I. A korai univ. termodin. I.

termikus egy. → kvantumstat.

betöltési szám, degeneráció

$$f_i = \frac{g_i}{e^{\frac{\varepsilon_i-\mu_i}{T_i}} \pm 1} \qquad \pm \; \frac{F}{B}$$

fázistércella $h^3 = (2\pi)^3$ $(\hbar = 1)$

részecskeszám

$$N_i = \frac{g_i}{(2\pi)^3}\int d^3\vec{r}\, d^3\vec{p}\; f_i(\vec{r},\vec{p}) = \frac{g_i}{(2\pi)^3}\int d^3\vec{r}\, d^3\vec{p}\, \frac{1}{e^{\frac{\varepsilon_i-\mu_i}{T_i}} \pm 1}$$

energia

$$E_i = \frac{g_i}{(2\pi)^3}\int d^3\vec{r}\, d^3\vec{p}\, \frac{\varepsilon_i}{e^{\frac{\varepsilon_i-\mu_i}{T_i}} \pm 1}$$

nyomás

$$P_i = \frac{g_i}{(2\pi)^3}\int d^3\vec{r}\, d^3\vec{p}\, \frac{P}{e^{\frac{\varepsilon_i-\mu_i}{T_i}} \pm 1} = \frac{g_i}{(2\pi)^3}\int d^3\vec{p}\, \frac{p^2}{3\varepsilon}\, \frac{1}{e^{\frac{\varepsilon_i-\mu_i}{T_i}} \pm 1}$$

disp. rel.

$\varepsilon_i^2 = p_i^2 + m_i^2 \quad (c=1)$

$\varepsilon\, d\varepsilon = p\, dp$

$\frac{d\varepsilon}{dp} = \frac{p}{\varepsilon}$

dim: $V \sim R^3(t)$, $p \sim R^{-1}(t)$, $p \sim V^{-1/3}$

$\frac{dp}{dV} = -\frac{1}{3}\frac{p}{V}$

$P = -\frac{d\varepsilon}{dV} = -\frac{d\varepsilon}{dp}\frac{dp}{dV} = \frac{1}{3}\frac{p^2}{\varepsilon V}$

I. főtétel

a) Rel. $T \gg m, \mu$ $\quad p \gg m \Rightarrow \varepsilon = p$

részecskeszám-sűrűség

$\frac{p}{T_i} = x \quad p^2 dp = T_i^3 x^2 dx$

$$\boxed{n_i = \frac{g_i}{(2\pi)^3}\int d^3p\, \frac{1}{e^{\frac{p}{T_i}} \pm 1}} = \frac{g_i}{2\pi^2}T_i^3 \int_0^\infty \frac{x^2 dx}{e^x \pm 1} = \begin{cases} + \;\frac{3}{4\pi^2}\zeta(3)\, g_f T_f^3 \\ - \;\frac{1}{\pi^2}\zeta(3)\, g_a T_a^3 \end{cases}$$

$d^3p = 4\pi\int p^2 dp$

$+ : \frac{3}{2}\zeta(3)$

$- : 2\,\zeta(3)$

$\zeta(3) \approx 1{,}20206\ldots$

energiasűrűség

$$\boxed{\rho_i = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^3 dp}{e^{\frac{p}{T_i}} \pm 1}} = \frac{g_i}{2\pi^2}T_i^4 \int_0^\infty \frac{x^3 dx}{e^x \pm 1} = \begin{cases} + \;\frac{\pi^2}{30}\frac{7}{8}\, g_f T_f^4 \\ - \;\frac{\pi^2}{30}\, g_a T_a^4 \end{cases}$$

$+ : \frac{7}{8}\frac{\pi^4}{15}$

$- : \frac{\pi^4}{15}$

nyomás

$$\boxed{P_i = \frac{1}{3}\frac{g_i}{2\pi^2}\int_0^\infty \frac{p^3 dp}{e^{\frac{p}{T_i}} \pm 1} = \frac{1}{3}\rho_i}$$

$p = w\rho$ állapotegyenlet

$w_{sug(Rel)} = 1/3$

teljes en. sűrűség

$$\rho = \sum_i \rho_i = \frac{\pi^2}{30}\left(\sum_f \frac{7}{8} g_f T_f^4 + \sum_a g_a T_a^4\right) = \frac{\pi^2}{30}T^4 \underbrace{\left(\frac{7}{8}\sum_f g_f\left(\frac{T_f}{T}\right)^4 + \sum_a g_a\left(\frac{T_a}{T}\right)^4\right)}_{\parallel\; g_*}$$

$$\boxed{\rho \equiv \frac{\pi^2}{30} g_* T^4} \qquad T \equiv T_\gamma$$

effektív degenerációs fok g_*

	gen.	polar.	szín	iz	anti.	stat.
e, μ	3	2	1	1	2	7/8
ν	3	1	1	1	2	7/8
q	3	2	3	2	2	7/8
$\gamma, W^\pm, Z^0$	1	2	1	4	1	1
g	1	2	8	1	1	1
H	1	1	1	4	1	1

pl.:
$T \gtrsim 300\,\text{GeV}$ $\quad g_* \approx 106{,}75$
$T \approx 10\,\text{MeV}$ $\quad g_* = 10{,}75$
$T = 10^{-4}\,\text{eV}$ $\quad g_* = \ldots\; \gamma + \nu$

(b) Nemrel. $T \ll m,\ (m-\mu)$ $\quad p \ll m \Rightarrow \varepsilon^2 = p^2+m^2 \approx \varepsilon \approx m + \frac{p^2}{2m}$ $\quad \frac{\sqrt{\pi}}{4}$

részecskeszám-sűrűség

$$n_i \approx \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,dp}{e^{\frac{m_i+\frac{p^2}{2m_i}-\mu_i}{T_i}}} = \frac{g_i}{2\pi^2} e^{-\frac{m_i-\mu_i}{T_i}} \int e^{-\frac{p^2}{2m_iT_i}} p^2\, dp = \frac{g_i}{2\pi^2} e^{-\frac{m_i-\mu_i}{T_i}} (2m_iT_i)^{3/2}\int_0^\infty e^{-x^2}x^2\,dx$$

$\frac{p^2}{2m_iT} = x^2 \quad dp = \sqrt{2m_iT}\,dx$

$x^2 = \frac{p^2}{2m_iT}$

$$\boxed{n_i = g_i\left(\frac{m_iT_i}{2\pi}\right)^{3/2} e^{-\frac{m_i-\mu_i}{T_i}}}$$

energiasűrűség

$$\boxed{\rho_i} = \frac{g_i}{2\pi^2}\int e^{-\frac{m_i+\frac{p^2}{2m_i}-\mu_i}{T_i}}\left(m_i+\frac{p^2}{2m_i}\right)p^2\,dp = m_i\cdot n_i + \frac{g_i}{2\pi^2}\frac{1}{2m_i}e^{-\frac{m_i-\mu_i}{T_i}}\int_0^\infty e^{-\frac{p^2}{2m_iT}}p^4\,dp =$$

$$= m_in_i + \frac{1}{2m_i}(2m_iT_i)^{5/2}\frac{g_i}{2\pi^2}e^{-\frac{m_i-\mu_i}{T_i}}\underbrace{\int_0^\infty e^{-x^2}x^4\,dx}_{\frac{3}{2}\frac{\sqrt{\pi}}{4}} = m_in_i + \frac32 T_in_i =$$

$$= \left(m_i+\tfrac32T_i\right)n_i \approx m_in_i\left(1+\tfrac32\tfrac{T_i}{m_i}\right) \approx \boxed{m_in_i} \qquad (T \ll m)$$

nyomás

$$\boxed{P_i} = \frac{g_i}{2\pi^2}\int_0 e^{-\frac{m_i+\frac{p^2}{2m_i}-\mu_i}{T_i}} \frac{p^2}{3\left(m_i+\frac{p^2}{2m_i}\right)}p^2\,dp \approx \frac{g_i}{2\pi^2}e^{-\frac{m_i-\mu_i}{T_i}}\frac{1}{3m_i}\int_0^\infty e^{-\frac{p^2}{2m_iT}}p^4\,dp =$$

$3m_i\left(1+\frac{p^2}{2m_i^2}\right)$ $\quad p \ll m$ $\qquad \frac{2}{3}\cdot\frac{1}{2m_i}$

$$= \frac23\left(\frac32 n_iT_i\right) = \boxed{n_iT_i}$$

$p = w\rho$

$n_iT_i = w\, n_im_i \qquad \frac{T_i}{m_i} = w \ll 1 \Rightarrow \boxed{w_{nr}\approx 0}$

Lecsatolódás

(a) Rel. $N_D = \frac{4\pi}{3}R_D^3(t_D)\frac{g}{2\pi^2}\int_0^\infty\frac{p^2\,dp}{e^{\frac{p-\mu}{T_D}}\pm1}$ $\quad$ D decoupling $\quad \varepsilon = p \sim a^{-1} \sim R^{-1}$, $a\sim R$

$\frac{p}{p_D} = \frac{R_D}{R(t)} \qquad p = p_D\frac{R_D}{R}$

$$N = \dots = \frac{4\pi}{3}R^3(t)\frac{g}{2\pi^2}\int_0^\infty \frac{p_D^2\frac{R_D^3}{R^3}\,dp_D}{e^{\frac{p_D\frac{R_D}{R}-\mu}{T}}\pm1} = \frac{4\pi}{3}R_D^3\frac{g}{2\pi^2}\int_0^\infty\frac{p_D^2\,dp_D}{e^{\frac{p_D-\frac{R}{R_D}\mu}{T\frac{R}{R_D}}}\pm1}$$

$e^{\frac{p_D-\frac{R}{R_D}\mu}{T\frac{R}{R_D}}}$

$N=N_D \Rightarrow T_D = T\frac{R}{R_D} \quad \mu_D = \mu\frac{R}{R_D} \quad \boxed{\mu = \mu_D\frac{T}{T_D}}$

$T\sim R^{-1}$

(b) NemRel $N_D = \frac{4\pi}{3}R_D^3\frac{g}{2\pi^2}\int_0^\infty p^2 e^{-\frac{\frac{p^2}{2m}-(\mu_D-m)}{T_D}}\,dp$

$p\sim R^{-1}$

$p = p_D\frac{R_D}{R} \Rightarrow (\varepsilon \sim R^{-2})$

$$N = \frac{4\pi}{3}R^3(t)\frac{g}{2\pi^2}\int_0^\infty p_D^2\frac{R_D^3}{R^3}\, e^{-\frac{\frac{p_D^2}{2m}\frac{R_D^2}{R^2}-(\mu-m)}{T}}\,dp_D = \frac{4\pi}{3}R_D^3\frac{g}{2\pi^2}\int_0^\infty p_D^2\, e^{-\frac{\frac{p_D^2}{2m}-\frac{R^2}{R_D^2}(\mu-m)}{T\frac{R^2}{R_D^2}}}\,dp_D =$$

$N=N_D \Rightarrow T_D = T\frac{R^2}{R_D^2} \quad \frac{T_D}{T} = \frac{R^2}{R_D^2}$ $\qquad (\mu_D-m) = \frac{R^2}{R_D^2}(\mu-m)$

$T\sim R^{-2}$ $\qquad \boxed{\mu = m+\frac{T}{T_D}(\mu_D-m)}$

I. A korai univ. termodin. II

entrópia

$$d\mathcal{E}_i = T_i dS_i - p_i dV_i + \mu_i dN_i$$

$$\left(\frac{\partial \mathcal{E}}{\partial V}dV + \frac{\partial \mathcal{E}}{\partial T}dT\right) = T\left(\frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial T}dT\right) - p\left(dV + \frac{\partial V}{\partial T}dT\right) + \mu\left(\frac{\partial N}{\partial V}dV + \frac{\partial N}{\partial T}dT\right)$$

$$\frac{\partial \mathcal{E}}{\partial V} = \frac{\mathcal{E}}{V} \qquad \frac{\partial S}{\partial V} = \frac{S}{V} \qquad \frac{\partial N}{\partial V} = \frac{N}{V}$$

$$dV\underbrace{\left(\frac{\mathcal{E}}{V} - T\frac{S}{V} + p - \mu\frac{N}{V}\right)}_{0} + dT\underbrace{\left(\frac{\partial \mathcal{E}}{\partial T} - T\frac{\partial S}{\partial T} + p\frac{\partial V}{\partial T} - \mu\frac{\partial N}{\partial T}\right)}_{0} = 0$$

$\frac{\mathcal{E}}{V} = \varepsilon_i \quad \frac{S}{V} = s \quad \frac{N}{V} = n$

$$\underline{s_i = \frac{\varepsilon_i + p_i - \mu_i n_i}{T_i}}$$

NR részecskék $p_i = 0 \quad \varepsilon_i = m_i n_i$

$$s_i \approx \frac{(m_i - \mu_i) n_i}{T_i} \quad \text{[illegible]}$$

$T_i \to 0 \quad \mu_i \to m_i$

$$\left(s_i \approx \frac{T_0}{T_0}\frac{(m_i - \mu_0) n_i}{T_i} \approx \frac{m_i - \mu_0}{T_0} n_i\right)$$

Rel:

$p_i = \frac{1}{3}\varepsilon_i \quad \mu \ll T$

$$s = \sum_i s_i \approx \sum_i \frac{4}{3}\frac{\varepsilon_i}{T_i} = \frac{4}{3}\frac{\pi^2}{30}\left(\sum_b g_b T_b^3 + \frac{7}{8}\sum_f g_f T_f^3\right) = \frac{2\pi^2}{45}T^3 \underbrace{\left(\sum_b g_b\left(\frac{T_b}{T}\right)^3 + \frac{7}{8}\sum_f g_f\left(\frac{T_f}{T}\right)^3\right)}_{g_{*s}}$$

$$\boxed{s = \frac{2\pi^2}{45} g_{*s} T^3}$$

$$S = \frac{4\pi}{3}R^3 \frac{2\pi^2}{45} g_{*s} T^3 \sim 10^{89}$$

$R_0 \sim 4\cdot 10^{26}\,\mathrm{m} \quad T \approx 2{,}73\,\mathrm{K}$

$\underline{dS \ll S}$

$\Downarrow$

$g_{*s} R^3 T^3 = \text{áll} \Leftarrow$ Univ. entrópiája váll.

most

ha $g_{*s} = \text{áll} \Rightarrow T \sim R^{-1}$

(korai univ.-ban nem!)

pl:

$T_1 = 10\,\mathrm{MeV} \quad \gamma, e^\pm, \nu_e$

$T = 0{,}5\,\mathrm{MeV} \quad T_\nu \sim R^{-1} \quad kT \neq 0{,}5\,\mathrm{MeV} \quad e^+e^- \to 2\gamma$

$T < 0{,}1\,\mathrm{MeV}$

$$g_{*s,\gamma e} T_1^3 R_1^3 + g_{*s,\nu} T_1^3 R_1^3 \quad \text{[illegible]} \; =$$

$$g_{*s,\gamma} T^3 R^3 + g_{*s,\nu} T_\nu^3 R^3$$

$T_\nu R = T_1 R_1$

$$g_{*s,\gamma e} T_1^3 R_1^3 = g_{*s,\gamma} T^3 R^3$$

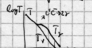
$2 + \frac{7}{8}\cdot 2\cdot 2 = \frac{11}{2} \qquad 2$

$$\frac{11}{2} T_1^3 R_1^3 = 2 R^3 T^3$$

$$\frac{11}{4} T_\nu^3 R^3 = R^3 T^3$$

$$\boxed{\frac{T_\nu}{T} = \left(\frac{4}{11}\right)^{1/3}}$$

$$T_\nu^{\text{ma}} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\text{ma}} = \underline{1{,}94\,\mathrm{K}} \qquad (T_\gamma^{\text{ma}} = 2{,}73\,\mathrm{K})$$



$$\underline{g_*}(\text{ma}) = 2 + \frac{7}{8}\,6\left(\frac{T_\nu}{T_\gamma}\right)^4 = 3{,}36 \qquad \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

$$n_{\nu_e}(\text{ma}) = \frac{6}{4\pi^2}\zeta(3) T_\nu^3 = 112\,\frac{1}{\mathrm{cm}^3}$$

$$\underline{g_{*s}}(\text{ma}) = 2 + \frac{7}{8}\cdot 6\left(\frac{T_\nu}{T_\gamma}\right)^3 = 3{,}909$$

$$s(\text{ma}) = \frac{2\pi^2}{45} g_{*s} T^3 = 2897\,\frac{1}{\mathrm{cm}^3}$$

$$n_\gamma(\text{ma}) = \frac{2}{\pi^2}\zeta(3) T^3 = 411\,\frac{1}{\mathrm{cm}^3}$$

$$S = \frac{4\pi}{3}R^3 s = 7{,}7\cdot 10^{89}$$

II A standard univerzum kozmológiája II.

Univ. homogén, izotróp $\Rightarrow$ Robertson–Walker metrika

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Theta^2 + r^2\sin^2\Theta\, d\phi^2\right]$$

$k=0$ sík
$k=1$ zárt
$k=-1$ nyílt

Friedmann:

$$\boxed{\begin{aligned}\left(\frac{\dot R}{R}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{R^2} + \frac{\Lambda}{3}\\ \frac{\ddot R}{R} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}\end{aligned}}$$

$H(t) \equiv \frac{\dot R}{R}$ Hubble-paraméter

$k=0$
$\Lambda=0$

$$\left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G}{3}\rho \;\Big/\; \frac{d}{dt}$$

$$2\frac{\dot R}{R}\left(\frac{\dot R}{R}\right)^{\cdot} = \frac{8\pi G}{3}\dot\rho$$

$$2\frac{\dot R}{R}\left(\frac{\ddot R}{R} - \frac{\dot R^2}{R^2}\right) = \frac{8\pi G}{3}\dot\rho$$

$$\dot\rho = \frac{3}{4\pi G}\frac{\dot R}{R}\left(-\frac{4\pi G}{3}(\rho+3p) - \underbrace{\frac{8\pi G}{3}\rho}_{-2\cdot\frac{4\pi G}{3}\rho}\right) = -\frac{3}{4\pi G}\frac{\dot R}{R}\frac{4\pi G}{3}(\rho+3p+2\rho) =$$

$$\underline{\dot\rho = -3\frac{\dot R}{R}(\rho + p)} \qquad (k=0,\ \Lambda=0)$$

Sugárzás dominálta univerzum $\quad p = \frac{1}{3}\rho$

$$\dot\rho = -3\frac{\dot R}{R}\frac{4}{3}\rho \Rightarrow \dot\rho R + 4\rho\dot R = 0 \rightarrow \frac{1}{R^3}\left(\dot\rho R^4 + 4\rho R^3\dot R\right) = 0 \rightarrow \frac{1}{R^3}\left(\rho\cdot R^4\right)^{\cdot} = 0$$

$\rho R^4 =$ áll. $\qquad \rho_r \sim R^{-4}$ sugárzásra

Anyag dominálta univerzum $\quad p = 0$

$$\dot\rho = -3\frac{\dot R}{R}\rho \qquad \frac{1}{R^2}\left(\dot\rho R^3 + 3\rho R^2\dot R\right) = 0 = \frac{1}{R^2}\left(\rho R^3\right)^{\cdot} \qquad \rho R^3 = \text{áll.}$$

$\rho_m \sim R^{-3}$ anyagra

$$H^2(t) = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \qquad H_0 = h\cdot 100\,\frac{\text{km/s}}{\text{Mpc}} \qquad h = 0{,}72\pm..$$

$$\boxed{d_H = \underset{=}{R_0}\cdot R(t)} \qquad \frac{R_{ma}}{R(t)} \equiv z(t) + 1 \qquad z(t_{ma}) = 0$$

$R_{ma} = 1$

$$\frac{\lambda(t_{ma}) - \lambda(t)}{\lambda(t)} = z(t)$$
$$\Downarrow$$
$$\frac{R_{ma}}{R(t)} = \frac{\lambda_{ma}}{\lambda(t)}$$

Sugárzás dom.

$\rho_m \sim R^{-4}$

$\left(\frac{\dot R}{R}\right)^2 \sim R^{-4}$

$\frac{\dot R}{R} \sim R^{-2}$

$\dot R \sim R^{-1}$

$R\,dR \sim dt$

$R^2 \sim t$

$\boxed{R \sim t^{1/2}}$

anyag dom.

$\rho_m \sim R^{-3}$

$\left(\frac{\dot R}{R}\right)^2 \sim R^{-3}$

$\frac{\dot R}{R} \sim R^{-3/2}$

$\sqrt{R}\,dR \sim dt$

$R^{3/2} \sim t$

$\boxed{R \sim t^{2/3}}$

kozm. áll.

$\rho \sim$ const

$\left(\frac{\dot R}{R}\right)^2 \sim c$

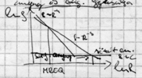
$\frac{\dot R}{R} \sim c$

$\frac{dR}{R} \sim c\,dt$

$\ln R \sim ct$

$\boxed{R \sim e^{ct}}$

anyag és sug. egyensúlya



Horizonttávolság FRW metrika (dt), $d\varphi, d\Theta = 0$

$$d_H = \int_0^t ds = R(t)\int_0^{R_0}\frac{dr}{\sqrt{1-kr^2}} = R(t)\int_0^t\frac{dt'}{R(t')}$$

$ds = 0$ fotonra $\Rightarrow \frac{dr}{\sqrt{1-kr^2}} = \frac{dt}{R(t)}$

Sug: $R \sim t^{1/2}$ $\quad d_H(t) = \sqrt{t}\int_0^t \frac{dt'}{\sqrt{t'}} = 2t$

$\left[2\sqrt{t'}\right]_0^t = 2\sqrt{t}$

anyag: $R \sim t^{2/3}$ $\quad d_H(t) = t^{2/3}\int_0^t \frac{dt'}{t'^{2/3}} = 3t$

$\left[3(t')^{1/3}\right]_0^t = 3t^{1/3}$

Egészen anyag $\underline{d_H(ma)} = 3\cdot(c)\cdot t_{ma} = 3\cdot 13{,}7\cdot 10^9\,\text{év}\cdot c = 4\cdot 10^{26}\,m$

$(H^2 =)$ $\quad \varrho_{krit}(t) = \frac{3H^2(t)}{8\pi G}$ kritikus sűrűség $(k=0)$

$\varrho_{krit}(ma) = \frac{3H_0^2}{8\pi G} = \frac{3h^2\left(100\frac{km/s}{Mpc}\right)^2}{8\pi\cdot 6{,}67\cdot 10^{-8}\frac{cm^3}{g\,s^2}} \approx 1{,}88\cdot 10^{-29}\,h^2\,\frac{g}{cm^3}$ $\quad h = 0{,}72$

sűrűségparaméter $\Omega \equiv \frac{\varrho}{\varrho_{krit}}$ $\quad \Omega_i = \frac{\varrho_i}{\varrho_{krit}}$ $\quad i$ = barion, Λ, m, r ...

$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1$

MREQ anyag-sugárzás egyenlőség $\quad \frac{\varrho_m}{\varrho_r} \sim R$ $\quad \frac{\left(\frac{\varrho_m}{\varrho_r}\right)_{ma}}{\underbrace{\left(\frac{\varrho_m}{\varrho_r}\right)_{MREQ}}_{=1}} = \frac{R_{ma}}{R_{MREQ}} = z_{MREQ}+1$

$\varrho_r(ma) = \frac{\pi^2}{30}\underbrace{g_*(ma)}_{\sim 3{,}36}T_{ma}^4 = \frac{\pi^2}{30}\cdot 3{,}36\cdot(2{,}725K)^4 =$

$= 7{,}8\cdot 10^{-34}\frac{g}{cm^3}$ $\quad \varrho_m(ma) = \Omega_m(ma)\cdot\varrho_{krit}(ma) \approx 0{,}3\cdot 1{,}88\cdot 10^{-29}(0{,}72)^2\frac{g}{cm^3} =$

exp WMAP: 0,3 $\quad = 2{,}92\cdot 10^{-30}\frac{g}{cm^3}$

$z_{EQ}+1 = \frac{\varrho_m(ma)}{\varrho_r(ma)} \approx 3850$ $\quad T \sim R^{-1}$ $\quad +3100s$

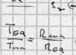
$\frac{T_{EQ}}{T_{ma}} = \frac{R_{ma}}{R_{EQ}} = z_{EQ}+1 = 3850$ $\qquad T_{EQ} = \underbrace{T_{ma}}_{\frac{2{,}725}{11600}\,eV}\cdot 3850 \simeq 0{,}9\,eV$

$R \sim t^{2/3}$ $\quad \underbrace{\frac{R_{ma}}{R_{EQ}}}_{z_{EQ}+1} = \frac{t_{ma}^{2/3}}{t_{EQ}^{2/3}} \Rightarrow t_{EQ} = \frac{t_{ma}}{(z_{EQ}+1)^{3/2}} = \frac{13{,}7\cdot 10^9\,\text{év}}{(3850)^{3/2}} \approx$

≈ 58600 év

Rekombináció KMHS (CMBR) kialakulása

$e^- + p \rightleftharpoons H + \gamma$ $\qquad$ Surface of last scattering (SLS)



H ionizációs en. $B_H = 13{,}6\,eV$

$n_e = n_p^* \approx n_p$ $\qquad n_p + n_H = n_B^* \approx n_B$

$(p + {}^4He + \ldots)$ $\qquad$ (sok p) $\quad$ (${}^4He$ is)

Kém. egyensúly: $T_e = T_p = T_H = T$, $\quad \mu_e + \mu_p = \mu_H$

$$\left\{\begin{array}{l} n_e = 2\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{m_e-\mu_e}{T}} \\ n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-\frac{m_p-\mu_p}{T}} \\ n_H = 4\left(\frac{m_H T}{2\pi}\right)^{3/2} e^{-\frac{m_H-\mu_H}{T}} \end{array}\right. \rightarrow \begin{array}{l} e^{\frac{\mu_e}{T}} = n_e\frac{1}{2}\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{m_e}{T}} \\ e^{\frac{\mu_p}{T}} = n_p\frac{1}{2}\left(\frac{2\pi}{m_p T}\right)^{3/2} e^{\frac{m_p}{T}} \end{array}$$

$m_p \approx m_H$ (kiv. exp)

$$n_H = 4\left(\frac{m_H T}{2\pi}\right)^{3/2} e^{-\frac{m_H}{T}}\cdot n_e n_p\frac{1}{4}\left(\frac{2\pi}{m_e T}\right)^{3/2}\left(\frac{2\pi}{m_p T}\right)^{3/2} e^{\frac{m_e+m_p}{T}} =$$

$$= n_p^2\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{m_p+m_e-m_H}{T}} = \boxed{m_p+m_e-m_H = B_H} = n_p^2\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}$$

(II) A korai univ. termodin. (II)

Rekombináció

Anarcionális ionizáció: $X \equiv \frac{n_p}{n_p + n_H}$

$$\frac{n_H}{n_p^2} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}$$

$X \to 0$ semleges $\quad n_H \gg n_p$

$X \to 1$ ionizált $\quad n_H \to 0$

$X \approx \frac{n_p}{n_B}$

$$\frac{n_H}{n_p^2} = \frac{n_B - n_p}{n_p^2} = \frac{1 - \frac{n_p}{n_B}}{\frac{n_p^2}{n_B}} = \frac{1-X}{X^2 n_B}$$

$\eta \equiv \frac{n_B}{n_\gamma} \qquad \eta(\text{ma}) \approx 6 \cdot 10^{-10}$

$$\frac{1-X}{X^2} = n_B \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T} = \eta \cdot \underbrace{\frac{2}{\pi^2} \zeta(3) T^3}_{n_\gamma} \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}$$

Pl.: $X = 0{,}1$

$$\frac{1-X}{X^2} = \frac{0{,}9}{0{,}01} \approx 90 \qquad \overset{\text{nagy}}{90} = \underset{\text{kicsi}}{\eta}\ \underbrace{\frac{2}{\pi^2} \zeta(3) T^3 \left(\frac{2\pi}{m_e T}\right)^{3/2}}_{\sim O(1)} \underset{\text{nagy számnak kell lennie}}{e^{B_H/T}}$$

$B_H \gg T$

$$90 \approx 6 \cdot 10^{-10} \cdot \frac{2}{\pi^2} \cdot 1{,}202 \cdot T^3 \left(\frac{2\pi}{511 \text{keV}}\right)^{3/2} T^{-3/2} e^{B_H/T}$$

$$T^{3/2} e^{B_H/T} = 90 \cdot \frac{\pi^2}{2\eta} \zeta(3) \left(\frac{m_e}{2\pi}\right)^{3/2} \simeq 8{,}3 \cdot 10^{18}$$

$[T, m_e, B_H] = \text{eV}$

$(T \sim 10 \text{eV})$

$$\boxed{\frac{3}{2} \ln T + \frac{13{,}6}{T} = 43{,}56}$$

$$\left(T = \frac{13{,}6}{-\frac{3}{2}\ln T_0 + 43{,}56}\right)$$

iterációval:

$T = 1 \text{eV} \to T = \frac{13{,}6}{43{,}56 - \ln 1} \approx 0{,}31$

$T = 0{,}31 \text{eV} \to T = \frac{13{,}6}{43{,}56 - \frac{3}{2}\ln 0{,}31} \approx 0{,}3$

$\ldots \boxed{T = 0{,}299 \text{eV}} \ldots$

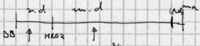
Rekombináció:

$$\boxed{T_{rek} = 0{,}26 \text{eV}}$$

$$t_{rek} = \frac{t_{ma}}{(z_{rek}+1)^{3/2}} \qquad \frac{R_{ma}}{R_{rek}} = \frac{T_{rek}}{T_{ma}} = \frac{0{,}26}{\left(\frac{2{,}725}{11600}\right)} \sim 1100$$

$\frac{R_{ma}}{R_{rek}} = z+1$

$$t_{rek} = \frac{13{,}6 \cdot 10^9 \text{év}}{(1100)^{3/2}} \approx 383000 \text{ év}$$



$T = 1 \text{MeV}$

$$\frac{t_{1MeV}}{t_{ma}} = \frac{T_{ma}^2}{(1 \text{MeV})^2} = \frac{(0{,}9 \text{eV})^2}{(1 \text{MeV})^2} \approx 0{,}81 \cdot 10^{-12}$$

$$t_{1MeV} = 58600 \cdot 365 \cdot 86400 \cdot 0{,}81 \cdot 10^{-12} \approx 1{,}5 \text{s}$$

$t > 1000 \text{s}$

$R \sim t^{1/2}$

$T \sim R^{-1}$

$t^{1/2} \sim T^{-1}$

$t \sim T^{-2}$

$R \sim t^{2/3}$

$T \sim R^{-1}$

$t = \frac{t_{ma}}{(z_*+1)^{3/2}}$

$z = 1 \to 5 \cdot 10^9 \text{év}$

$z = 5 \to 1 \cdot 10^9 \text{év}$

(Univ.) kozmológia időskálája

	t	T	
Planck	$t_P \approx 10^{-44}$ s	10^{19} GeV	~ kvantumgravitációs QG
Infláció	10^{-34} s	10^{14} GeV	~ nagy egyesítés GUT
Elektrogyenge	10^{-10} s	10^{2} GeV	~ EW (feltevés)
QGP → hadron	25 μs	200 MeV	~ (mérhető)
	1 s	1 MeV	
WFO	1,15 s	0,8 MeV	– gyenge lefagyás
BBN	100 s (269 s)	0,1 MeV (0,07 MeV)	nukleoszintézis (vége)
MREQ	58600 év	0,9 eV	~ anyag-sug. egyensúly
Rekombináció	385 000 év	0,26 eV	
ma	$13{,}7 \cdot 10^{9}$ év	$\sim 10^{-4}$ eV	$\frac{2{,}725}{11600}$ eV

III Inflációs kozmológia

Nagy Bumm problémái:

① Simasági probléma

$\Lambda=0 \qquad H^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2} \qquad -\frac{k}{R^2} = H^2\Big(1 - \underbrace{\frac{8\pi G}{3H^2}}_{\rho_{krit}^{-1}}\rho\Big) = H^2(1-\Omega)$

$(1-\Omega)_t = \frac{-k}{H^2R^2}$

matterdom.: $R \sim t^{2/3} \quad \dot R \sim \frac{2}{3}t^{-1/3} \quad H = \frac{2}{3}t^{-1}$

rad. dom.: $R \sim t^{1/2} \quad \dot R \sim \frac{1}{2}t^{-1/2} \quad H = \frac{1}{2}t^{-1}$

$\Big\} \; H \sim t^{-1}$

matterd.: $(\Omega-1)_t \sim t^{2/3} \quad (t^{-2}\cdot t^{4/3} = t^{-2/3})$

$$(\Omega-1)_{rek} = (\Omega-1)_{ma}\cdot\left(\frac{t_{rek}}{t_{ma}}\right)^{2/3} = (\Omega-1)_{ma}\left(\frac{383\,000}{13{,}7\cdot10^9}\right)^{2/3} \simeq 10^{-3}$$

rad. d.: $(\Omega-1)_t \sim t \quad (t^{-2}\cdot t = t^{-1})$

$$(\Omega-1)_t = (\Omega-1)_{EQ}\underbrace{\frac{t}{t_{EQ}}}_{\frac{1}{(z_t^*+1)^2}\sim\left(\frac{R_{EQ}}{R}\right)^{-2}} = (\Omega-1)_{ma}\frac{1}{z_{EQ}+1}\cdot\left(\frac{z_{EQ}+1}{z_t+1}\right)^2 = (\Omega-1)_{ma}\frac{z_{EQ}+1}{(z_t+1)^2}$$

$z_t+1 = (z_{EQ}+1)(z_t^*+1) \qquad \frac{R_0}{R} = \frac{R_0}{R_{EQ}}\cdot\frac{R_{EQ}}{R}$

Pl: BBN $\quad z_{BBN}+1 = 4\cdot10^8 \qquad (\Omega-1)_{BBN} = (\Omega-1)_{ma}\cdot 2{,}4\cdot10^{-14}$

miért kell ilyen nagyon közel, Ω-nak nagyon pontosabban 1-nek kell lennie... finomhangolás...

② Horizont probléma

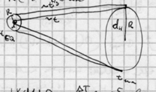
$d_H = 4\cdot10^{26}\,m = R_{ma}\cdot R_0)$

$d_H \sim t$

$R \sim t^{2/3}$ (m.d)

$$\frac{d_H}{R} = \frac{d_H(t)}{d_H(ma)}\cdot\underbrace{\frac{d_H(ma)}{R(ma)}}_{=1}\cdot\frac{R(ma)}{R(t)} = \frac{t}{t_{ma}}\cdot\frac{t_{ma}^{2/3}}{t^{2/3}} = \left(\frac{t}{t_{ma}}\right)^{1/3}$$

$$\frac{d_H(EQ)}{R(EQ)} = \left(\frac{t_{EQ}}{t_{ma}}\right)^{1/3} = \frac{1}{(z_{EQ}+1)^{1/2}} = \frac{1}{\sqrt{1100}} \simeq \frac{1}{33}$$



A jelenlegi horizontban lévő térfogaton belül 33000 irányban kauzálisan szétcsatolt tartomány van, de ezek mégis ugyanolyan hőmérsékletűek!

WMAP: $\frac{\Delta T}{T} \simeq 10^{-5}$ legnagyobb hőmérsékletkülönbség, (mindössze Gauss-eloszlást követ)

③ Mágneses monopólusok hiánya...

Klasszikus skalártér   Alan Guth

$k=0 \quad \Lambda=0 \qquad \phi \quad V(\phi) \qquad ds^2=dt^2-R^2(t)\,dl^2$

$$g_{\mu\nu}=\begin{pmatrix}1&&&0\\&-R^2&&\\&&-R^2&\\0&&&-R^2\end{pmatrix} \qquad g^{\mu\nu}=\begin{pmatrix}1&&&0\\&-\frac{1}{R^2}&&\\&&-\frac{1}{R^2}&\\0&&&-\frac{1}{R^2}\end{pmatrix} \qquad g=\det g_{\mu\nu} \quad \sqrt{-g}=R^3$$

$$\mathcal{L}_0=\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi) \qquad S=\int d^4x\,\underbrace{\sqrt{-g}\,\mathcal{L}_0}_{R^3\mathcal{L}_0=\mathcal{L}}$$

Euler-Lagrange egy.

$$\left(\cancel{\frac{d}{dt}}\right) \qquad \boxed{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)-\frac{\partial\mathcal{L}}{\partial\phi}=0}$$

$$\mathcal{L}_0=\frac{1}{2}\dot\phi^2-\frac{1}{2R^2}(\vec\nabla\phi)^2-V(\phi) \qquad \mathcal{L}=R^3\left(\frac{1}{2}\dot\phi^2-\frac{1}{2R^2}(\vec\nabla\phi)^2-V(\phi)\right)$$

$$\frac{\partial\mathcal{L}}{\partial\phi}=-R^3V' \qquad \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}=\begin{pmatrix}R^3\dot\phi\\-R\vec\nabla\phi\end{pmatrix} \qquad \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)=R^3\ddot\phi+3R^2\dot R\dot\phi-R\Delta\phi$$

$$\text{E-L}: \quad R^3\ddot\phi+3R^2\dot R\dot\phi-R\Delta\phi+R^3V'=0 \quad /\cdot R^{-3}$$

$$\ddot\phi+3\frac{\dot R}{R}\dot\phi-\frac{\Delta\phi}{R^2}+V'=0$$

Ha ϕ homogén $\Delta\phi=0$

$$\boxed{\ddot\phi+3H\dot\phi=-V'} \qquad \left(H^2=\frac{8\pi G}{3}\rho \quad m_{pl}^*=\frac{1}{\sqrt{8\pi G}}\right)$$

Hubble-drag (fékezés) tágulás miatt

$$\boxed{H^2=\frac{1}{3m_{pl}^{*2}}\rho} \qquad \rho\to T^{00} \qquad T^{\mu\nu}=\partial^\mu\phi\,\partial^\nu\phi-g^{\mu\nu}L=$$

$$=(p+\rho)u^\mu u^\nu-g^{\mu\nu}p$$

együttmozgó r. $\quad u^\mu=(1,0,0,0)$

$$T^{00}=\rho$$

$$p=\frac{R^2}{3}(T^{11}+T^{22}+T^{33})$$

(hom.)

$$\rho=\dot\phi^2-\left(\frac{1}{2}\dot\phi^2-\frac{1}{2R^2}\cancel{(\vec\nabla\phi)^2}-V\right)=\frac{1}{2}\dot\phi^2+V$$

$$p=\frac{R^2}{3}\left(\frac{1}{R^4}(\vec\nabla\phi)^2+\frac{3}{R^2}\left(\frac{1}{2}\dot\phi^2-\frac{1}{2R^2}(\vec\nabla\phi)^2-V\right)\right)=$$

$$=\frac{R^2}{3}\left(\frac{3}{2R^2}\dot\phi^2-\frac{1}{2R^4}\cancel{(\vec\nabla\phi)^2}-\frac{3}{R^2}V\right)=\frac{1}{2}\dot\phi^2-V$$

(hom.)

$$\boxed{\begin{aligned}H^2&=\frac{1}{3m_{pl}^2}\left(\frac{1}{2}\dot\phi^2+V(\phi)\right) &(1)\\ \ddot\phi+3H\dot\phi&=-V'(\phi) &(2)\end{aligned}}$$

$$\frac{d}{dt}(1) \Rightarrow 2H\dot H=\frac{1}{3m_{pl}^2}\left(\dot\phi\underset{(2)}{\ddot\phi}+V'(\phi)\dot\phi\right)=\frac{1}{3m_{pl}^2}\left(\dot\phi(-3H\dot\phi-V')+V'\dot\phi\right)=$$

$$=\frac{1}{3m_{pl}^2}\left(-3H\dot\phi^2\right)$$

$$\dot H=-\frac{1}{2m_{pl}^2}\dot\phi^2$$

tfh. választható monoton miatt.

$$\frac{\dot H}{\dot\phi}\approx H'$$

H és ϕ t-ben monoton nő

$$H'=-\frac{1}{2m_{pl}^2}\dot\phi \Rightarrow \dot H=-\frac{1}{2m_{pl}^2}\left(-2m_{pl}^2H'\right)^2=-2m_{pl}^2H'^2$$

III Inflációs kozmológia

(1) $H^2 = \frac{1}{3m_{pl}^2}\left(\frac{1}{2}\dot\phi^2 + V(\phi)\right)$

(2) $\ddot\phi + 3H\dot\phi = -V'$

(3) $\dot H = -\frac{1}{2m_{pl}^2}\dot\phi^2$

(4) $\dot\phi = -2m_{pl}^2 H'$

(5) $\dot H = -2m_{pl}^2 H'^2$

$(4) \Rightarrow H'' = -\frac{1}{2m_{pl}^2}\frac{d\dot\phi}{dt}\frac{dt}{d\phi} = -\frac{1}{2m_{pl}^2}\frac{\ddot\phi}{\dot\phi}$

$$\varepsilon \equiv 2m_{pl}^2\left(\frac{H'}{H}\right)^2 \overset{(4)(1)}{=} 2m_{pl}^2\frac{\frac{1}{4m_{pl}^4}\dot\phi^2}{\frac{1}{3m_{pl}^2}\left(\frac{1}{2}\dot\phi^2+V\right)} = 3\frac{\frac{1}{2}\dot\phi^2}{\frac{1}{2}\dot\phi^2+V}$$

$\varepsilon \ll 1 \Rightarrow \frac{1}{2}\dot\phi^2 \ll V$

$$\eta \equiv 2m_{pl}^2\frac{H''}{H} = -\frac{\ddot\phi}{H\dot\phi}$$

$\eta \ll 1 \Rightarrow |\ddot\phi| \ll |H\dot\phi|$

Slow roll $\varepsilon, \eta \ll 1$

$H^2 = \frac{1}{3m_{pl}^2}V(\phi)$ (a)

(b) $(a)' = 2HH' = \frac{1}{3m_{pl}^2}V'(\phi)$

$3H\dot\phi = -V'$

(c) $(b)' = 2H'^2 + 2HH'' = \frac{1}{3m_{pl}^2}V''(\phi)$

$\frac{(b)}{(a)} = 2\frac{H'}{H} = \frac{V'}{V} \Rightarrow \varepsilon = 2m_{pl}^2\left(\frac{V'}{2V}\right)^2 = \boxed{m_{pl}^2\frac{V'^2}{2V^2} \ll 1}$

$\frac{(c)}{(a)}$: $2\frac{H''}{H} + 2\frac{H'^2}{H^2} = \frac{V''}{V}$

$$\underbrace{2m_{pl}^2\frac{H''}{H}}_{\eta} = m_{pl}^2\frac{V''}{V} - \underbrace{2m_{pl}^2\frac{H'^2}{H^2}}_{\varepsilon}$$

$$\eta + \varepsilon = \boxed{m_{pl}^2\frac{V''}{V} \ll 1}$$

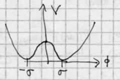
ez a két feltétel a pot.-ra.

GUT inf.

elektrogyenge és erős egyesülése 10^{14-15} GeV körül

spontán szim. sértés

$V = \frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$



$M^2_{GUT} = V''(\sigma)$

$V' = \lambda\phi(\phi^2 - \sigma^2)$

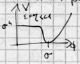
$M_{GUT} \sim \sigma \sim 10^{14-15}$ GeV ($m_{pl} \sim 10^{19}$ GeV)

$V'' = 3\lambda\phi^2 - \lambda\sigma^2$ $V''(\sigma) = 2\lambda\sigma^2 \sim \sigma^2$

$\varepsilon \approx \frac{m_{pl}^2\phi^2}{\sigma^4}$ ha $\phi \ll \sigma$ $\varepsilon \ll 1$ ✓ jó

$\varepsilon + \eta \approx \frac{m_{pl}^2}{\sigma^2} > 1$ nem jó slow rollra

módosított pot.



minden derivált $\approx 0 \Rightarrow \varepsilon + \eta \ll 1$

$\dot V = V'\dot\phi = -3H\dot\phi^2$ $\varepsilon \ll 1 \Rightarrow \frac{1}{2}\dot\phi^2 \ll V$

$\dot V$ kicsi! $|-3H\dot\phi^2| \ll |HV|$ $H^2 \sim V$

$H^2 = \left(\frac{\dot R}{R}\right)^2 \approx \text{áll}$

legyen $H\Delta t = 100$

$R_{\text{infl. vége}} = R_{\text{infl. kezdete}} \cdot \underbrace{e^{100}}_{\sim 10^{43}}$

$\frac{dR}{R} \approx \text{const}\, dt$

$\ln R \approx c\Delta t \Rightarrow \boxed{R = R_0 e^{H\Delta t}}$ $\quad H^2 \approx \frac{V}{m_{pl}^2} \approx \frac{m_{GUT}^4}{m_{pl}^2} = \frac{(10^{14-15}\,GeV)^4}{(10^{19}\,GeV)^2} \approx 10^{20}\,GeV^2$ $\quad 10^{68}\,\frac{1}{s^2}$

$H = 10^{34}\,1/s \qquad H \sim t^{-1}$ $\quad$ Univ. életkora: $T \approx H^{-1} \quad T = 10^{-34}\,s$

Pl: $H\Delta t = 100 \Rightarrow \Delta t \approx 10^{-32}\,s$

$$\left.\begin{aligned} t_{\text{tágulás kezdete}} &= 10^{-34}\,s \\ t_{\text{infláció vége}} &= 10^{-32}\,s \\ H &= 10^{34}\,1/s \end{aligned}\right\} R_{iv} = R_{ik} \cdot 10^{43}$$

$\Delta V \ll 3\underbrace{H\Delta t}_{100} V$

ez alatt a pot. tényleg nem változott.

① Simasági probléma megold.

$(\Omega - 1)_t = \frac{k}{R^2 H^2}$ minél simábban nyugszik, Ω annál pontosabban 1...

$(\Omega-1)_{i.v.} = \frac{k}{R_{iv}^2 H^2} = \frac{k}{R_{ik}^2 H^2}\left(\frac{R_{ik}}{R_{iv}}\right)^2 = (\Omega-1)_{i.k.} \underbrace{\left(\frac{R_{ik}}{R_{iv}}\right)^2}_{\sim (10^{-43})^2 \sim 10^{-86}}$

$(\Omega-1)_{ik}$ bármi lehet, mert $(\Omega-1)_{iv}$ ennek 10^{-86} szorosa lesz

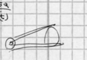
② Horizont probléma

$\frac{d_H(t_{REK})}{R(t_{REK})} = \frac{1}{\sqrt{1100}} \qquad \frac{d_H(t)}{R(t)} = \frac{1}{\sqrt{z_t + 1}}$

$k\ 10^5$
$M\ 10^6$
$G\ 10^9$

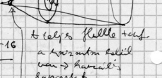
$\frac{R(iv)}{d_H(iv)} = \frac{R(iv)}{R(ma)} \frac{R(ma)}{d_H(ma)} \frac{d_H(ma)}{d_H(iv)} \simeq 10^{25}$ $\qquad z_t + 1 = (z_{ea}+1)(z^*+1)$ $\quad z^* + 1 = \frac{R_{ea}}{R(t)}$

$\frac{1}{z_{iv}+1} \approx \frac{1}{4{,}2 \cdot 10^{25}} \qquad \frac{t_{ma}}{t_{iv}} = \frac{13{,}7 \cdot 10^9\,\text{év}}{10^{-32}\,s} \approx 4{,}28 \cdot 10^{50}$



$z_{iv} + 1 = 3850 \cdot \frac{R_{ea}}{R_{iv}} = 3850 \cdot 1{,}1 \cdot 10^{22} = 4{,}2 \cdot 10^{25}$

$\frac{R_{ea}}{R_{iv}} \approx \frac{T_{iv}}{T_{ea}} \simeq \frac{10^{13}\,GeV}{0{,}9\,eV} \simeq 1{,}1 \cdot 10^{22}$



$\frac{R_{ik}}{d_{Hik}} = \frac{10^{-43} R_{iv}}{10^{-2} d_{Hiv}} \simeq 10^{-41} \overbrace{\frac{R_{iv}}{d_{Hiv}}}^{\sim 10^{25}} \simeq 10^{-16}$

A teljes Hubble tart. a horizonton belül van → hozzáis kapcsolat

④ $\underset{\frac{1}{H} \cdot d_H}{(H\Delta t \approx 100)}$ CMBR fluktuációt gaussosak → infl.-val kapcsolat, infláton kvantum fluktuációk segítségével az Univ.

IV Bariogenezis

anyag - antianyag aszimmetria

A belátható univ.-ból $10^{22} M_\odot$-nyi anyagot látunk, antianyagot nem. megoldások?

ⓐ Barion aszimm. $\bar B, B$ térben szétvált

ⓑ Barion aszimmetria a kezdetektől

ⓒ kezdetben szimm → mai időig aszimm. → aszimmetria

dinamikus szimm. sértés

Az univ. tágulásával bizonyos reakciók leállhat, (annihilációs ritkává lesz) a részecskék kifagynak (kiesnek a term. egyensúlyból)

$\Gamma_{ann.} > H$ addig egyensúly $B+\bar B \to \gamma$

$\Gamma < H$ leáll a folyamat (kifagyás), beáll valamilyen egyensúly

$$X := \frac{m}{T} \qquad Y := \frac{n}{s} \qquad Y = \frac{g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}}{\frac{2\pi^2}{45} g_{*s} T^3} \sim \left(\frac{m}{T}\right)^{3/2} e^{-\frac{m}{T}}$$

(nő időben) együttmozgó térfogatban részecske gyakoriság

$$x^{3/2} e^{-x}$$

PL: ma proton

$$e^{-\frac{10^3\,\mathrm{MeV}}{\frac{1}{11600}\,\mathrm{eV}}} \approx e^{-10^{13}}$$

ma $Y=0$ lenne, ha egyensúlyban maradnak

A gyakoriság változása ($O \approx$ ex-ben)

Boltzmann-egyenlet

$$\Gamma_A = n_{EQ}\langle\sigma_A v\rangle$$

$$\langle\sigma_A v\rangle = \sum_{X,\bar X}\langle\sigma_{A\bar A\to X\bar X} v\rangle$$

$$\frac{x}{Y_{EQ}}\frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{EQ}}\right)^2 - 1\right]$$

x_F freezeout   hideg részecske $(x \gg 1)$

$$\langle\sigma_A v\rangle = \sigma_0\left(\frac{T}{m}\right)^n \qquad \frac{1}{x^n}$$

$n=0$ s-hullám annihiláció
$=1$ p-hullám annihiláció

ⓐ Barion aszimm. esetén $\boxed{n_B = n_{\bar B}}$ $p+\bar p$ $\langle\sigma_A v\rangle = c_1 m_\pi^{-2}$ $m_\pi \approx 135$ MeV, $c_1 \approx 1$

freezeout Boltzmann egyenletből

$$x_F = \underbrace{\ln\left(0{,}038(n+1)\frac{g}{\sqrt{g_*}}\, m_{Pl}\cdot m\cdot\sigma_0\right)}_{a} - (n+1/2)\ln a$$

MeV-ben

$$Y_\infty = \frac{3{,}79\,(n+1)\,x_f^{n+1}}{\left(\frac{g_{*s}}{\sqrt{g_*}}\right) m_{pl}\, m\, \sigma_0}$$

gyakoriság freezeout után

$x < x_f$: $Y \sim Y_{EQ}$

$x = x_f$ $Y = Y_{EQ}(x_f)$

$x > x_f$ $Y \to Y_\infty$

pl.: $p+\bar{p}$ hadron szint.

$n=0$ $c_1 \approx 1$

$x_f \simeq \underline{42}(+\overbrace{\ln c_1}^{\approx 0}) \Rightarrow T_f \simeq 22\,\mathrm{MeV}$

$Y_\infty \simeq 7\cdot 10^{-20} \left(c_1^{-1}\right)$

mért adatok $Y_{mért} = \frac{n_B}{s} = \frac{n_\gamma \eta}{s} = \frac{411\ 1/cm^3}{2897\ 1/cm^3}\eta \simeq \frac{\eta}{7} \simeq$

$\simeq 10^{-10}$ $\eta_{mért} \simeq 6\cdot 10^{-10}$ $\eta = \frac{n_B}{n_\gamma}$

problémák: - ugyanannyi p mint $\bar{p}$

- 10 nagyságrendnyi eltérés a gyakoriságban!

(b) Barion aszimmetrikus eset

$n_B \gg n_{\bar{B}} \Rightarrow Y_{\bar{B}} \approx 0$ $Y_\infty(\bar{B}) = 10^{18} e^{-9\cdot 10^5}$

nagyon kicsi

$Y_\infty \to n_{\bar{B}} = Y_\infty s_{ma}$

$n_{\bar{B}} \cdot \underbrace{\frac{4\pi}{3}(d_H(ma))^3}_{V \approx 4\cdot 10^{26}} = 7{,}5\cdot 10^{107} e^{-9\cdot 10^5}$ $\ln(n_{\bar{B}}V) = 248 - 9\cdot 10^5$

$Y^{mért} \approx 10^{-10}$

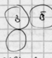
a végén nem maradt antibarion $Y_\infty \simeq 10^{-10}$ $x_F = ?$

$$Y_\infty = \frac{3{,}79\,(n+1)\,x_F^{n+1}}{\left(\frac{g_{*s}}{\sqrt{g_*}}\right) m_{pl}\, m\, \sigma_0} \cdot n=0 \quad g_* = g_{*s} = 10{,}75$$

$x_F \simeq 24{,}5 \leftrightarrow T_f \simeq 38\,\mathrm{MeV}$

Gal. klaszter szinten szét kell válniuk az anyagnak és antianyagnak

(megfigyelés)



$10^{14} M_\odot$ mi tartja távol folyamat

szétválasztaná a B-t a $\bar{B}$-től

a horizonton belüli barionok kerültek egy doménbe:

$d_H(T = 38\,\mathrm{MeV}) = 2t \simeq 1{,}4\cdot 10^{-3}\,s\,(c) \simeq 4{,}2\cdot 10^5\,m$

$t \sim T^{-2}$ $\frac{1}{T^2}\,s \approx 7\cdot 10^{-4}\,s$ $\rho = \frac{\pi^2}{30} g_* \underset{10{,}75}{T^4} \underset{38\,MeV}{\approx} 1{,}7\cdot 10^{12}\,\frac{g}{cm^3}$

$M = \rho \frac{4\pi}{3} d_H^3 = 5{,}2\cdot 10^{35}\,g$

$m_B < \eta M \simeq 3\cdot 10^{26}\,g$ $M_\odot \approx 2\cdot 10^{33}\,g$ kevés, mivel egy $M_\odot$ sem...!

(2)

(IV) Bariogenezis

(C) Dinamikus barionszám sértés

barion szám. → 0 anyag → B szám. kell

1GeV-nél $n_q \approx n_{\bar q} \approx n_\gamma \quad \frac{n_{\bar q}}{n_q} \approx \frac{60\,m}{60\,m+1}$ ennyi aszimmetria kell

Szaharov feltételek – a barion aszimmetria kialakulásához

- barionszám sértés
- C, CP sértés (idő iránya)
- egyensúlyból való kitérés

↳ egyensúlyban S_{max}, ha $\mu = 0 \quad \mu_B = -\mu_{\bar B}$

kemodinamika is minden ellentétes az mindenkit, így ki kell jönni az egyensúlyból $\begin{pmatrix} A \to a & a \to c + \dots \\ \bar A \to \bar a & \bar a \to \bar c + \dots \end{pmatrix}$

3 modell

(1) GUT bariogenezis $10^{14}-10^{15}$ GeV-en.

leptokvarkok X (nagy tömegű)

kit bomlás	arány	ΔB
$X \to qq$	r	$2/3$
$X \to \bar q \bar l$	$1-r$	$-1/3$
$\bar X \to \bar q \bar q$	$\bar r$	$-2/3$
$\bar X \to q l$	$1-\bar r$	$1/3$

CPT miatt $\quad r + (1-r) = \bar r + (1-\bar r)$

ha C és CP sérül $\quad r \neq \bar r$

összes (nettó) barionszám változás $\quad \varepsilon = r\frac{2}{3} - (1-r)\frac{1}{3} - \frac{2}{3}\bar r + \frac{1}{3}(1-\bar r)$

$\varepsilon = r - \frac{1}{3} - \bar r + \frac{1}{3} = r - \bar r$

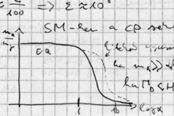
magas hőmérsékleten $\quad T \gg m_X c^2 \Rightarrow n_X \approx n_{\bar X} \approx n_\gamma$

$\frac{n_B}{s} \approx \frac{n}{7} = \frac{\varepsilon n_X}{s} = \frac{\varepsilon \overset{\shortparallel}{n_X}}{g_* n_\gamma} \approx \frac{\varepsilon}{100} \Rightarrow \varepsilon \approx 10^{-8}$

$s = \frac{2\pi^2}{45} g_{*S} T^3 \Rightarrow s \approx 7 n_\gamma g_*$

$n_\gamma \approx \frac{2}{\pi^2}\zeta(3) T^3$

SM-ben a CP sértés kicsi

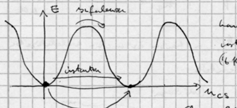


(3)

unitaritás
$\alpha = \alpha_{GUT} \approx \frac{1}{45}$ de Higgsre $\alpha \ll \alpha_{GUT}$ lehet

elvileg működhet...

② Elektrogyenge bariogenezis   Y–M elméletnek többszörös vákuuma van



barionszám sértés!
$(T=0)$
instanton átmenet valószínűsége
('t Hooft) $e^{\left(-\frac{4\pi}{\alpha_W}\right)}$ $\alpha_W \approx 0{,}03$
$\approx 10^{-180}$

$\Delta B = 3$
$\Delta L = 3$ $B-L =$ áll

Chern–Simons szám (winding number)

szfaleron átmenet $T \neq 0$

(2 vákuumállapotot összekötő egyensúlyi inflexiós pontja)

$$\Gamma_{sp} = T^4 e^{-\frac{E_{sp}}{T}} \qquad E_{sp} = \frac{\overset{5-10}{c}}{\underset{0{,}03}{\alpha}} \underset{80\,GeV}{M_W} \approx 8-10\,TeV$$

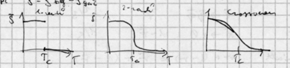
elektrogyenge skála $T \simeq 100\,GeV$

$$\Gamma_{sp} \sim e^{-100}$$ így is kis valószínűség

EW fázisátmenetnél $\rightarrow M_W = 0 \Rightarrow \Gamma \sim \alpha^4 T^4$ jó nagy ráta

részletes vizsgálatnál kiderül, hogy 1. rendű fázisátmenet kell

pl. $\varrho = \varrho_{liq} - \varrho_{gáz}$



SM szerint, ha $m_H < 75\,GeV$, akkor lesz elsőrendű
(Fodor Zoltán)
kísérletileg! $m_H > 115\,GeV$ SUSY így is tudja (MSSM)

③ Bariogenezis leptogenezis révén   leptonszám → barionszám

$B-L =$ áll $B+L$ vált.

TFH majorana neutrínó $\boxed{N = \bar{N}}$ nehézből $10^{12}\,GeV$ ~~kéne~~

Seesaw-mechanizmus: nagy tömegű majorana ν tudna kis tömegű ν-t generálni

(1) $N \rightarrow \ell + \dots$ ha $\Gamma_\ell \neq \Gamma_{\bar{\ell}}$ (CP sértés) $\rightarrow$ aszimmetria

(2) $N = \bar{N} \rightarrow \bar{\ell} + \dots$ $\left.\begin{array}{l} L = 10 \\ B = 20 \end{array}\right\} \begin{array}{l} B-L = 10 \\ B+L = 30 \end{array}$

④ szfaleron révén leptonszámból generál barionszámot
$\left.\begin{array}{l} B-L = 10 \\ B+L = 50 \end{array}\right\} \begin{array}{l} L = 20 \\ B = 30 \end{array}$ itt is aszimmetria lesz

(V) BBN I.

$T > 200\,\mathrm{MeV}$ q, g sugárzás dominálta kor
$T < 200\,\mathrm{MeV}$ p, n

$R \sim \sqrt{t}$ $T \sim R^{-1}$ $T^3 R^3 g_* = \text{áll.}$

$k=0$ Λ elhanyagolható $H^2 = \frac{8\pi G}{3} \rho_r$ $\rho_r = \frac{\pi^2}{30} g_* T^4$ $\dot{R} \sim \frac{1}{2}\frac{1}{\sqrt{t}}$

$$H = \sqrt{\frac{8\pi G}{3}\frac{\pi^2}{30}} \sqrt{g_*}\, T^2 = \frac{1}{2} t^{-1} \qquad H = \frac{\dot R}{R} = \frac{1}{2} t^{-1}$$

$$t = \frac{1}{2}\sqrt{\frac{45}{4\pi^3}} \frac{1}{\sqrt{G}} \frac{1}{\sqrt{g_*}} T^{-2} = \frac{1}{2}\sqrt{\frac{45}{4\pi^3}} \frac{1}{\sqrt{g_*}} \frac{1{,}22\cdot 10^{19}\,\mathrm{GeV}}{\mathrm{MeV}^2}\left(\frac{T}{\mathrm{MeV}}\right)^{-2} =$$

$\frac{1}{\sqrt{G}} = m_{Pl} = 1{,}22\cdot 10^{19}\,\mathrm{GeV}$ $\mathrm{MeV}^{-1} = \frac{1}{1{,}52\cdot 10^{21}}\,\mathrm{s}$

$$= \frac{1}{2}\sqrt{\frac{45}{4\pi^3}} \frac{1}{\sqrt{g_*}} \frac{1{,}22\cdot 10^{22}}{1{,}52\cdot 10^{21}}\left(\frac{T}{\mathrm{MeV}}\right)^{-2} \mathrm{s} = 2{,}42 \cdot \frac{1}{\sqrt{g_*}}\left(\frac{T}{\mathrm{MeV}}\right)^{-2} \mathrm{s}$$

Pl.:	T	g_*	t	
	1 MeV	10,75	0,74 s	
WFO	0,8 MeV	10,75	1,15 s	gyenge lefagyás
NS	0,07 MeV	3,36	269 s	nukleoszintézis

WFO (Weak freeze-out) $\frac{\Gamma_w}{H} < 1$ $\Gamma_w \sim T^5$, $H \sim T^2$ $n \leftrightarrow p$ gyenge lecsatolódás $\left(\frac{\Gamma}{H}\right)_{wd} \approx \left(\frac{T}{1{,}5\,\mathrm{MeV}}\right)^3$ weak decoupling

$n + \nu \leftrightarrow p + e^-$ $\left(\frac{\Gamma}{H}\right)_{WFO} \simeq \left(\frac{T}{0{,}8\,\mathrm{MeV}}\right)^3$ egyensúlyból kiesik a folyamat

$$\eta(T) = \frac{n_B(T)}{n_\gamma(T)} = \frac{N_B(T)}{\frac{4\pi}{3} R^3 \frac{2}{\pi^2}\zeta(3) T^3} = \frac{N_B(m_n)}{\frac{4\pi}{3}R(m_n)^3 \frac{2}{\pi^2}\zeta(3) T_{m_n}^3 \frac{g_{*s}(m_n)}{g_{*s}(T)}} =$$

$$g_{*s}(T)\, R(T)^3 T^3 = g_{*s}(m_n) R^3(m_n) T_{m_n}^3 \qquad N_B(T) = N_B(m_n)$$

$$= \frac{n_B(m_n)}{n_\gamma(m_n)} \frac{g_{*s}(T)}{g_{*s}(m_n)} = \eta(m_n) \underbrace{\frac{g_{*s}(T)}{g_{*s}(m_n)}}_{\mathcal{O}(1)} \qquad \begin{cases} g_{*s}(1\,\mathrm{MeV}) = 10{,}75 \\ g_{*s}(m_n) \simeq 3{,}909 \end{cases} \Downarrow \quad \eta(T) \approx \eta(m_n)$$

$$\eta(m_n) = \frac{n_B}{n_\gamma} \qquad n_B = \frac{N_B}{V} = \frac{M_B}{V m_N} = \frac{\rho_B}{m_N} = \frac{\rho_c \Omega_B}{m_N} \qquad \rho_c = \frac{3H_0^2}{8\pi G}$$

$$\eta = \frac{\rho_c \Omega_B}{n_\gamma m_N} = \frac{1{,}22\cdot 10^{-23} h^2 \frac{g}{cm^3} \Omega_B}{411 \frac{1}{cm^3}\cdot 1{,}67\cdot 10^{-24} g} = 2{,}74\cdot 10^{-8} \Omega_B h^2$$

$0 < \Omega_B \lesssim 1$ $0 < h < 1$ $\Omega_B \cdot h^2 \sim \mathcal{O}(1)$ $\eta \lesssim 2{,}74\cdot 10^{-8}$ $\boxed{\eta \ll 1}$

Nukleáris statisztikus egyensúly — kémiai reakció egyensúly

$p, n \leftrightarrow A$ $Z\cdot p + (A-Z) n \leftrightarrow A(Z, A-Z)$

$$\Downarrow$$

$$Z \mu_p + (A-Z)\mu_n = \mu_A$$

$$\boxed{\begin{array}{l}\text{NR nérnegyenlőség} \\ n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m-\mu}{T}}\end{array}} \qquad n_A = g_A\left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-\frac{m_A-\mu_A}{T}} \qquad g_p = g_n = 2$$

$$n_p = g_p\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-\frac{m_p-\mu_p}{T}} \qquad m_p \approx m_n \approx m_N$$

$$n_n = g_n\left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-\frac{m_n-\mu_n}{T}} \qquad \text{(nem kitevőben)}$$

$$e^{\frac{\mu_A}{T}} = e^{\frac{Z\mu_p + (A-Z)\mu_n}{T}} = n_p^Z \frac{1}{2^Z}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}Z} n_n^{A-Z} \frac{1}{2^{A-Z}}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A-Z)} e^{\frac{Z m_p + (A-Z) m_n}{T}}$$

$m_A \approx m_N A$ $\quad \underline{n_A = g_A A^{3/2}\left(\frac{m_N T}{2\pi}\right)^{3/2} n_p^Z n_n^{(A-Z)} 2^{-A}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}A} e^{B_A/T}}$

$B_A = Z m_p + (A-Z) m_n - m_A$ kötési energia

tömegarány: $X_A \equiv \frac{A \cdot n_A}{n_B}$ $\quad \sum_A X_A = 1 = \frac{n_p + n_n + \sum_{A=2}^{\infty} A n_A}{n_B}$

$X_p = \frac{n_p}{n_B}$ $\quad X_n = \frac{n_n}{n_B}$ $\qquad n_B = \eta n_\gamma = \eta \frac{2}{\pi^2}\zeta(3)T^3$

$$X_A = \frac{A^{5/2}}{n_B} g_A \left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A-1)} 2^{-A} (n_B X_p)^Z (n_B X_n)^{A-Z} e^{B_A/T} = g_A A^{5/2}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A-1)} 2^{-A} n_B^{A-1} X_p^Z X_n^{A-Z} e^{B_A/T}$$

$$X_A = g_A A^{5/2} 2^{-A}\left(\frac{2}{\pi^2}\zeta(3)\right)^{A-1} \eta^{A-1}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A-1)} T^{3(A-1)} X_p^Z X_n^{A-Z} e^{B_A/T} =$$

$$= \underbrace{g_A A^{5/2} 2^{-A}\left(\frac{2}{\pi^2}\zeta(3)\right)^{A-1}(2\pi)^{\frac{3}{2}(A-1)} X_p^Z X_n^{A-Z}}_{\sim \mathcal{O}(1)} \underbrace{\left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} e^{B_A/T}}_{\text{hőmérséklet függés}}$$

$X_A \simeq \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} e^{B_A/T}$ $\quad$ A nem elhanyagolható, ha $X_A \simeq 1$

$X_A = 1 \Rightarrow T_{NS}^A$

---

n/p tömegarány $\quad \frac{X_n}{X_p} = \frac{2\cancel{\left(\frac{m_n T}{2\pi}\right)^{3/2}} e^{-\frac{m_n - \mu_n}{T}}}{2\cancel{\left(\frac{m_p T}{2\pi}\right)^{3/2}} e^{-\frac{m_p - \mu_p}{T}}} \approx e^{-\frac{\overbrace{(m_n - m_p)}^{Q}}{T}} e^{\frac{\mu_n - \mu_p}{T}} =$

$= \underline{e^{-\frac{Q}{T}} e^{\frac{\mu_e - \mu_\nu}{T}}}$ $\quad Q = 1{,}293\,\mathrm{MeV}$ $\quad n + \nu \leftrightarrow p + e^-$ ~~NSE~~

$\mu_n + \mu_\nu = \mu_p + \mu_e$

$n_{e^-} - n_{e^+} = n_p^* \approx n_B = \eta n_\gamma = \eta \underbrace{\frac{2}{\pi^2}\zeta(3)T^3}_{\text{kicsi}}$ $\quad \underline{\mu_n - \mu_p = \mu_e - \mu_\nu}$

Rel: $\quad n_{e^-} - n_{e^+} = \frac{2}{2\pi^2}\left[\int_0^\infty \left(\frac{1}{e^{\frac{p-\mu}{T}}+1} - \frac{1}{e^{\frac{p+\mu}{T}}+1}\right) p^2 dp\right] =$ $\quad (p + {}^4He + \ldots)$

$\mu_{e^-} = -\mu_{e^+} = \mu$ $\qquad$ kicsi (ηn_γ -ből) $\Rightarrow \frac{\mu^3}{T^3} \ll \frac{\mu}{T}$

$$= \frac{1}{\pi^2}\left[\frac{\mu^3}{3} + \frac{\pi^2 T^2 \mu}{3}\right] = \frac{T^3}{3\pi^2}\left[\frac{\mu^3}{T^3} + \pi^2\frac{\mu}{T}\right] \simeq \underbrace{\frac{1}{3}\frac{\mu}{T}}_{\text{kicsi}} T^3$$

NRel: $\quad n_{e^-} - n_{e^+} = 2\left(\frac{m_e T}{2\pi}\right)^{3/2}\left(e^{-\frac{m_e - \mu_e}{T}} - e^{-\frac{m_e + \mu_e}{T}}\right) = 4\left(\frac{m_e T}{2\pi}\right)^{3/2} \mathrm{sh}\frac{\mu_e}{T} e^{-\frac{m_e}{T}} =$

$= \eta \frac{2}{\pi^2}\zeta(3) T^3$ $\qquad \frac{1}{T^{3/2}} \mathrm{sh}\frac{\mu}{T} e^{-\frac{m}{T}} = \underbrace{\eta}_{\sim 6\cdot 10^{-10}} \frac{2}{\pi^2}\zeta(3)\frac{1}{4}\left(\frac{2\pi}{\underbrace{m_e}_{511\,\mathrm{keV}}}\right)^{3/2} \approx 4{,}98\cdot 10^{-14}$ $\quad [m_e, T, \mu]$ - keV

pl:

$T = 10\,\mathrm{keV}$ $\quad \mu = 246\,\mathrm{keV}$ $\qquad T > 30\,\mathrm{keV}$ $\quad \underline{\frac{\mu}{T} \ll 1}$

$T = 20\,\mathrm{keV}$ $\quad \mu = 10{,}6\,\mathrm{keV}$

$T = 30\,\mathrm{keV}$ $\quad \mu = 0{,}061\,\mathrm{keV}$

$n_\nu - n_{\bar\nu} = \frac{1}{2\pi^2}\left[\int(\ldots) p^2 dp\right] = \frac{T^3}{6\pi^2}\left[\pi^2\frac{\mu}{T} + \left(\frac{\mu}{T}\right)^3\right] = \eta \frac{2}{\pi^2}\zeta(3)T^3$ $\qquad \frac{n_\nu - n_{\bar\nu}}{n_\gamma} = \eta \ll 1$

(2) $\quad \frac{n_\nu - n_{\bar\nu}}{n_\gamma} = \frac{1}{12\zeta(3)}\left[\pi^2\frac{\mu}{T} + \left(\frac{\mu}{T}\right)^3\right] = \eta$ $\qquad \frac{\mu_\nu}{T} \ll 1$

(V-VI) **BBN II.** $X_A \approx \left(\frac{T}{m_N}\right)^{3/2(A-1)} \eta^{A-1} e^{B_A/T} \simeq 1 \Rightarrow T^A_{NS}$

	A	B_A	J_A	g_A	T^A_{NS}
^2H	2	2,22	1	3	0,07
^3H	3	8,48	1/2	2	—
^3He	3	7,72	1/2	2	0,11
^4He	4	28,3	0	1	0,28

$$\frac{3}{2}(A-1)\ln\left(\frac{T}{m_N}\right)+(A-1)\ln\eta+\frac{B_A}{T}=0$$

$$\boxed{T^A_{NS}=\frac{B_A/(A-1)}{\frac{3}{2}\ln\frac{m_N}{T}+\ln\eta^{-1}}}$$

pl.: $A=4$ ^4He $T^{(0)}=1\,MeV$ $T^{(1)}=\frac{28{,}3\,MeV/3}{\frac{3}{2}\ln\left(\frac{938\,MeV}{1\,MeV}\right)-\ln\left(6\cdot10^{-10}\right)}\simeq 0{,}2996\,MeV$

$T^{(2)}=\frac{28{,}3\,MeV/3}{\frac{3}{2}\ln\left(\frac{938\,MeV}{0{,}2996\,MeV}\right)-\ln(6\cdot10^{-10})}\simeq 0{,}283\,MeV$ $T^{(3)}=\ldots=0{,}282\,MeV\ \ldots$

$T^{^4He}_{NS}=0{,}28\,MeV$ $T^{^3He}_{NS}=0{,}11\,MeV$ $\boxed{T^{^2H}_{NS}=0{,}07\,MeV}$!!!

$n\to p+e^-+\bar\nu_e$ $\cdot\ {}^4He+t\leftrightarrow {}^7Li+\gamma$

$n+p\leftrightarrow d+\gamma$ ${}^4He+{}^3He\leftrightarrow {}^7Be+\gamma$

$d+p\leftrightarrow {}^3He+\gamma$ ${}^7Be+n\leftrightarrow {}^7Li+p$

${}^3He+n\leftrightarrow t+p$ ${}^7Li+p\leftrightarrow 2\times{}^4He$

$d+d\leftrightarrow t+p$

$d+d\leftrightarrow {}^3He+n$

$d+t\leftrightarrow {}^4He+n$

$d+{}^3He\leftrightarrow {}^4He+p$

bomlások

${}^3H(t)\to{}^3He+e^-+\bar\nu_e$ 12 év

$e^-+{}^7Be\to{}^7Li+\nu_e$ 53 nap

$T=0{,}5\,MeV$-nál kiesnek az egyensúlyból

$T=0{,}07\,MeV$-nál indul be a BBN

az összes $n\to{}^4He$-ban kötődik le (a többi elhanyagolható)

$\left(\frac{X_n}{X_p}\right)_{EQ}\simeq e^{-\frac{Q}{T}}$ $T>0{,}8\,MeV$-nál $\frac{X_n}{X_p}=\left(\frac{X_n}{X_p}\right)_{EQ}$ $\left[\frac{\Gamma_{n+\nu\leftrightarrow p+e^-}}{H}\approx\left(\frac{T}{0{,}8\,MeV}\right)^3\right]$

$T_{WFO}=0{,}8\,MeV$ $n+\nu\to p+e^-$

$0{,}07\,MeV<T<0{,}8\,MeV$ $n\to p+e^-+\bar\nu_e$ n bomlik $\tau_n=(887\pm2)s$

$n(t)=n(t_{WFO})e^{-\frac{(t-t_{WFO})}{\tau_n}}$

$T=T^d_{NS}=0{,}07\,MeV$-en az összes el nem bomlott $n\to{}^4He$

$\boxed{T>0{,}8\,MeV\quad X_p+X_n=1\quad \frac{X_n}{X_p}=e^{-\frac{Q}{T}}}\Rightarrow X_p+X_pe^{-Q/T}=1$ $X_p=\frac{1}{1+e^{-Q/T}}$ $X_n=\frac{e^{-Q/T}}{1+e^{-Q/T}}$

WFO $T=0{,}8\,MeV$ $t=1{,}15\,s$ $X_p(t_{WFO})=0{,}83$ $X_n(t_{WFO})=0{,}17$ $\frac{X_n}{X_p}\sim 1/5$

NS t $\underline{X_n}(T^d_{NS}\to t^d_{NS}=269\,s)=X_n(t_{WFO})\cdot e^{-\frac{t^d_{NS}-t_{WFO}}{\tau_n}}=X_n(1{,}15\,s)\,e^{-\frac{269\,s-1{,}15\,s}{887\,s}}\underline{\simeq 0{,}122}$

$\frac{X_n}{X_p}\sim 1/7$ $\underline{X_{^4He}}=\frac{1}{2}\cdot4\cdot X_n(t^d_{NS})\simeq\underline{0{,}25}$

$A=4$ $n_{^4He}=\frac{n_n}{2}$ kell anyi $\forall n\to{}^4He$ ^4He: $2n$, $2p$ van benne

<u>75% H, 25% ^4He</u> <1% ^2H, ^3He, ^7Li, ..

<u>$X_{^4He}(\eta, N_\nu, \tau_n)$ függés</u>

(1) fgtlen. — η nő, akkor $\Rightarrow$ T^d_{NS} is nő $\Rightarrow X_n(T^d_{NS})$ nő $\Rightarrow X_{^4He}$ nő

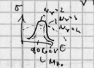
$\tau_n(\eta)$ — τ_n nő $\Rightarrow X_n(T^d_{NS})$ nő $\Rightarrow X_{^4He}$ nő

$N_\nu(\eta)$ — N_ν nő $\Rightarrow g_*$ nő $\Rightarrow t_{WFO}, t_{NS}$ csökken $\Rightarrow X_n(t^d_{NS})$ nő $\Rightarrow X_{^4He}$ nő

N_ν szám:

$e^-e^+ \to (Z^0)$ látható

$\nu\bar{\nu}$ ha láthatatlan hatáskeresztmetszet



$m_\nu < 45\,GeV$

Z^0-ban annihiláló ν

BBN reliktum ν $m \leq 1\,MeV$ $\forall$-ra

(CERN $m \leq 46\,GeV$, a Z_0-hoz annihilálódik $Z^0 \to \nu + \bar{\nu}$)

$N_\nu = 2$ $\quad \tilde{g}_*(0{,}8\,MeV) = 2 + \frac{7}{8} \cdot 4 + \frac{7}{8} \cdot 4 = 9 \qquad g_* = 10{,}75$

$g_*(0{,}07\,MeV) = 2 + \frac{7}{8} \cdot 4 \cdot \left(\frac{4}{11}\right)^{4/3} = 2{,}9284 \qquad g_* = 3{,}36$

$\frac{\tilde{\Gamma}}{H} \approx \left(\frac{T}{0{,}8\,MeV}\right)^3 \frac{1}{\sqrt{a}} \qquad \tilde{g}_* = a\, g_* \qquad a = 0{,}8372093 \quad T = 0{,}8\,MeV$

$\sqrt{a} = 0{,}91498 \qquad \sqrt[3]{\sqrt{a}} = 0{,}97082$

$\frac{\tilde{\Gamma}}{H} \approx \left(\frac{T}{0{,}7766}\right)^3 \qquad \tilde{T}_{ext} = 0{,}7766\,06 \qquad t = 2{,}417 \cdot \frac{1}{\sqrt{g_*}} \left(\frac{T}{MeV}\right)^{-2} s$

$\tilde{t}_{ext} = 1{,}3356\,s \qquad t_{ext} = 1{,}155 = t_f$

$X_n(\tilde{t}_f) = 0{,}1591 \qquad X_n(t_f) = 0{,}1657$

$\tilde{t}_{NS} = 289{,}2\,s \qquad t_{NS} = 269\,s$

$\tilde{X}_n(t_{NS}) = 0{,}115 \qquad X_n(t_{NS}) \simeq 0{,}1225$

$Y_p(= X_{n,He}) = 0{,}23 \qquad Y_p = 0{,}245 \qquad$ 6% eltérés!

②

VII H égető csillagok

- termikus reakcióráta

$a+b \to c+d$

N_a $\Delta t, \Delta A$ $N_a \sigma$ $\Delta t = \frac{ds}{v}$

$$j_a = \frac{N_a}{\Delta t \Delta A} = \frac{N_a}{\frac{ds}{v}\Delta A} = \frac{N_a v}{V} = n_a v$$

ráta $R = \frac{1}{1+\delta_{ab}} j_a N_b \sigma \frac{1}{V} = \frac{1}{1+\delta_{ab}} n_a n_b \langle \sigma v \rangle$

pl. reakcióráta

Maxwell

$$\phi(\vec v_1)d^3 v_1 = \left(\frac{m_1}{2\pi kT}\right)^{3/2} e^{-\frac{m_1 v_1^2}{2kT}} d^3 v_1 \qquad \boxed{\int \phi(\vec v_1) d^3 v_1 = 1}$$

$\phi(v_2)d^3 v_2 = \dots \Rightarrow$ TKP rsz. $\vec v = \vec v_2 - \vec v_1$ $\quad \vec V = \frac{m_1 \vec v_1 + m_2 \vec v_2}{m_1 + m_2}$

$$\phi(\vec v_1)\phi(\vec v_2) d^3 v_1 d^3 v_2 = \phi(\vec V)\phi(\vec v) d^3 V d^3 v$$

$$\phi(\vec V) d^3 V = \left(\frac{m_1+m_2}{2\pi kT}\right)^{3/2} e^{-\frac{(m_1+m_2)V^2}{2kT}} d^3 V \qquad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\phi(\vec v) d^3 v = \left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\frac{\mu v^2}{2kT}} d^3 v$$

TKP rsz-ben

$$\langle A \rangle = \underbrace{\int d^3 V \phi(\vec V)}_{1} \int d^3 v\, \phi(\vec v) A(\vec v)$$

termikus átlag

$$\int \phi(\vec v) d^3 v = \left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \int_0^\infty e^{-\frac{\mu v^2}{2kT}} v^2 dv = \left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \int_0^\infty \left(\frac{2kT}{\mu}\right)^{3/2} e^{-x^2} x^2 dx =$$

$$= \left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \left(\frac{2kT}{\mu}\right)^{3/2} \underbrace{\int_0^\infty e^{-x^2} x^2 dx}_{\sqrt{\pi}/4} = 1$$

$x^2 = \frac{\mu v^2}{2kT}$ $\quad dx = \sqrt{\frac{\mu}{2kT}}\, dv$ $\quad v = \sqrt{\frac{2kT}{\mu}}\, x$

$$\langle \sigma v \rangle = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int d^3 v\, \sigma v\, e^{-\frac{\mu v^2}{2kT}} = \left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \int \sigma(E)\, e^{-E/kT} \underset{v^2}{\underbrace{\frac{2E}{\mu}}} \underset{v\,dv}{\underbrace{\frac{dE}{\mu}}} =$$

$\sigma = \sigma(E)$ $\quad E = \frac{1}{2}\mu v^2$ $\quad v^2 = \frac{2E}{\mu}$ $\quad v\,dv = \frac{1}{\mu} dE$

$$= \left(\frac{\mu}{2\pi kT}\right)^{3/2} 4\pi \frac{2}{\mu^2} \int_0^\infty \sigma(E) e^{-E/kT} E\, dE = \boxed{\sqrt{\frac{8}{\mu\pi}} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT} dE}$$

$$Y_i = \frac{n_i}{n_B} \qquad \frac{\partial Y_i}{\partial t} = \sum_j c_j^i Y_j + \sum_{j,k} c^i_{jk} Y_j Y_k - \sum_{j,k} c^k_{ij} Y_k \underset{(i)}{Y_j}$$

$\gamma + j \to i + \dots$ $\quad c^i_j$ $\qquad j + k \to i + \dots$ $\quad c^i_{jk}$ $\qquad c^i_{jk} = \langle \sigma v \rangle_{jk \to i}$

$e^- + j \to i$ $\qquad i + j \to k + \dots$ $\quad c^k_{ij}$

$e^- + i \to j + \dots$ $\quad c^j_i$ $\qquad$ pl. ${}^4He + {}^3H \to {}^7Li + \gamma$

σ - ra:

EM : $T = \frac{2\pi}{\hbar} |\langle \Psi_f | \hat O | \Psi_i \rangle|^2 \delta(E_f)$ $\quad E_1, E_2, E_3 \dots$ $M_1, M_2, M_3 \dots$ $\quad \hat O = \hat E_1, \hat E_2, \dots \hat M_1, \dots$

(1) gyenge: $\hat O_F = \sum_{i=1}^{A} t_\pm^i$ $\quad t_+|p\rangle = |n\rangle$, $t_-|n\rangle = |p\rangle$ $\quad$ Fermi $J_i = J_f$ $\quad$ Gamow-Teller $\hat O_\mu = \sum_{i=1}^{A} t_\pm^i \sigma_\mu^i$ $\quad J_i = J_f, J_f \pm 1$ $\quad \cancel{J_i = J_f = 0}$

erős: $\hat{O} = \hat{S}$ pl.: ${}^2H + {}^3H \to {}^4He + n$ $\quad \Psi \xrightarrow{r\to\infty} h_i^-(k_i r_i) + \sum_{j=1}^{N_c} S_{ij} h_j^+(k_j r_j)$
bejövő $\quad$ kifutó és kül.

és a Jenkel-fv.

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töltött rész. $\quad V(r) = Z_1 Z_2 \frac{e^2}{r}$ $\quad e^2 = 1{,}44$ MeV fm $\quad \alpha = \frac{e^2}{\hbar c} = \frac{1{,}44 \text{ MeV fm}}{197 \text{ MeV fm}}$

$\sigma = P(E)\cdot\sigma_m \qquad \sigma_m \sim \frac{4\pi}{k^2}\sum_{\ell=0}(2\ell+1)\sin^2\delta_\ell(E)$

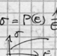
$P(E) \approx e^{-\frac{1}{\hbar}\int\sqrt{2\mu(V(r)-E)}} \approx \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right) \equiv$

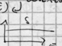
$\equiv \exp(-2\pi\eta(E)) \qquad E = \frac{1}{2}\mu v^2$

$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v} \sim \frac{1}{\sqrt{E}}$

$\sigma = P(E)\frac{1}{E}S(E)$ asztrofizikai S faktor

$S(E) = E\sigma\frac{1}{P(E)}$

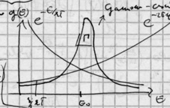




$\langle\sigma v\rangle_{\text{töltött rész.}} = N\int_0^\infty \underbrace{e^{-E/kT - 2\pi\eta}}_{\text{k. özel. kb. Gaussal}} S(E)\,dE \qquad \langle E\rangle = \int \ldots = \frac{3}{2}kT$

$f(E) \sim g(E)$

Csillag hőmérséklete a $\frac{3}{2}kT$



$g(E) = I_{max}\, e^{-\left(\frac{E_0}{\Delta/2}\right)^2}$ (Steepest descent)

$f'(E_0) = g'(E_0) = 0 \Rightarrow E_0$ $\qquad E_0 = 1{,}22\cdot(Z_1^2 Z_2^2 \mu T_6^2)^{1/3}$ keV

$\left.\begin{array}{l} f(E_0) = g(E_0) \\ f''(E_0) = g''(E_0)\end{array}\right\} \begin{array}{l} I_{max} \\ \Delta \end{array}$ $\qquad \Delta = 0{,}75\ (\text{---}\ T_6^5)^{1/6}$ keV

$I_{max} = \exp\left(-\frac{E_0}{kT}\right) T_6\ [10^6 K]$

Nap működése $\quad kT = 1{,}3$ keV $\quad (1{,}5\cdot 10^6 K)$

	E_0 [keV]	$\Delta/2$ [keV]	I_{max}	
pp	5,5	3,2	$1{,}1\cdot 10^{-6}$	ezek mennek
${}^3He + {}^3He$	22,0	6,3	$4{,}5\cdot 10^{-23}$	
${}^3He + {}^4He$	23,0	6,4	$5{,}5\cdot 10^{-23}$	
${}^{12}C + {}^4He$	56,0	9,8	$3\cdot 10^{-57}$	ezek nem mehetnek
${}^{16}O + {}^{16}O$	237	20,2	$6{,}2\cdot 10^{-239}$	

② $S(E) \approx S(E_0) = $ const $\quad \langle\sigma v\rangle = \left(\frac{2}{\mu}\right)^{1/2}\frac{\Delta}{(kT)^{3/2}}S(E_0)\,e^{\left(-\frac{3E_0}{kT}\right)}$ $\quad \frac{3E_0}{kT} = \tau \sim T^{-1/3} \quad \Delta \sim T^{5/6}$

$\langle\sigma v\rangle \sim T^{-2/3} e^{-T^{-1/3}} = T^n$ pontonként $\quad e^{-T^{-1/3}} = T^{n+2/3} \quad (f = c(\,)^n)\ n = \frac{\tau - 2}{3}$ $\quad \frac{\Delta}{(kT)^{3/2}} \sim T^{-2/3}$

VII H égető csillagok

csillagok ⟶ csillagközi tér

tömeg $\frac{M_*}{M_\odot}$	élettartam T [év]	végállapot
1	10^{10}	fehér törpe WD
8	$5\cdot 10^{7}$	SN
30	$2\cdot 10^{6}$	BH

elemek szintézise

kisugárzott en. tömeg

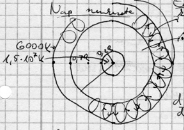
$E \sim M$

$E = L\cdot T$

luminozitás · élettartam

$T \sim M^{-2,5}$ $\quad L \sim M^{3,5}$

Nap szerkezete



$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dp(r)}{dr} = \frac{-GM(r)\rho(r)}{r^2}$$

$M(r)$: r sugarон belüli tömeg

$\rho(r)$: sűrűség

$T(r)$: hőmérséklet

$p(r)$: nyomás

$L(r)$: luminozitás

$\mathcal{E}(r)$: termikus en. sűrűség (egységnyi tömegre)

a: S-B áll. $\quad \mathcal{K}$: opacitás

c: fénys. $\quad \gamma$: fajhők aránya

$c_{V,p} = \left(\frac{\partial Q}{\partial T}\right)_{V,p}$ $\quad \frac{dI}{dx} = -\mathcal{K}\rho I$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\,\mathcal{E}(r)$$

$$\frac{dT(r)}{dr} = -\frac{3}{4ac}\frac{\mathcal{K}(r)\rho(r)}{T(r)^3}\frac{1}{4\pi r^2}L(r) \qquad r < 0{,}74 R_\odot$$

$$\frac{dT(r)}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T(r)}{p(r)}\frac{dp(r)}{dr} \qquad r > 0{,}74 R_\odot$$

$4\pi r^2 dp = \frac{-GM(r)\rho(r)4\pi r^2 dr}{r^2}$ m — kis gömbhéj tömege — gravit. felület

nyomás

$dL = 4\pi r^2 dr\,\mathcal{E}$

en. termelés

$\mathcal{E}(r, \rho, T, X_p, Y, Z, \sigma)$

H He M összetétel

kezdetben H (71%) He (27%) M (2%)

most H (34%) He (64%) M (2%)

en. termelés

$4p \to {}^4He$ hogyan?

$p+p \to \emptyset \quad p+{}^4He \to \emptyset$

${}^4He + {}^4He \to \emptyset$

③

PP $\quad p+p \to d + e^+ + \nu_{pp}$, $\quad p + e^- + p \to d + \nu_{pep}$

leglassabb

$d + p \to {}^3He + \gamma$

$\downarrow$ 86% $\qquad \downarrow$ <1% $\qquad$ EM 14%

${}^3He + {}^3He \to {}^4He + 2p$ erős

${}^3He + p \to {}^4He + e^+ + \nu_{hep}$ gyenge

${}^4He + {}^3He \to {}^7Be + \gamma$

99% $\qquad$ 1%

${}^7Be\,e^- \to {}^7Li + \nu_{7Be}$ $\qquad$ ${}^7Be + p \to {}^8B + \gamma$

${}^7Li + p \to {}^8Be$ $\qquad$ ${}^8B \to {}^8Be + e^+ + \nu_{8B}$

${}^8Be \to 2\,{}^4He$ $\qquad$ ${}^8Be \to 2\,{}^4He$

spektrum

$\nu_{pp}, \nu_{hep}, \nu_{8B}$ folytonos

ν_{pep}, ν_{7Be} diszkrét (2 (gerjesztett alap))

$\underline{CNO}$ ciklus $\quad {}^{12}C + p \to {}^{13}N + \gamma \longrightarrow {}^{13}N \longrightarrow {}^{13}C + e^+ + \nu_e$

$\Uparrow \qquad\qquad\qquad\qquad\qquad \Downarrow$

${}^{15}N + p \to {}^{12}C + {}^{4}He \qquad\qquad {}^{13}C + p \to {}^{14}N + \gamma$

$\Uparrow \qquad\qquad\qquad\qquad\qquad \Downarrow$

${}^{15}O \to {}^{15}N + e^+ + \nu_e \qquad \Longleftarrow \qquad \boxed{{}^{14}N + p \to {}^{15}O + \gamma}$ leglassabb

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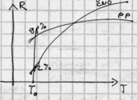
Ráták:

		$T = T_\odot$	$T = 10T_\odot$	$T = 100T_\odot$
napban 98	$p + p \to d + e^+ + \nu_e$	$T^{3,7}$	$T^{1,4}$	$T^{0,25}$
2	${}^{14}N + p \to {}^{15}O + \gamma$	$T^{19,1}$	$T^{8,5}$	$T^{5,36}$



$\langle \sigma v \rangle \propto T^n$

$n = \frac{\tau - 2}{3} \qquad \tau = \frac{3E_0}{kT}$

$4p \to {}^4He + 2e^+ + 2\nu_e + Q$

$28{,}3\,MeV + 4 \cdot 0{,}511\,MeV - 2 \cdot 1{,}3\,MeV$

$2(m_n - m_p) = 2 \cdot 1{,}293\,MeV$

$Q = 27{,}76\,MeV \to \left.\begin{array}{l} 97\%\ \text{foton} \\ 3\%\ \nu \end{array}\right\}$ viszi el

$E_{foton} \approx 27\,MeV \qquad k_\odot = 1366\,\frac{W}{m^2}$ 1AU-nál

$R = \frac{k_\odot \cdot 4\pi R_{NF}^2}{27\,MeV} \simeq 10^{38}\,1/s \qquad I_\nu = 2 \cdot 10^{38}\,1/s$ neutrínó keletkezési ráta

$\Phi = \frac{I_\nu}{4\pi R^2} \simeq 6 \cdot 10^{10}\,\frac{\nu}{s\,cm^2}$

$A = 1{,}1 \cdot 10^5\,km^2 \qquad \eta = 0{,}1$ hatásfokú napelem kell 15 TW termeléséhez. Ez kb. Izland területe.

A Nap másodpercenként $4{,}3 \cdot 10^9\,kg$ tömeget veszít el sugárzással.
Ez kb. a Gízai piramis tömege.