$

$$\mathcal{M} = -\frac{ie}{\sqrt{2k^0}}\, \overline{u}(p') \left[\mathcal{M}_0(p', p-k) \frac{i(\not p - \not k + m)}{(p-k)^2 - m^2} \gamma^\mu \varepsilon_\mu^*(k) + \gamma^\mu \varepsilon_\mu^*(k) \frac{i(\not p' + \not k + m)}{(p'+k)^2 - m^2 + i\varepsilon} \mathcal{M}_0(p'+k, p) \right] u(p)$$

Soft foton $\quad |k| \ll |q| = |p' - p|$

$$(p-k)^2 - m^2 = -2p\cdot k \qquad k = \omega(1, \hat{k})$$

$$(p'+k)^2 - m^2 = 2p'\cdot k$$

közelítések $\quad \mathcal{M}_0(p', p-k) = \mathcal{M}_0(p', p) \qquad$ számlálókban $\not k$-t elhagyom (3)

$$(\not p - m)\gamma^\mu u(p) = \left[2p^\mu + \gamma^\mu \cancel{(-\not p + m)}\right] u(p)$$

$$\overline{u}(p')\gamma^\mu(\not p' + m) = \overline{u}(p')\, 2p'^\mu$$

$$\mathcal{M} = \overline{u}(p')\, \mathcal{M}_0(p', p)\, u(p) \left(\frac{p'\cdot\varepsilon^*}{p'\cdot k} - \frac{p\cdot\varepsilon^*}{p\cdot k} \right) \frac{-ie}{\sqrt{2\omega}}$$

$$d\sigma = d\sigma(p \to p')\, \frac{e^2}{2\omega} \sum_{\lambda=1,2} \left| \varepsilon_\lambda \left(\frac{p'}{p'\cdot k} - \frac{p}{p\cdot k} \right) \right|^2$$

$$d(\text{prob}) = \frac{d^3k}{(2\pi)^3}\cdot\frac{e^2}{2\omega}\sum_{\lambda=1,2}\left|\qquad\right|^2$$

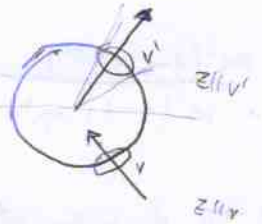
$\boxed{\text{QED II}}$

[Peskin-Schröder-ben van benne]

IR probléma

$$J(v,v') = \int \frac{d\Omega_k}{4\pi}\left\{\frac{2(1-v v')}{(1-\hat{k}\cdot v)(1-\hat{k} v')} - \cancel{\frac{m^2/E^2}{(1-\hat{k}\cdot v)^2}} - \cancel{\frac{m^2/E^2}{(1-\hat{k}\, v')^2}}\right\}$$

ultrarel. határeset: $\boxed{E', E \gg m}$ $\rightarrow$ $|v|, |v'| \sim 1$



$$J(v,v') \approx \int_{\hat{k} v = v' v}^{1} d(\cos\theta)\,\frac{1-v\cdot v'}{(1-v\cos\theta)(1-v v')}$$ $\hat{k} \approx v$

addig nyitom a két kúpot (amíg összeérnek)

$$+\int_{\hat{k}\cdot v' = v v'} d\cos\theta\,\frac{1-v\cdot v'}{(1-v\cdot v')(1-v'\cos\theta)}$$

$\hat{k}\cdot v = 1 - x(1-v v')$ $\quad x>0 \quad$ x ne legyen sokkal nagyobb, mint 1

$$\Rightarrow J(v,v') = \log\left(\frac{1-v' v}{1-|v|}\right) + \log\left(\frac{1-v'\cdot v}{1-|v'|}\right) = \log\frac{(E'E - p'\cdot p)^2}{EE'(E-p)(E'-p')} =$$

$p = |p|$, $p' = |p'|$ $\quad E = p + \frac{m^2}{2p} + O\left(\frac{1}{p^2}\right) + \dots \qquad E(E-p) \approx \frac{E+p}{2}(E-p) = \frac{m^2}{2}$

$$= 2\log\frac{p\cdot p'}{m^2/2} = 2\log\left(-\frac{q^2}{m^2}\right)$$

$-2p\cdot p' \approx q^2$

$q = p - p'$

$q^2 = \underbrace{p^2 + p'^2}_{\cancel{2m^2}} - 2p\cdot p'$

IR div.

$$d\sigma(p \to p' + \gamma(k)) = d\sigma(p\to p')\frac{\alpha}{\pi}\int_{\mu}^{|q|}\frac{d\omega}{\omega}\log\left(-\frac{q^2}{m}\right) \overset{q^2\to\infty}{\simeq} d\sigma(p\to p')\underbrace{\frac{\alpha}{\pi}\log\left(\frac{-q^2}{\mu^2}\right)\log\left(-\frac{q^2}{m^2}\right)}_{\text{Sudakov dupla logaritmus}}$$

μ: IR reguláló

óra

óra

IR divergencia a vertex függvényben

$$\gamma^\mu + \Gamma^\mu(p',p) \qquad q = p'-p \qquad \sigma \text{ végső} \qquad F_1^R = F_1^b(q^2) - F_1(0)$$

$$\gamma^\mu F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu} q^\nu F_2(q^2) \qquad \hookrightarrow F_2(0) = \frac{\alpha}{2\pi}$$

eddig a nem renormált

$$F_1(q^2) = \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\cdot\left\{\log\frac{m^2(1-z)^2}{m^2(1-z)^2-q^2xy} + \frac{m^2(1-4z+z^2)+q^2(1-x)(1-y)}{m^2(1-z)^2-q^2xy+\mu^2 z}\right.$$

$$\left. - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2+\mu^2 z}\right\}$$

$\mu \to 0$ divergenciák kell!

divergencia: $z \sim 1$ körül $\Rightarrow x, y \sim 0$ $\qquad x = 1-y-z$

$$\simeq \frac{\alpha}{2\pi}\int_0^1 dz \int_0^{1-z} dy \left[\frac{-2m^2+q^2}{m^2(1-z)^2 - q^2 y(1-z-y)+\mu^2} - \frac{-2m^2}{m^2(1-z^2)+\mu^2}\right] =$$

$y = (1-z)\xi \qquad w = 1-z$

$$= \frac{\alpha}{2\pi}\int_0^1 d\xi\ \frac{1}{2}\int_0^1 \underbrace{d(w^2)}_{2w\,dw}\left[\frac{-2m^2+q^2}{\left[m^2-q^2\xi(1-\xi)\right]w^2+\mu^2} - \frac{-2m}{m^2w^2+\mu^2}\right] =$$

$$= \frac{\alpha}{4\pi}\int_0^1 d\xi\left[-\frac{2m^2+q^2}{m^2-q^2\xi(1-\xi)}\log\frac{m^2-q^2\xi(1-\xi)}{\mu^2} + 2\log\frac{m^2}{\mu^2}\right] = \ldots$$

nem annyira fontos most

$$F_1(q^2) = -\frac{\alpha}{2\pi} f_{IR}(q^2)\log\frac{(m^2 \text{ vagy } -q^2)}{\mu^2}$$

$$f_{IR}(q^2) = \int_0^1 \frac{m^2-q^2/2}{m^2-q^2\xi(1-\xi)}\,d\xi - 1$$

pl.

atom

hard scattering

$$\gamma_\mu \longrightarrow \gamma_\mu\overbrace{\left(1+F_1(q^2)\right)}^{|1+F_1|^2 \sim 1+2F_1} + \frac{i}{2m}\sigma_{\mu\nu}q^\nu F_2(q^2)$$

(most az interferenciával foglalkozunk)

$$\frac{d\sigma}{d\Omega} \longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_0\left[1 - \overset{2F_1}{\frac{\alpha}{\pi}} f_{IR}(q^2)\log\left(\frac{-q^2 \text{ vagy } m^2}{\mu^2}\right) + O(\alpha)\right]$$

Legyen $-q^2 \to \infty$ $\qquad q^2 \gg m^2$

$\int_0^1 \to$ nulla körül + 1 körül
$\frac{1}{\xi} \qquad \frac{1}{1-\xi}$

$$f_{IR}(q^2) = \log \frac{-q^2}{m^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi} \log\left(-\frac{q^2}{m^2}\right) \log\left(-\frac{q^2}{\mu^2}\right)\right]$$

valójában nem ez a mért hatáskeresztmetszet

$$\left(\frac{d\sigma}{d\Omega}\right)_{mért} = \left(\frac{d\sigma}{d\Omega}\right)(e \to e) + \underbrace{\left(\frac{d\sigma}{d\Omega}\right)(e \to e\gamma)}_{\left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{\alpha}{\pi} \log\left(-\frac{q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right)\right)}$$

$E < E_{lim}$

detektor felbontása miatt nem tudom eldönteni, jött-e szoft foton is

IR div kiesik

A Hatás ker. függ a detektor felbontóképességétől

$$\left(\frac{d\sigma}{d\Omega}\right)\left(e \to e + \gamma(E<E_l)\right) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{\alpha}{\pi} \log\left(-\frac{q^2}{m^2}\right) \log\left(\frac{E_l^2}{\mu^2}\right)\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{mért} = \left(\frac{d\sigma}{d\Omega}\right)(e \to e) + \left(\frac{d\sigma}{d\Omega}\right)\left(e \to e + \gamma(E<E_l)\right) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(-\frac{q^2}{E_l^2}\right)\right)$$

ha $\boxed{-q^2 \gg m^2}$

praktikusan: mindig jó

elméletben: tud negatív lenni, ha E_l nagyon kicsi, ekkor magasabb rendben kell számolni

Bloch-Nordsieck, Epstein-Schröder felösszegzés

All: akkor is kiesik az IR div, ha $-q^2 \gg m^2$ nem teljesül

$$\left(\frac{d\sigma}{d\Omega}\right)_{mért} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(\frac{-q^2 \text{ vagy } m^2}{\mu^2}\right) + \frac{\alpha}{2\pi} J(\underline{v}, \underline{v}') \log \frac{E_l^2}{\mu^2} + O(\alpha^2)\right]$$

ha $-q^2 \gg m^2$ $\underbrace{2 f_{IR}(q^2) = J(\underline{v}, \underline{v}')}_{\text{ez igazából mindig igaz}} \approx \log\left(-\frac{q^2}{m}\right)$ —

$$J(\underline{v}, \underline{v}') = \int \frac{d\Omega_k}{4\pi} \left(\frac{2 p p'}{\left(\frac{k}{\omega} \cdot p\right)\left(\frac{k}{\omega} \cdot p'\right)} - \frac{m^2}{\left(\frac{k}{\omega} \cdot p\right)^2} - \frac{m^2}{\left(\frac{k}{\omega} \cdot p'\right)^2}\right)$$

kis szorzatok

2. integrál

$$\int \frac{d\Omega_k}{4\pi} \frac{1}{(\hat{k}\cdot p)^2} = \frac{1}{2}\int_{-1}^{1} d(\cos\theta) \frac{1}{(p^0 - p\cos\theta)^2} = \frac{1}{(p^0)^2-(\underline{p})^2} = \frac{1}{m^2}$$

$$\hat{k} = (1, \hat{k}) \qquad \int \frac{d\Omega_k}{4\pi} \frac{1}{(\hat{k}\cdot p)(\hat{k}\cdot p')} = \int_0^1 d\xi \int \frac{d\Omega_k}{4\pi} \frac{1}{\left[\xi \hat{k} p + (1-\xi)\hat{k}\cdot p'\right]^2}$$

$$\frac{1}{AB} = \int_0^1 d\xi \frac{1}{[\xi A + (1-\xi) B]^2}$$

$$\frac{1}{\hat{k}(\xi p + (1-\xi)p')}$$

$$\int d\xi \frac{1}{(\xi p' + (1-\xi)p)^2} = \int_0^1 d\xi \frac{1}{m^2 - \xi(1-\xi) q^2} \qquad q = p - p'$$

$$2pp' = 2m^2 - q^2$$

$$J(v, v') = \int_0^1 \frac{2m^2 - q^2}{m^2 - \xi(1-\xi) q^2} d\xi - 2$$

$$f_{IR}(q^2) = \int_0^1 d\xi \frac{m^2 - q^2/2}{m^2 - q^2 \xi(1-\xi)} - 1 \qquad J(v, v') = 2 f_{IR}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{mért} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(-\frac{q^2}{E_l^2}\right) + O(\alpha^2)\right]$$

Bloch – Nordsieck $\quad \forall\ \alpha$ rendig felösszegezve az IR színs tagokat

$$\left(\frac{d\sigma}{d\Omega}\right)_{mért} = \left(\frac{d\sigma}{d\Omega}\right)_0 \underbrace{e^{-\frac{\alpha}{\pi} f_{IR}(q^2) \log\left(-\frac{q^2}{E_l^2}\right)}}_{0 \le \ < 1}$$

$E_l = 0 \to \left(\frac{d\sigma}{d\Omega}\right)_{mért} = 0$ lenne $\Rightarrow$ az e^- infrarészecske (infraparticle)

Renormálás tetszőleges rendig

$$\mathcal{L} = -\frac{1}{4} F_0^2 + \underset{\substack{\text{kis} \\ \text{tömeg} \\ \text{4 foton}}}{\frac{1}{2}\mu_0^2 A_0^2} - \frac{1}{2}\lambda_0 (\partial A)^2 + i\bar{\psi}_0 (\partial\!\!\!/ - m_0)\psi - e_0 \bar{\psi}_0 A\!\!\!/_0 \psi_0$$

regularizálni kell

csup ... pert. e_0-al

$m\ (m_0, e_0, \lambda_0, \mu_0)^\Lambda \qquad A_0 = \sqrt{Z_3}\, A$

$e\ (m_0, e_0, \lambda_0, \mu_0)^\Lambda \qquad \psi_0 = \sqrt{Z_2}\, \psi$

$\lambda\ (m_0, e_0, \lambda_0, \mu_0)^\Lambda \qquad e_0 = \frac{\sqrt{Z_3}\, Z_2}{Z_1} e = \sqrt{Z_3}\, e$

$\mu_0 (\qquad)$

$$G_0(p_1 \dots p_{2n}, k_1 \dots k_n, \mu_0, m_0, e_0, \lambda_0, \Lambda) = Z_2^{n} Z_3^{\ell/2} \underbrace{G_0(p_1 \dots p_{2n}, k_1 \dots k_2, \mu, m, e, \lambda)}_{\Lambda \to \infty \text{ limesz van}}$$

$$\mathcal{L} = -\frac{1}{4}\overbrace{Z_3 F^2}^{=F_0^2} + \frac{1}{2}\mu_0^2 Z_3 A^2 - \frac{1}{2}\lambda_0 Z_3 (\partial A)^2 + Z_3\left(\bar\Psi i\partial\!\!\!/\Psi - (m-\delta m)\bar\Psi\Psi\right) - Z_1 e\bar\Psi A\!\!\!/\Psi$$

$$= -\frac{1}{4}F^2 + \frac{1}{2}\mu^2 A^2 - \frac{1}{2}\lambda(\partial A)^2 + i\bar\Psi(\partial\!\!\!/ - m)\Psi - e\bar\Psi A\!\!\!/\Psi + \boxed{\mathcal{L}_{c.t.}}$$

ellentagok (counter term)

ezeket is e-ben perturbációként kezeljük

renormált pert.szám: e szerint fejtjük sorba

Hatványszámolás

a gráf összes belső impulzusa → L

$$I = \int d^4q_1 \dots d^4q_L \frac{1}{p_1^2 - m^2}\cdots\frac{1}{p_I^2 - m^2}\Big(\qquad\Big)$$

$p_1 \dots p_I$ = kifejezhetők q_L-el és a külső vonalak impulzusaival (Kirchoff I tv.)

$q \to \lambda q$ $\quad I \to \lambda^{\omega(G)} I$

$\lambda \gg 1$

$$\omega(G) = 4L - 2I_B - I_F + \sum_{v=1}^{V}\delta_v$$

bozon prop; vertexek; v vertex deriváltjainak száma (QED-ben nincs ilyen)

$$\omega(G)\begin{cases} <0 & I \text{ naivan divergens} \\ =0 & I \text{ (log) divergens} \\ >0 & I \text{ divergens}\end{cases}$$

1PI: 1 rész. csep

primitív divergenciák ⇒ 1 PI gráf "amputált gráf"

Tétel G $\quad \forall \gamma \subseteq G$, γ 1PI $\quad \omega(\gamma) < 0 \Rightarrow G$ konvergens

↳ ezért működik a rekurzív renormálás (Collins: renormalization)

QED II

$$\omega(G) = 4L + \sum_{v=1}^{V} \delta_v - I_F - 2I_B$$

$$L = I_B + I_F - V + 1$$

$$\omega(G) - 4 = 3I_F + 2I_B + \sum_v (\delta_v - 4)$$

$$2I_F + E_F = \sum_{v=1}^{V} f_v \qquad 2I_B + E_B = \sum_{v=1}^{V} b_v$$

külső fermionvonalak száma

$$\omega(G) - 4 = \sum_v \Big(\overbrace{\tfrac{3}{2} f_v + b_v - \delta_v - 4}^{\omega_v}\Big) - \tfrac{3}{2} E_F - E_B (-\delta)$$

hány impulzust lehet kifaktorizálni

$$\Pi_{\mu\nu}(k) = \underbrace{(k^\mu k^\nu - \eta^{\mu\nu} k^2)}_{\delta=2} \Pi(k^2)$$

$\psi \cdot \pi A \; \partial \,?$

$\omega = 2$ $\quad$ $\omega = 0$

$$\omega(G) = 4 - \sum_v [g_v] - \tfrac{3}{2} E_F - E_B - \delta$$

$\int d^4x \, \mathcal{L}$ dim = 4

$\omega_v + [g_v] = 4 \qquad \omega_v - 4 = -[g_v]$

QED $\quad [e] = 0$

$$\omega = 4 - \tfrac{3}{2} E_F - E_B - \delta$$

$E_B = 2, \quad E_F = 0$

$\Pi_{\mu\nu}$ $\quad \delta = 2 \quad \omega = 0$

$E_F = 2 \; ; \; E_B = 0$

$\sum \; \delta m + Z(p\!\!\!/ - m) + O((p\!\!\!/ - m)^2)$

$\omega = 1$ lin div

kirelliz? szimm. miatt igazából $\omega = 0$

$E_F = 2 \quad E_B = 1$

$\omega = 0$

$E_F = 0 \quad E_B = 4$

$\omega = 0$ lenne, de $\delta = 4 \to \omega = -4$

ez nem divergens

QED II

QED:

$Z_1,\ Z_2,\ Z_3,\ \delta m$

(λ, μ)

pl. $A\bar{\Psi} i\partial\!\!\!/ \Psi - B\bar{\Psi}\Psi$

$A, B \sim O(e^2)$ elsőrendben nem ad járulékot

1. renormálási feltételek $\quad \mathcal{L}_0 = \boxed{\mathcal{L}_R} + \mathcal{L}_{ct}$

a fizikát ezzel parametrizálit

ezt használjuk

$$\Sigma_R(p\!\!\!/)\Big|_{p\!\!\!/=m} = 0 \qquad \frac{i}{p\!\!\!/ - m - \Sigma(p) - i\varepsilon} \qquad m_R = m_{phys}$$

$A = A_1 + A_2 + \dots$

$\quad\ \ e^2 \quad e^4$

$$\frac{\partial \Sigma_R(p\!\!\!/)}{\partial p\!\!\!/}\Big|_{p\!\!\!/=m} = 1$$

$$\Gamma^\mu(p,p)\Big|_{p\!\!\!/=m} = \gamma^\mu$$

első rendben

(Thomson hatáskeresztmetszet $\rightarrow \alpha$

$e+\gamma \rightarrow e+\gamma \qquad \alpha = \frac{e_R^2}{4\pi}$)

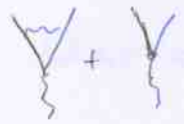
lehetne ezt is pl. $\quad \Sigma(p)\Big|_{p=0} = -m$

$$\frac{\partial \Sigma(p\!\!\!/)}{\partial p\!\!\!/}\Big|_{p=0} = 1$$

$$\Gamma^\mu(p,p)\Big|_{p\!\!\!/=0} = \gamma^\mu$$

renormált mennyiségben Λ-t már el lehet $\Lambda\to\infty$ hagyni

$$-i\,\Sigma_0(p,\Lambda, m_R, e_R) \quad - i(A_1 p\!\!\!/ + B_1) = -i\,\Sigma_R(p, \cancel{\Lambda}, m_R, e_R)$$



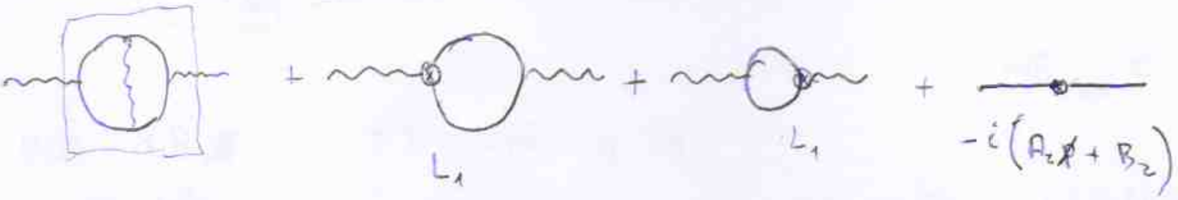
$L\bar{\Psi}\gamma^\mu A_\mu +$

$$\Sigma_0(p = m_R, \Lambda, m_R, e_R) = -B_1$$

$$\frac{\partial \Sigma_0}{\partial p\!\!\!/}\Big|_{p\!\!\!/=m_R} = -A$$

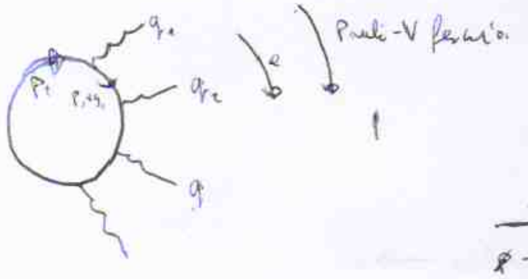
$$\Gamma^\mu{}_\mu(p,p) \rightarrow -L\gamma^\mu$$

$$L_1\gamma^\mu = \Gamma^\mu(p,p)\Big|_{p\!\!\!/=m_R}$$



Bebizonyítjuk, hogy az eljárás konzisztens

Pauli-Villars tetszőleges rendben (Itzykson - Zuber)



$$\frac{1}{\not{p}-M+i\varepsilon} \to \frac{\not{p}+M}{p^2-M^2+i\varepsilon}$$

polinomok

$$I=\sum_{j=0}^{s} C_j \int d^4p \, \mathrm{tr} \left(\frac{1}{\not{p}-M_j+i\varepsilon}\gamma_\mu \frac{1}{\not{p}+\not{q}_1-M_j+i\varepsilon} \cdots \gamma_{2\mu}\right) = \sum_{j=0}^{s} C_j \int \frac{d^4p}{(2\pi)^4} \frac{P_n(p^2, p\cdot q_i, q_i^2)+M_j^2 P_{n-1}(p^2, p\cdot q_i, q_i^2)+\dots}{Q_{2n}(p^2, p\cdot q_i, q_i^2)+M_j^2 Q_{n-1}(p^2, p\cdot q_i, q_i^2)+\dots}$$

racionalizálás

tr

$M_0=m$ $\quad C_0=1$ $\quad$ a többi fiktív

$$\frac{P_n}{Q_{2n}} + M_j^2\left(\frac{P_{n-1}}{Q_{2n}} - \frac{P_n Q_{2n-1}}{Q_{2n}^2}\right)+\dots$$

teljes $\sim p^{-2n}$ $\quad p\to\infty$

M_j^{2k} elég $\sim p^{-2n-2k}$ $\quad p\to\infty$

$$\sum_{j=0}^{s} C_j = 0 \qquad \sum_{j=0}^{s} C_j M_j^2 = 0$$

$\sim$ elég ha, $k=2$

legalább $p^{-\dots}$-nal csökken

$M_1^2 = m^2+2\Lambda^2 \qquad C_1=1$

$M_2^2 = m^2+\Lambda^2 \qquad C_2=-2$

Λ levágás, $\Lambda\to\infty$-re visszakapjuk az eredeti integrált

$\partial_\mu j^\mu_{(x)}=0$

Fotonra:

$$G_{\varrho\sigma}(k) = -i\left(\frac{g_{\varrho\sigma} - k_\varrho k_\sigma/\mu^2}{k^2-\mu^2+i\varepsilon} - \frac{k_\varrho k_\sigma/\mu^2}{k^2-\mu^2/\lambda}\right)$$

$$G_{\varrho\sigma}(k) \to G_{\varrho\sigma}(k,\mu) - G_{\varrho\sigma}(k,\mu_1)$$

$\boxed{\mu_1 \sim \Lambda}$

QED II

$$\mathcal{L}_{reg} = \frac{1}{\mu_1^2-\mu^2}\left\{-\frac{1}{4}\left(\partial_\varrho A_\sigma - \partial_\sigma A_\varrho\right)\left(\Box+\mu^2+\mu_1^2\right)\left(\partial^\varrho A^\sigma - \partial^\sigma A^\varrho\right)\right.$$

$$\left. + \frac{\mu^2\mu_1^2}{2}A^2 - \frac{\lambda}{2}\,\partial A\left(\lambda\Box+\mu^2+\mu_1^2\right)\partial\cdot A\right\} + \sum_{j=0}^{3}\bar{\Psi}_j\left(i\not\partial - e\not A - M_j\right)\Psi_j$$

ADA

$\partial_\mu j^\mu = 0$ mértékinv

λ, μ^2, μ_1^2

$j=2,3$ "rossz" előjel a hamiltonban $\Rightarrow$ bozonként kell kezelni

de nem baj, mert ezek $\Lambda \to \infty$ majd eltűnnek az elméletből

Óra

dec. 20, jan. 3., jan 10., + jan 18, + jan 27

$$\partial^{\rho}_x \langle 0 | T j^{\mu}(x) \psi(x_1) \bar{\psi}(y_1) \ldots \psi(x_n) \bar{\psi}(y_n) A_{\rho_1}(z_1) \ldots A_{\rho_l}(z_l) | 0 \rangle =$$

$$= \sum_{i=1}^{n} \langle 0 | T \left\{ [j_0(x), \psi(x_i)] \delta(x^0 - x^0_i) \bar{\psi}(y_i) + \psi(x_i) [j_0(x), \bar{\psi}(y_i)] \delta(x^0 - y^0_i) \right\}$$

itt van valami

$\psi \ldots$

$$\partial^{\rho}_x \langle 0 | T j_{\rho}(x) \psi(x_1) \bar{\psi}(y_1) \ldots A_{\rho_1}(z_1) | 0 \rangle = e \langle 0 | T \psi(x_1) \bar{\psi}(y_1) \ldots A_{\rho_l}(z_l) | 0 \rangle \left(\sum_{i=1}^{n} \delta^{(4)}(x - y_i) - \sum_{i=1}^{n} \delta^{(4)}(x - x_i) \right)$$

PV regularizált $\mathcal{L}$-ből számoltunk!

PV eltávolítjuk ($\Lambda \to \infty$)

Feltesszük, hogy L-1 hurok rendig minden OK.

→ csak overall divergenciákkal kell foglalkoznom