

# Nemeggyensúlyi transport

nanorendszerekben

- ami egy emelt testi résznek (am minthoz leválik)
- felül vagy "széle" (anyagok kapcsolatba hozatalhoz kidolgozni)

1. óra  
Bevetés

## 1) Makroskopikus rendszerek

atomi < meso < makroszk.

nm

(némely más nm)

elektr. transzp. tul.-át fogunk nézni (előzőben)

Öhmikus vezetés:

metab  $\gg \lambda_{\text{de Broglie}} \leftarrow E_F$

-II-  $\gg l = v_F \cdot T$  szab. időben  $\rightarrow$  

-II-  $\gg L_F$ : fázisrelaxációs hossz  $\rightarrow T, B$

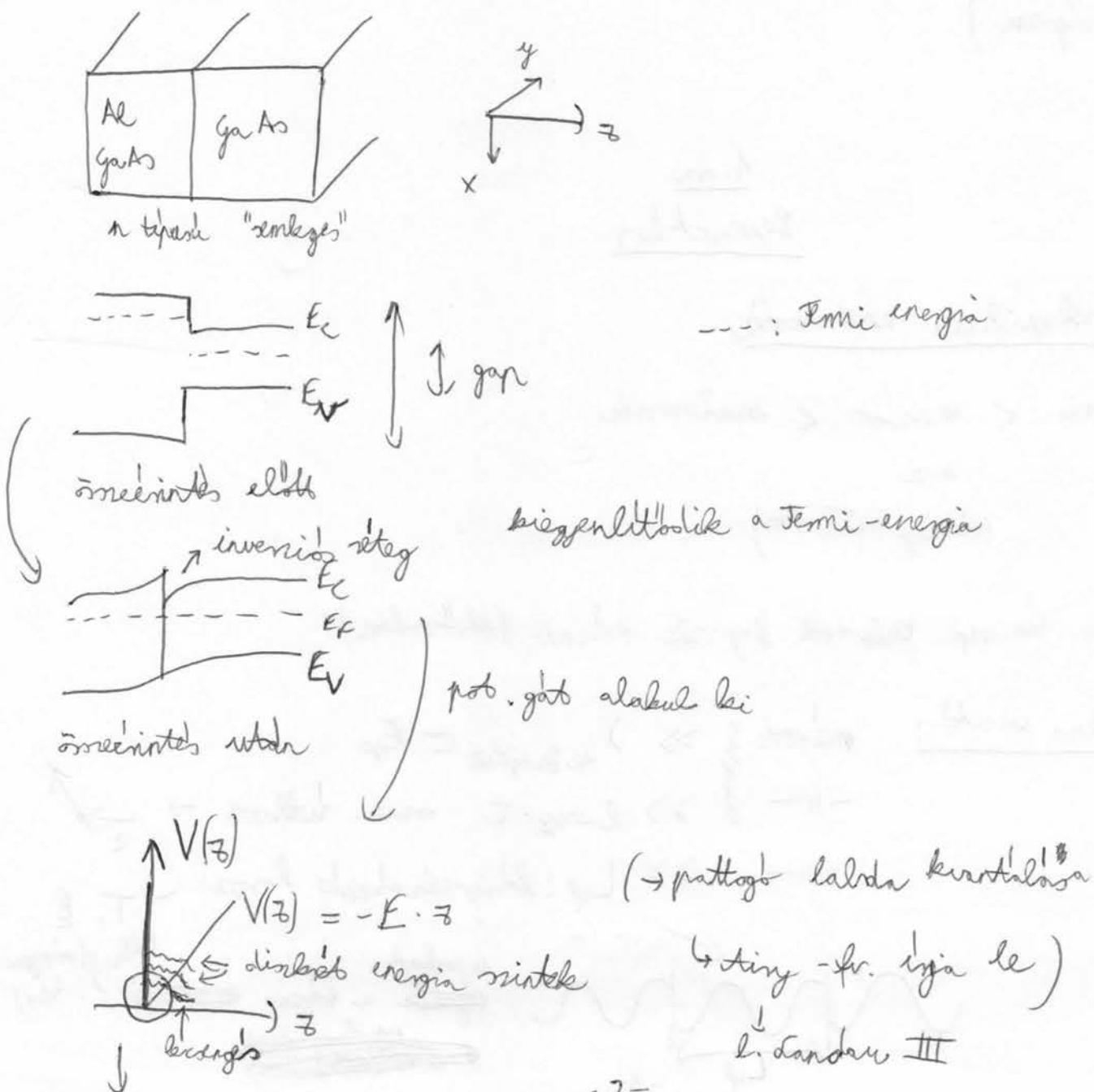


ugorhat  
~~ugrik a fázis~~  
~~ugrik át a fázis~~  
~~ugrik át a fázis~~  
 $(\Delta t_m)$  (magn. ter.)

## 2) Mikroszkopikus felvételről

- a) - nagy töltésű, kevés szennyező  
 - kevés kristályhiba  
 - vékonyréteg (2 dim.)  
 - nagy magékonyságú elektron (kicsi eff. tömege)  
 - kicsi  $e^-$  részleg  
 - b. nálak ritkán nagy!! ~ ( $\mu m$  nagyságrendben)  
 (készlel pontosságnak)

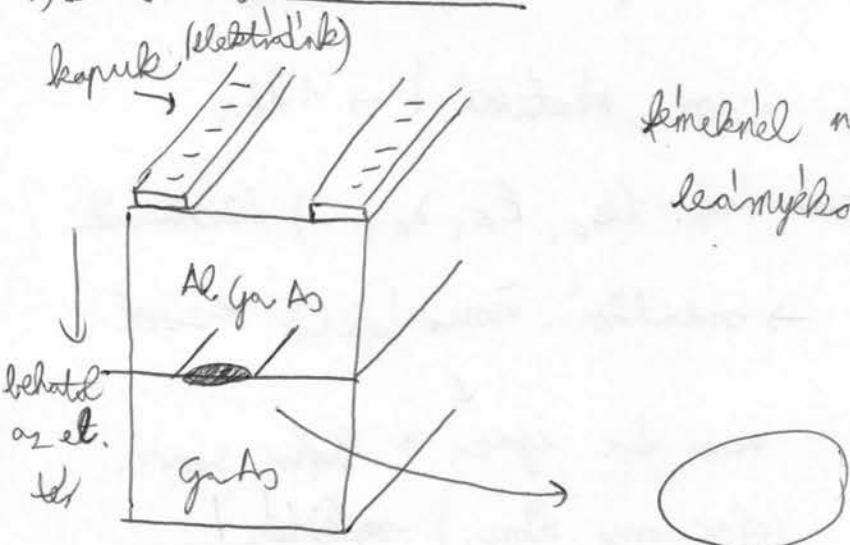
} összefügg



$\pi$   $e^-$  ledekorálódik a rotációsban ( $\Rightarrow$  irányban)

$$\Rightarrow 20 \text{ } e^- \text{ g}^{-1} = 20 \text{ EG}$$

b) Kisbóltsi megvalósítás:



Fémekkel nem lehet, mert  $\pi$   $e^-$ -től leámykölcsön!!! (allamirányban lehet)

kis karantna lefeljük záráni

a leírásban kv.  $\leftarrow$  a 2DEG -b  
mechanika belfel  
(néhány nm -re)

c) Tipikus paraméterek GaAs (100 felén)  $T < 10\text{K}$

- effektív tömeg:  $m_{eff} = 0,06 + m_e$
- hallszám:  $\rho_{2D} = \frac{m}{2\pi\hbar^2} = 0,28 \cdot 10^{11} \frac{1}{\text{cm}^2\text{meV}} \rightarrow$  constant!
- $e^-$  szűrő a stábján:  $n_s = 4 \cdot 10^{11} \frac{1}{\text{cm}^2} \leftarrow$  (kicsi)!
- Fermi - hullámzám:  $k_F = \sqrt{2\pi n_s} = 1,58 \cdot 10^6 \frac{1}{\text{cm}}$
- Fermi - energia:  $E_F = \frac{\hbar^2 k_F^2}{2m_{eff}} = 14 \text{ meV} !!!$
- Fermi - hullámhossz:  $\lambda_F = \frac{2\pi}{k_F} = 40 \text{ nm} = 400 \text{ \AA} !!!$
- átl. rész. hossz:  $l = v_F \cdot T = 10^2 - 10^4 \text{ nm} / \text{K} !!!$
- fáriskol. hossz:  $\sqrt{3-L} \leq 200 \text{ nm} / l \gg \lambda_F !!!$

Megj.: normál fém:  $\lambda_F = \sim \text{\AA}$   $E_F = \sim 1 \text{ eV}$

$\left( \text{fesz. emelte} \approx \frac{2\pi}{a} \right)$

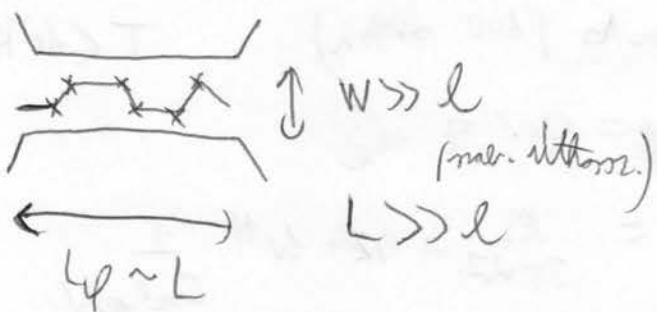
$\lambda_F = \frac{2\pi}{k_F} \approx a$  (H atomokkal felépítve a mű.-b)

Táto: elátk. önmérhető a mű. mértével!  $\rightarrow$  1 hfr.

- (2DEG) •  $n_s$  kicsi  $\Rightarrow$  elöljük jobb tel. ( $k_F, E_F, \lambda_F, \dots$ ) következik
- $E_F \approx 14 \text{ meV}$  kicsi  $\rightarrow$  nálunk önmérhető ellen
- nem les igaz a Bohr-Sommer.  
(alacsony kom.) sajátos!

### 3) Transport-tanulmányok

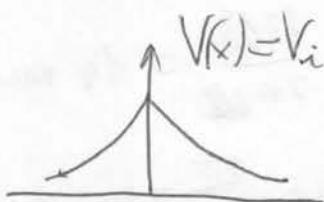
#### a) Diffúzió:



$e^-$  sebesség ütközések

Hogyan lehet, hogy 1 hfr.-el lehat, mégis van rengeteg ütközés?

pl.  $\delta$  pot.-on szorás



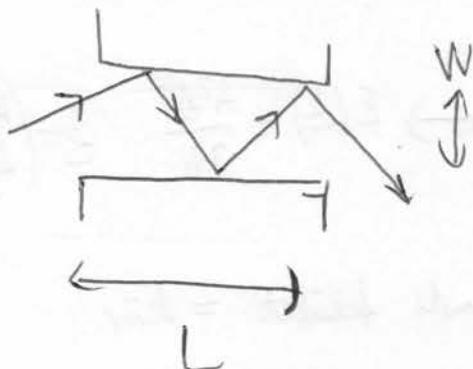
$V(x) = V_0 \cdot \delta(x)$   
 $\rightarrow$  lehat ütközések, de a hfr. folytatos!

⇒ kvantum kohärenz (nível  $L \approx L$ )



- gyenge lokalizáció
- univerzális vezetőképesség - fluktuációk ( $V \propto F$ )
- Tharonov-Bohm-effektus (ez lezajd a felületen)

b) ballistikus tart.



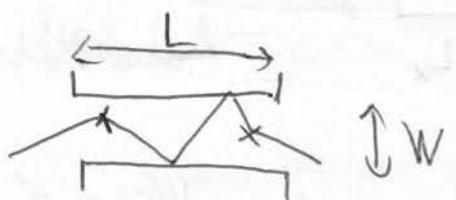
$$W, L < l$$

$$-\frac{\hbar^2}{2m} \Delta \Psi = E\Psi$$

$$\left( \Delta + \frac{2mE}{\hbar^2} \right) \Psi = 0$$

Kelvholz →  $(\Delta + k^2) \Psi = 0$   
egy. elosztásban +  $\Psi|_{\text{hat}} = \dots$

c) kvázi-ballistikus



$$W < l < L$$

van néhány közösségen, de nem való

ballistikus tart. -> fizikai részi

- van ellenállás, mert a részek (kontaktu) van viszonyai!



- önműködő ellenállás (Landau-Gütter - formula)

# Ravasz problemek

$$H\Psi = E\Psi$$

$$\hat{H} = \frac{(\vec{k} + e\vec{A})^2}{2m} + U(r)$$

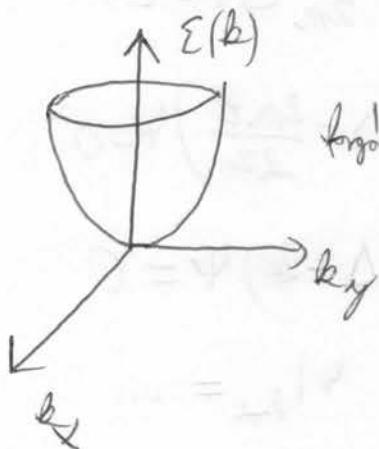
↑  
eff. tömeg

confining pot. (ezekkel pot. járás a körökön  
hosszú életre)

$$1) \text{ szab. el. 2D: } U(r)=0, \beta=0$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi = E\Psi$$

$$\Psi = \frac{1}{\sqrt{A}} \cdot e^{i\vec{k} \cdot \vec{r}} \Leftrightarrow \Sigma(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m} \quad \vec{k} = \begin{pmatrix} k_x \\ k_y \end{pmatrix}$$



fogási parabola

$\varepsilon = \text{kants felület} = \text{kör}$

$$\left( \frac{\sqrt{2m}\varepsilon}{\hbar^2} \right)^2 = k_x^2 + k_y^2$$

$"R"$



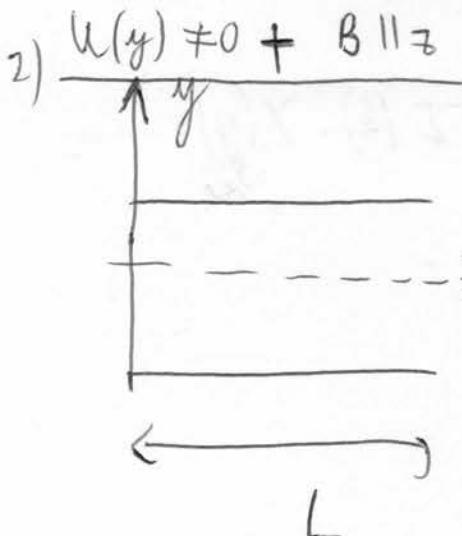
$k_y \rightarrow$  periodikus  
hosszfelület

$$N(\varepsilon) = \frac{R^2 \pi}{\frac{2\pi}{k_x} \cdot \frac{2\pi}{k_y}} \cdot 2 = \frac{2m\varepsilon}{\hbar^2} \pi \cdot \frac{k_x k_y}{(2\pi)^2} \quad \leftarrow N(\varepsilon) = \sum_{\vec{k}} \frac{1}{\varepsilon(\vec{k}) - \varepsilon}$$

$$S_{20} = \frac{1}{k_x k_y} \frac{dN}{d\varepsilon} = 2 \cdot \frac{2m\pi}{\hbar^2 \cdot (2\pi)^2} = \frac{m}{\pi \hbar^2} = \text{const}$$



$$N_S = S_{20} \cdot E_F = S_{20} \cdot \frac{\hbar^2 k_F^2}{2m} \Rightarrow k_F = \sqrt{2\pi n_S}, \quad v_F = \frac{\hbar k_F}{m}$$



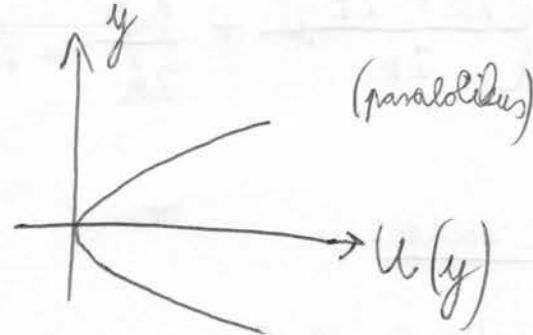
$$H = \frac{(A - qA)^2}{2m} + U(y)$$

$$q = -e$$

$$\underline{A} = \begin{pmatrix} -By \\ 0 \\ 0 \end{pmatrix} \quad \leftarrow \text{Standard-methode}$$

$$\underline{B} = \tau \underline{B} \underline{A} = \begin{pmatrix} 0 \\ 0 \\ \lambda_x A_y - \lambda_y A_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$$

$$H = \frac{(\lambda_x + eB \cdot y)^2}{2m} + \frac{\lambda_y^2}{2m} + U(y)$$



$y^1$  irányban a körök zártak  
be így a ZEG-ek

(változók inkább így:



(van min. pontok is)

↳  $y$  kmp. is van ( $\sim x$ ) → kisit bonyolultabb a megoldás módja

$\Rightarrow$   $\exists$  ismétel magn. h.

$$[\lambda_x, \hat{u}] = 0 \rightarrow \text{könig svr.-elk: } e^{ikx} \leftarrow \left( \frac{\partial}{i\lambda_x} \right) \varphi = \lambda_x \cdot \varphi$$

$\therefore$  el. h.  $\lambda_x = h \cdot k$

$$\varphi = e^{ikx}$$

$$\lambda_x = h \cdot k$$

$$\Rightarrow \psi(x, y) = e^{ikx} \cdot \chi(y)$$

$$H\psi = \left[ \frac{(hk + eBy)^2}{2m} + \frac{p_y^2}{2m} + U(y) \right] \cdot \chi_{nk}(y) = \varepsilon(k) \cdot \chi_{nk}(y)$$

spec. esetek:

a)  $U=0, B=0$

$$U = \frac{1}{2} m \omega_0^2 y^2$$

$$\left[ \frac{h^2 k^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2} m \omega_0^2 y^2 \right] \chi_{nk}(y) = \varepsilon_n(k) \cdot \chi_{nk}(y)$$

oscill.

$$\chi_{nk}(y) = e^{-\frac{m\omega_0}{h} y^2} \cdot H_n \left( \sqrt{\frac{m\omega_0}{h}} \cdot y \right)$$

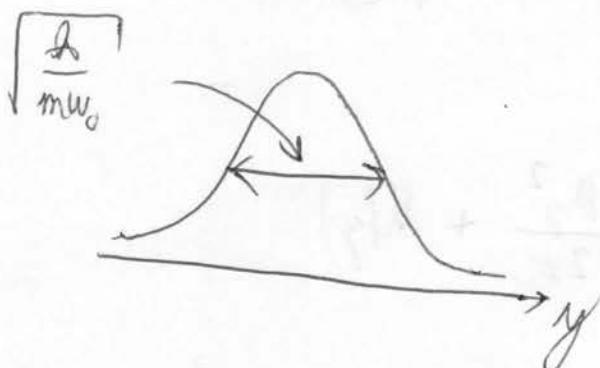
Gauss-gömb Kermit-pd.

↓  
karakter. form

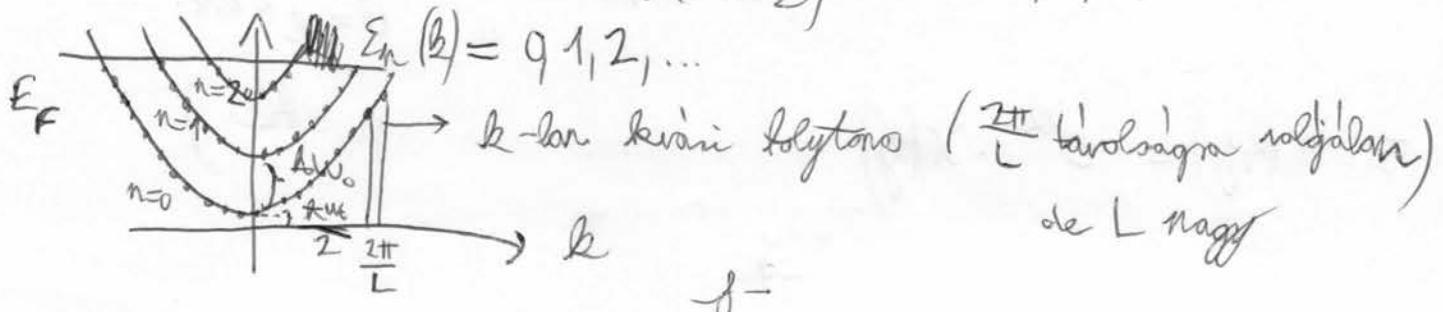
$$H_0(x) = \frac{1}{\pi^{1/4}}$$

$$H_1(x) = \frac{x}{\pi^{1/4}} \cdot x$$

$$H_2(x) = \frac{x^2 - 1}{\sqrt{2} \cdot \pi^{1/4}}$$



$$\varepsilon_n(k) = \frac{h^2 k^2}{2m} + \hbar \omega_0 \left( n + \frac{1}{2} \right) \quad n=0, 1, 2, \dots$$



$E_F$  meghat. hogy hányszámot használjanak

$$n = 0, \dots, M-1$$



$$\frac{1}{2} m_0 w_0^2 \left(\frac{w_p}{2}\right)^2 = E_F$$

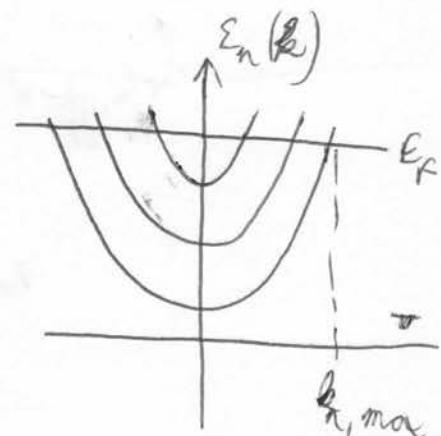
$$E_F \geq \hbar \omega_0 \left( n_{\max} + \frac{1}{2} \right)$$

$$n_{\max} \leq \frac{E_F}{\hbar \omega_0} - \frac{1}{2}$$

$$M(E_F) = n_{\max} + 1 = \text{Int} \left( \frac{E_F}{\hbar \omega_0} + \frac{1}{2} \right)$$

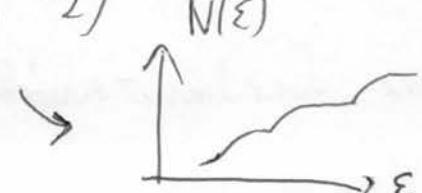
$$N(E_F) = 2 \sum_{n=0}^{M-1} \frac{\frac{1}{2} \cdot k_{n,\max}}{\frac{2\pi}{L}}$$

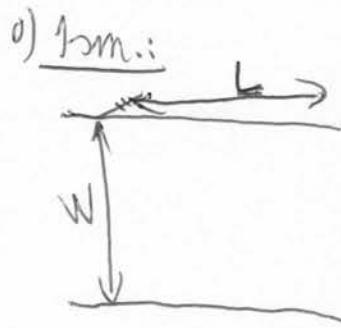
fül. miatt (par. zága)  
ellen fülv.-ra  
vannak az ill.-k



$$\frac{N(E_F)}{L} = \frac{2}{w_p} \cdot \sum_{n=0}^{M(E_F)-1} \sqrt{\left(\frac{k_F w_p}{\pi}\right)^2 - \frac{4 k_F w_p}{\pi} \left(n + \frac{1}{2}\right)}$$

$$\rho_{\text{par}}^0(\epsilon) = \frac{dN(\epsilon)}{d\epsilon}$$





2. ora

$$\hat{H} = \frac{(\Delta + q\hat{A})^2}{2m} + U(y)$$

↓ beräcks (confining) pot.

$$\underline{B} \neq 0 \quad + \quad \underline{A} = \begin{pmatrix} -By \\ 0 \\ 0 \end{pmatrix}, \quad \text{rot } \underline{A} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$$

$$H = \frac{(p_x + eBy)^2}{2m} + \frac{p_y^2}{2m} + U(y)$$

$$(p_x, \hat{A}) = 0 \rightarrow \hat{p}_x : \text{kk}$$

$$\psi(x, y) = e^{ikx} \cdot X(y)$$

↑  
ky  
↓

da  $B=0$ : 10 Schr. eignen

1) da  $B \neq 0$ , da  $U=0$

$$\left[ \frac{p_x^2}{2m} + \frac{(p_y + eBy)^2}{2m} \right] X_m(y) = \varepsilon_n(k) \cdot X_n(y)$$

mo.: Landau-nivek

$$\gamma_k = \frac{eB}{2m}, \quad \omega_c = \frac{eB}{m} \quad \text{iklotron frekvencia}$$

$$\left[ \frac{p_y^2}{2m} + \frac{1}{2} m \omega_c^2 (y + \gamma_k)^2 \right] X_{n,k}(y) = E_n(k) X_{n,k}(y)$$

$$X_{n,k}(y) = e^{-\frac{m\omega_c}{2}(y+\gamma_k)^2} \cdot H_n\left(\frac{m\omega_c}{2} \cdot (y+\gamma_k)\right)$$

Gauss-görbe      Hermite-polinom

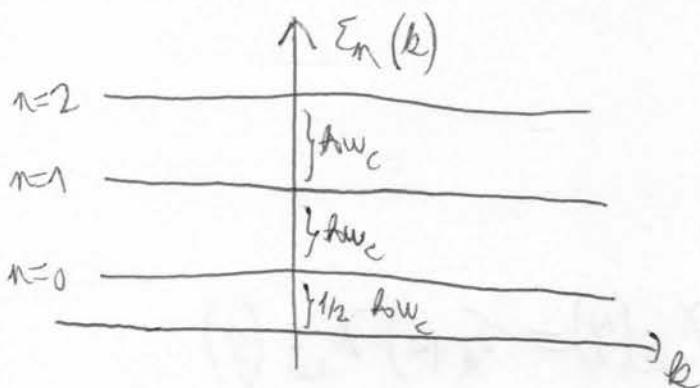
$$\underline{E_n(k) = \hbar \omega_c \cdot \left(n + \frac{1}{2}\right)}$$

$n = 0, 1, 2, \dots$

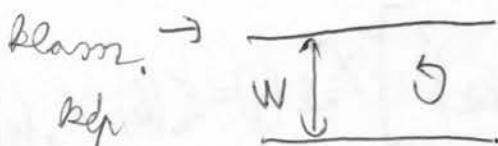
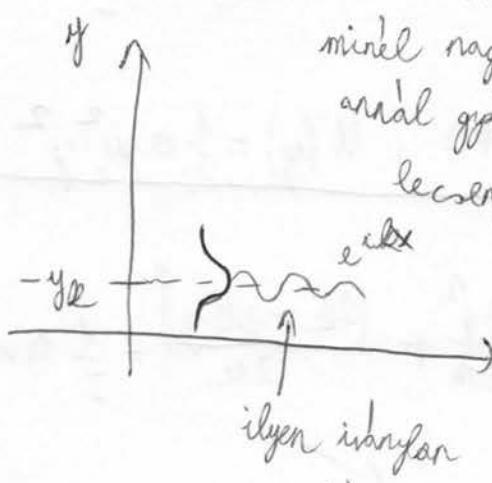
$$\frac{1}{\sqrt{B}} \sim \sqrt{\frac{\hbar}{m\omega_c}} = l_B$$

magnes hossz

minel nagyobb  $B$ ,  
annál gyorsabban  
leszeng a hfr.



$$v_S = \frac{1}{\hbar} \frac{\partial E_n(k)}{\partial k} = 0$$



nincs pot., ezért az  $e^-$  egy körfolyton halad  
↳ nem visz nettel áramot (nincs "drift")

- mintek degeneráciás, mert  $\times$  irányba periodikus  $\rightarrow$  határfejt.

$$\omega_k = \frac{2\pi}{L}$$

$$\rightarrow \Delta y_k = \frac{\Delta \omega_k}{eB}$$

$$N_0 = 2 \cdot \frac{W}{\Delta y_B} = 2 \cdot \frac{W}{\frac{\Delta 2\pi}{eB}} = 2 \cdot \frac{W \cdot L \cdot eB}{b \cdot 2\pi} = 2 \cdot \frac{\phi}{\frac{h}{e}} = 2 \cdot \frac{\phi}{\frac{h}{\phi_0}}$$

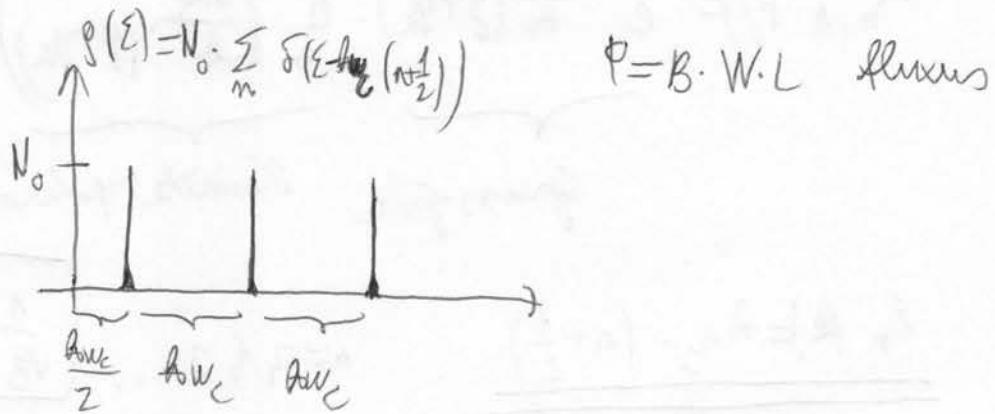
enzyi rész.

lehet a

Hörzben

~~m=0 ill. lass~~

$$\text{absol } \Phi = \frac{h}{e} \text{ flexurkantón}$$



$$2) \underline{B \neq 0}, \underline{U(y) = \frac{1}{2} m w_0^2 y^2}$$

$$\left[ \frac{p_y^2}{2m} + \frac{(h + eBy)^2}{2m} + \frac{1}{2} m w_0^2 y^2 \right] X_{nk}(y) = \varepsilon_n(\beta) \cdot X_{nk}(y)$$

teljes négyzetű alakítmánnal minősítjük az eloszó exet

$$\left[ \frac{p_y^2}{2m} + \frac{1}{2} m \frac{w_c^2 w_e^2}{\sqrt{2}} y_k^2 + \frac{1}{2} m \sqrt{2} \left( y + \frac{w_e^2}{\sqrt{2}} y_k \right)^2 \right] X_{nk}(y) = \varepsilon_n(\beta) \cdot X_{nk}(y)$$

$$w_c = \frac{eB}{m}, \quad \sqrt{2} = \sqrt{w_c^2 + w_0^2}, \quad y_k = \frac{h}{eB}$$

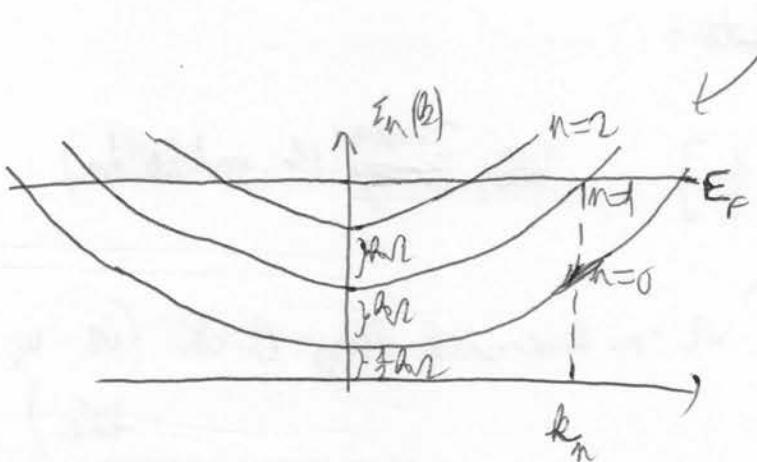
$$X_{nk}(y) = u_n \left( y + \frac{w_e^2}{\sqrt{2}} y_k \right), \quad u_n(x) = e^{-\frac{x^2}{2}} H_2(x)$$

$$q_r = \sqrt{\frac{m\omega_c}{\hbar}} y, \quad q_{\theta} = \sqrt{\frac{m\omega_c}{\hbar}} y_k$$

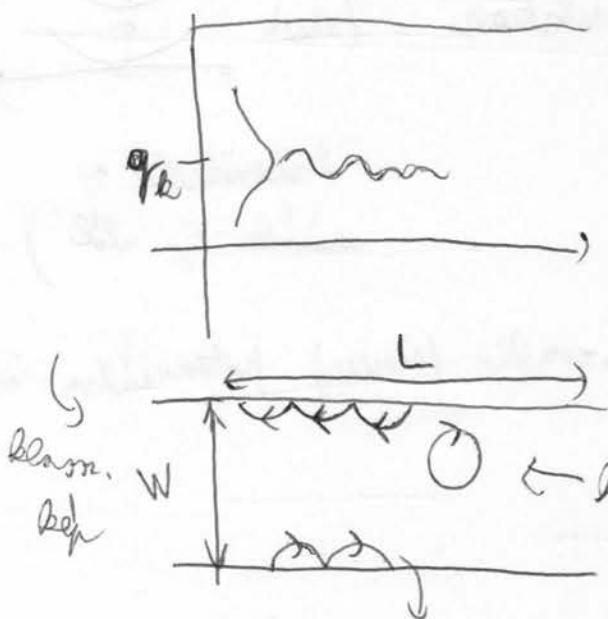
$$\Sigma_n(k) = \frac{1}{2} m \frac{w_0^2 w_c^2}{\sqrt{2}} \cdot y_k^2 + \left(n + \frac{1}{2}\right) \hbar \omega_c = \hbar \omega_c \left(n + \frac{1}{2}\right) + \frac{\hbar^2 k^2}{2m} \frac{w_0^2}{\sqrt{2}}$$

$$v_{\alpha} = \frac{1}{\hbar} \frac{\partial \Sigma_n(k)}{\partial k} = \frac{\hbar k}{m} \cdot \frac{w_0^2}{\sqrt{2}} \rightarrow \text{lin. k-lan!}$$

dirkelt nukle + parabola



Hullambr.:



↓ minél nagy  $k_n$ , annál jobban eltolódik ~ Gauss a berakts több. mielőre

↳ annál nagyobb les a csp. sebesség

skipping orbit  $\leftrightarrow$  edge state (Kvantum elnevezés)

$$l_B = \sqrt{\frac{\hbar}{m\omega_c}} \rightarrow \text{ha } B \text{ nagyobbra, kisebb a köríjűlök a hfr.-nek}$$

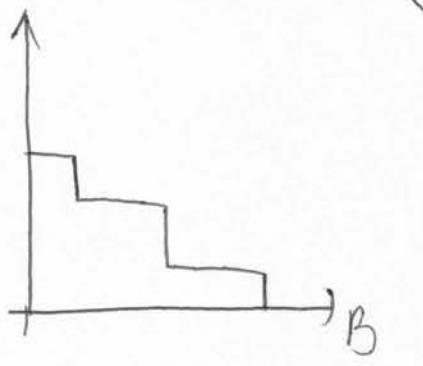
B nagy  
 ↳ ha van szörfpotencial  $\rightarrow$  az  $e^-$  átszabadhat műs imp. áll.-ba,  
 kivéve a skip. általá, műs kicsi az átlagos

$$\langle X_{n_2} | U | X_{n_1} \rangle \sim 0$$

||

↗ nincs műs a minta régi állapotokba is 'all.-ba'!  
 kvantum-Hall-effektus

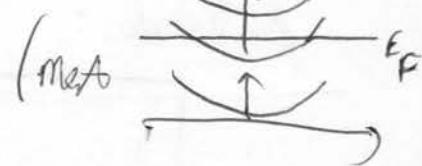
$$M(E_F) = \ln b \left[ \frac{e_F}{\hbar v_F} + \frac{1}{2} \right] \quad \text{all. } \frac{\pi^2}{h} \text{ (e. műsra)}$$



n-on keretnél függ B-től ( $\propto$   $v_F$ -n ker. függ B-től)

→ ha B nő, M csökken

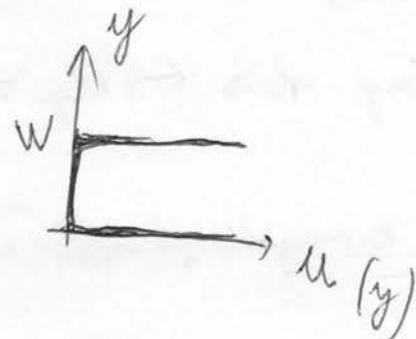
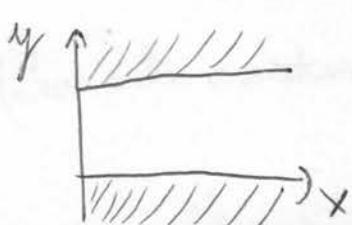
magnetic depopulation



kikenülnek a  
nincs  $E_F$  aleg)

(hasonlóan meg lehet vizálni hard-wall (meret) potenciálra is)

3) Hard wall pot.,  $B=0$ :



$$U(y) = \begin{cases} 0, & 0 < y < W \\ \infty, & y=0 \text{ vagy } y=W \end{cases}$$

$$H = \frac{p_x^2 + p_y^2}{2m}, \quad \Psi_{\text{harmonic}} = 0 \Rightarrow \Psi(y=0) = \Psi(y=W) = 0$$

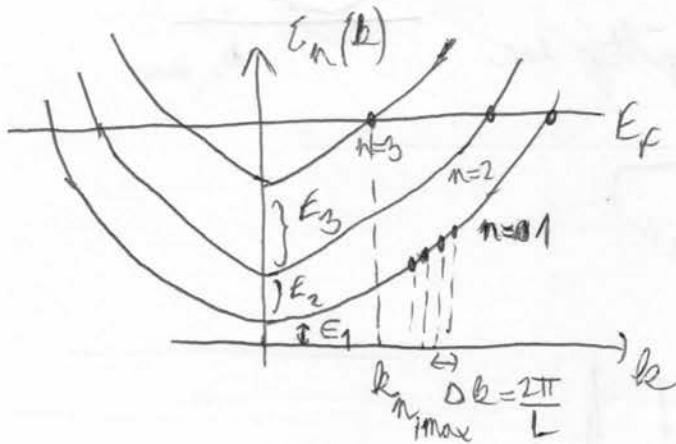
$$\boxed{H \Psi(x, y) = E \Psi(x, y)} \quad \text{normalizálás}$$

$$\Psi(x, y) = e^{ikx} \cdot \sin\left(\frac{n\pi y}{W}\right) \cdot \frac{1}{\sqrt{W}} \quad \text{ha } n=1, 2, 3, \dots$$

$(n=0 \text{ trivial mo.})$

$$E_n(k) = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m} \left(\frac{n\pi}{W}\right)^2 = E_1 + \frac{\hbar^2 k^2}{2m}$$

ahol  $E_n = E_1 \cdot n^2$ ,  $E_1 = \frac{\hbar^2}{2m} \cdot \left(\frac{\pi}{W}\right)^2$



↳ hasonlít a Landau-móddal, de negyzetesen ~~egy~~ többszöröszerűen terjednek a minélkül

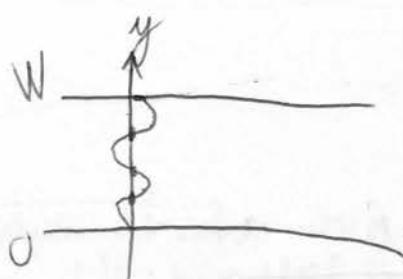
$$E_F \geq n_{\max}^2 \cdot E_1$$

$$\leftarrow M = n_{\max} = \ln \left[ \sqrt{\frac{E_F}{E_1}} \right] = \ln \left[ \sqrt{\frac{\frac{\hbar^2 k_F^2}{2m}}{\frac{\hbar^2}{2m} \left(\frac{\pi}{W}\right)^2}} \right] = \ln \left[ \frac{k_F W}{\pi} \right]$$

Keresztmódosítás

száma

$$\frac{\hbar^2 k_F^2}{2m} = E_F$$



→ azt kell leírni, mennyi komponens lehet a hard-wall-on belüli

normal ferde  $\lambda_F \sim \text{Å}$   $(\lambda = \frac{2\pi}{k} \frac{\lambda}{\pi/W})$

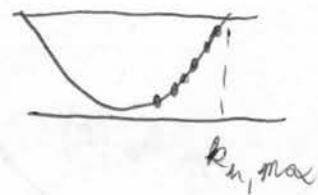
↓  
millios nagyságrendű (magyon nagy)

Alaposszámítás:

$$N(E_F) = 2 \cdot \sum_{n=1}^{m_{\max}} 2 \cdot \frac{k_{n,\max}}{\frac{2\pi}{L}} = 2 \cdot 2 \cdot \frac{L}{2\pi} \sum_{n=1}^{m(E_F)} \sqrt{(E_F - \epsilon_1 \cdot n^2) \frac{2m}{\hbar^2}}$$

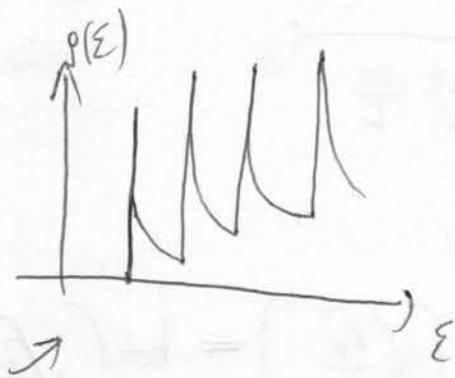
↑      ↑  
spin      min.  
szám      a parabolára

addit parabolán  
meneti pötty van



$$\Rightarrow \frac{N(E)}{L} = \frac{2}{W} \sum_{n=1}^{m(E)} \sqrt{\left(\frac{k_F \cdot W}{\pi}\right)^2 - n^2}$$

$$\rho(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon}$$



a confining (berendezés) miatt

a konst. ell. sér.-ell. illetve ugrásokat kaptunk

hard wall,  $B \neq 0$

Landau-métek

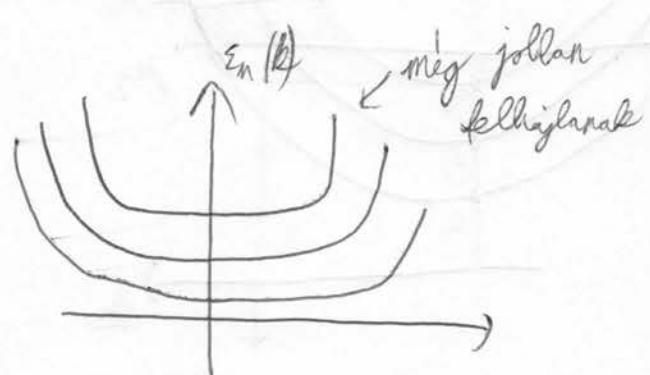
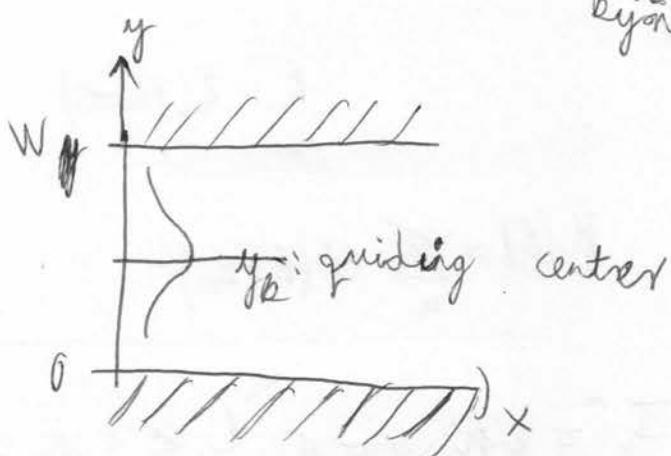
$B=0$  esetén megoldásait lineárikombináljuk, hogy a ~~termelhetetlen kiegészítők~~ megtaláljuk

csomkelt basis  $\Psi_{nk}(x,y) = \sqrt{\frac{2}{W}} \cdot \sin\left(\frac{n\pi y}{W}\right) \cdot e^{ikx}$

$$H = p_y + \frac{(h_0 b + eB_0 y)^2}{2m}$$

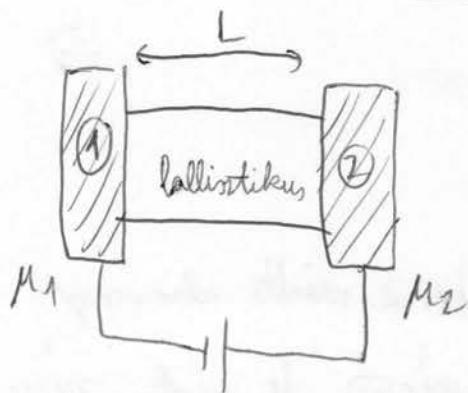
$$y \cdot \sin \frac{n\pi y}{W}$$

$\rightarrow H_{n,m}(k, B)$  (csomkolas es direktus k miatt alacsony  
k-mu ad jo eredményt, de azok sezen  
kijönnek)



- nullamfor, illapontositas jellegében hasonlít a parabolikus  
berzárás pot. megoldására

# Elektronos transport → vezetőképesség



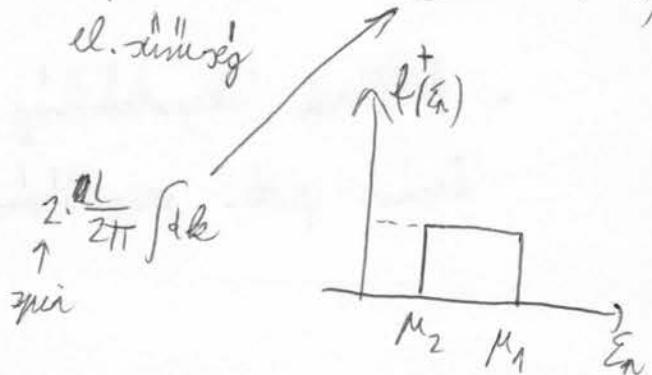
T=0 nőm.

félvessz: reflectionless contact

$$E_n = \epsilon_n (k=0)$$

$$M(\epsilon) = \sum_n \Theta(\epsilon - \epsilon_n)$$

$$I_n^+ = e N_0 \cdot v = e \cdot \frac{1}{L} \cdot \sum_k \frac{1}{2\pi} \frac{\partial \epsilon_n(k)}{\partial k} f^+(\epsilon_n)$$



$$\Theta = \frac{e}{L} \cdot 2 \cdot \frac{L}{2\pi} \cdot \frac{1}{\hbar} \int_{E_n}^{\infty} dk \frac{\partial \epsilon_n(k)}{\partial k} f^+(\epsilon_n) =$$

$$= \frac{2e}{\hbar} \int_{E_n}^{\infty} d\epsilon_n f^+(\epsilon_n)$$

$$I = \sum_n I_n^+ = \sum_n \frac{2e}{\hbar} \cdot \int_{E_n}^{\infty} d\epsilon f^+(\epsilon) = \frac{2e}{\hbar} \int_{-\infty}^{\infty} M(\epsilon) f^+(\epsilon) d\epsilon$$

Ha:  $M(\epsilon) = \text{const}$   $\forall \epsilon < \epsilon_1$

$$= \frac{2e}{h} M(\varepsilon_F) (\mu_1 - \mu_2)^{\text{bias}} = \frac{2e^2}{h} \cdot M (V_1 - V_2)$$

$$G_C = \frac{I}{V} = \left( \frac{2e^2}{h} \right) M (\varepsilon_F)$$

↑  
kontakthus  
miatti vezetőkép.

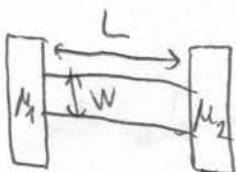
$$R_C = \frac{h}{2e^2} = 12,9 \text{ k}\Omega$$

(megj.: relativ. elm.  $\rightarrow C$  nagy }  
 kv. mech.  $\rightarrow h$  kicsi }  $\rightarrow$  ereket nem  
 látogatni

DE itt  $\frac{h}{2e^2} = 12,9 \text{ k}\Omega \rightarrow$  erő jól tudjuk  
 mérni )

3. óra

(im.)



$$I = G_C \cdot V \quad \text{ahol } V = \frac{\mu_1 - \mu_2}{e}$$

reflectionless contacts

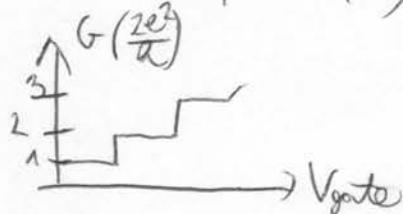
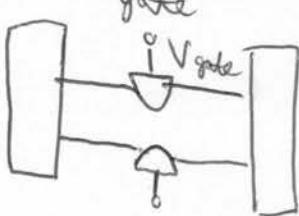
littlik, hogy:

$$G_C = \frac{2e^2}{h} \cdot M$$

$$R_C = \frac{h}{2e^2} = 12,9 \text{ k}\Omega$$

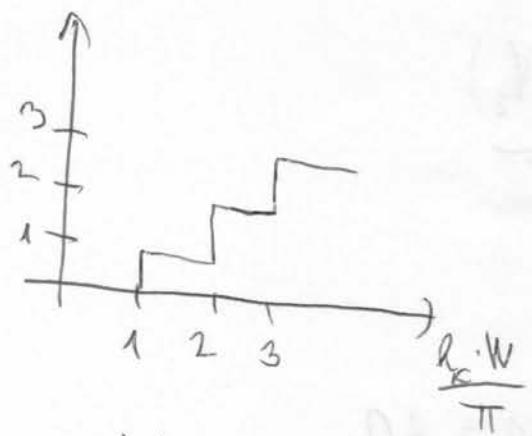
nyitott csatornák #

Kísérlet: B. F. Wees: PRL 60, 848 (1988)



az  $V_{\text{gate}}$ -tel bőven a kapcsolási részben hangsúlyozni (illetve  $W$  meghosszabbítani)  
 $\lambda_F = 30 \text{ nm}$   
 $W = 250 \text{ nm}$

$$M = \left[ \frac{L \cdot W}{\pi} \right] \rightarrow \text{egyszerűbb}$$



a beschränkt a  $E_F$  die es leisten kann, nicht von invasionslos  
ist, es kann egraaktive hard-wall a potencial

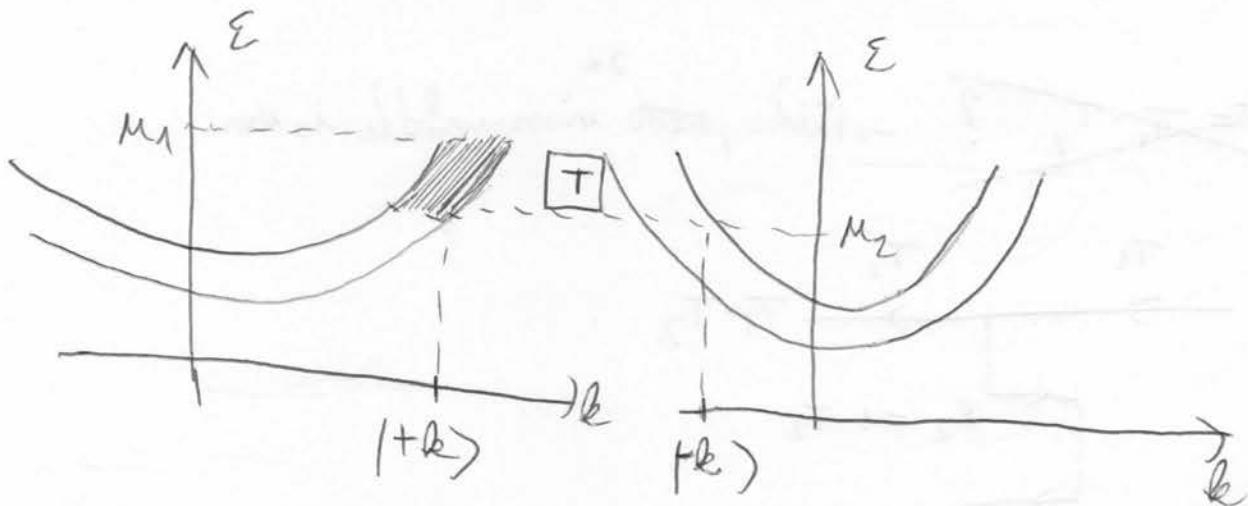
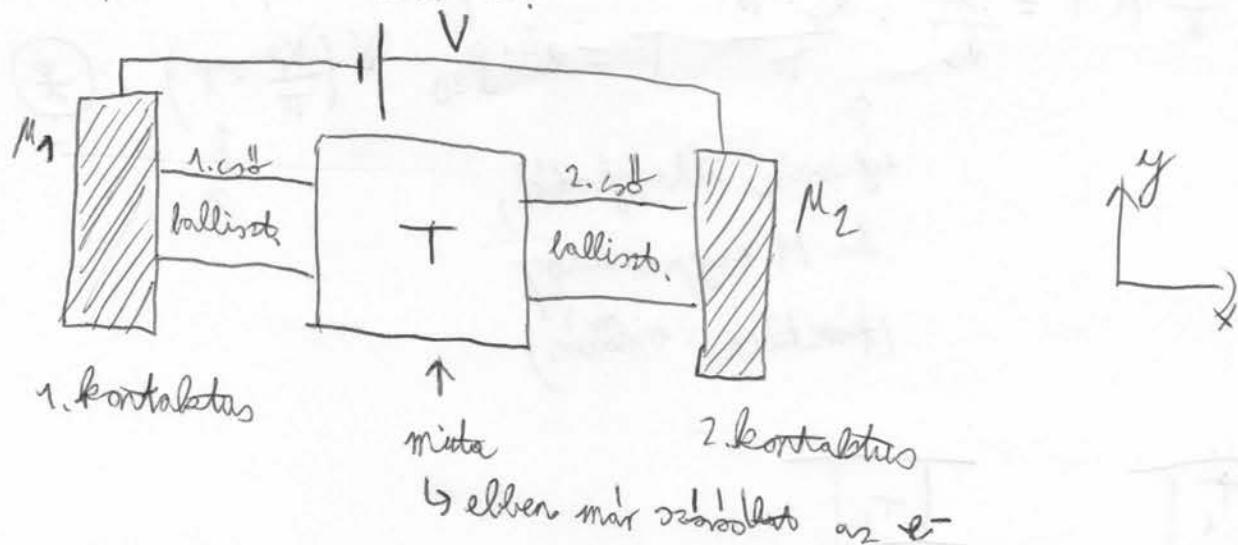
B - teber: PRB 31, 3625 (88)



$\Rightarrow$  magnetic depop. mitts (da  $B_{\text{ext}}$  reversibl var bez  $E_F$  alott) wökkén M

## Landauer - formula

Mit van hoe van viszszáns is?



1. csökkent a műtakai jövevénnyel

$$I_1^+ = \frac{2e}{h} \cdot M (\mu_1 - \mu_2)$$

→ műtakai kimenetű áram + irányla

$$I_2^+ = I_1^+ \cdot T = \frac{2e}{h} M \cdot T (\mu_1 - \mu_2)$$

viszszáns:  $I_1^- = \frac{2e}{h} M (\mu_1 - \mu_2) \cdot (1-T)$

net áram:  $I = I_1^+ - I_1^- = I_2^+ = \frac{2e}{h} M \cdot T (\mu_1 - \mu_2) = \frac{2e^2}{h} M \cdot T \cdot V$

(kr. meghatározás  
M és T-ben  
van)

$$\frac{I}{V} = G = \frac{2e^2}{h} M \cdot T$$

## Ohm - tör.

$$G = \frac{2e^2}{\pi} \cdot M \cdot T = \frac{2e^2}{\pi} \cdot \frac{R_F \cdot W}{\pi} \cdot T = e^2 \cdot \rho_{20} \cdot W \cdot \left( \frac{R_F}{\pi} \cdot T \right)$$

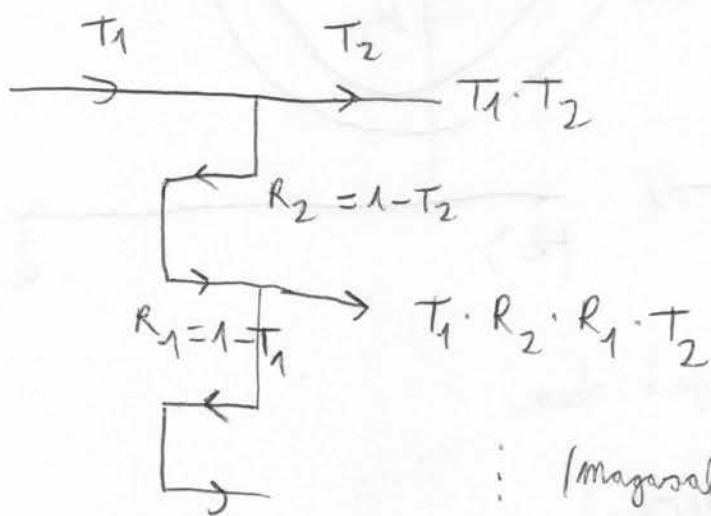
↑  $\frac{m}{4\pi^2 \cdot \text{mill}}$  ↑  $\frac{\text{ohm}}{m}$

↑  
egyszerű ellagnásból,  
ha M nagyon nagy  
(fémekben milliók)

(\*)



~~$T = T_1 \cdot T_2$  ?~~ → Rövid, mert viszonylagosan is rán



$$T_{12} = T_1 \cdot T_2 + T_1 \cdot T_2 \cdot R_1 \cdot R_2 + T_1 \cdot T_2 \cdot (R_1 \cdot R_2)^2 + \dots \stackrel{R_1 \cdot R_2 < 1}{=} \frac{T_1 \cdot T_2}{1 - R_1 \cdot R_2}$$

belátható, hogy

$$\frac{1 - T_2}{T_{12}} = \frac{1 - T_1}{T_1} + \frac{1 - T_2}{T_2}$$

$\Rightarrow$   $\frac{1 - T}{T}$  additív

$N$  dr számcenzum:

$$\frac{1-T(N)}{T(N)} = N \cdot \frac{1-T}{T} \quad (\text{additivitás miatt})$$

$$\Downarrow$$

$$T(N) = \frac{T}{N \cdot (1-T) + T}$$

$$N = v \cdot L$$

→ egységesi hosszon a számcenzumnak #

$$T(L) = \frac{L_0}{L+L_0} \quad \text{és} \quad L_0 = \frac{T}{v(1-T)}$$

||

$$\star \Rightarrow G = \frac{W}{L+L_0} e^2 \cdot \rho_{20} \cdot \frac{\nu_F \cdot L_0}{\pi} \quad \begin{matrix} := D \\ \text{other telj. hossz} \end{matrix} \quad \frac{\nu_F \cdot L_0}{\pi} = D \quad \cancel{\text{---}}$$

$\sigma$

Ohm-tv.

Einstein-relativitás

$\uparrow$  diff. sze.

$\uparrow$  elv fogj. belátni,

hogy ez tényleg a diff. allando

$$G = \frac{W}{L+L_0} \cdot \sigma$$

$$R = \rho \cdot \frac{l}{A}$$

$$G = \frac{1}{\rho} \cdot \frac{A}{l} = \sigma \cdot \frac{A}{l} \quad (20-\text{ban} \quad [A] = [l] = m \Rightarrow [G] = [\sigma])$$

diffrakt: (Einstein-rel. levez.)

$$F = -e \cdot D \cdot \text{grad } n = -e \cdot D \cdot \frac{\rho_{20} (M_1 M_2)}{L} \cdot \hat{x} = -e^2 D \cdot \rho_{20} \cdot \frac{M_1 M_2}{L} \hat{x} =$$

$$\approx -(\text{grad } V) = |E|$$

$$= -e^2 \cdot \rho_{20} \cdot D \cdot E \equiv \sigma E$$

$$\boxed{\sigma = e^2 \rho_{20} \cdot D}$$

Einstein-relativ

$$L^2 \sim \langle x^2 \rangle = D \cdot T \xrightarrow{\text{måndit i tiden} \rightarrow \text{relax. id}}$$

$$\underbrace{v_F \cdot t \cdot l}_{\ell} \quad \boxed{D = v_F \cdot l}$$

$$L_0 = ? \quad \cancel{\text{formel tillbörda}}$$

$\ell$ : (måndit i tiden)

$\ell \cdot r$  är ströccentrum  $\ell$  horison

$$(1-t) \cdot r \cdot \ell \approx 1 \rightarrow (\cancel{\ell}) \text{ måndit i tiden} 1 \text{ sända van}$$

~~(han lär inte)~~

~~██████████~~

$$L_0 = \frac{T}{\sqrt{1-t}} = T \cdot \ell \Rightarrow L_0 \sim \ell \Rightarrow L \gg L_0 \sim \ell$$

$L_0$  def. period

$$\Rightarrow G = \frac{W}{L} \cdot \sigma \approx \frac{W}{L} \cdot \sigma$$

Mit vettink bekräfta leverettsle?  $\rightarrow$  valorenvisges  
 ftransform). "preadattuk (har  $\rightarrow$  ref. mätaff ~~██████████~~  $\frac{1-t}{T} \rightarrow$   
 attuk  $\sigma$ ). Eben nioes beror en elektronik interferensija!

$\frac{1-T}{T}$  additiv:

$$G^{-1} = \frac{\lambda}{2e^2 M} \cdot \frac{1}{T} = \underbrace{\frac{\lambda}{2e^2 M}}_{G_c^{-1}} + \underbrace{\frac{\lambda}{2e^2 M} \cdot \frac{1-T}{T}}_{G_s''^{-1}}$$

(a) Landauer-form  
fan att. av  
ellenalls  
adödik  
"zwe")

kontakt-  
ellenalls

mötcentrum  
ellenallsa

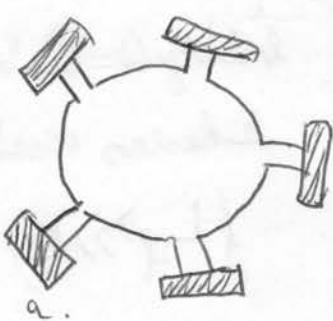
Töle terminal ( $T=0$ )

1) (Landauer - Büttiker - formula)

$$I = \frac{2e}{h} \cdot M \cdot (\mu_1 - \mu_2) = \frac{2e}{h} \cdot \bar{T} \cdot (\mu_1 - \mu_2), \quad \bar{T} = M \cdot T$$

$$0 < T < 1$$

DE  $\bar{T}$  mår nem



$$I_a = \frac{2e}{h} \cdot \sum_b (\bar{T}_{a \leftarrow b} \cdot M_b - \bar{T}_{b \leftarrow a} \cdot M_a)$$

$$I_a = \sum_b (G_{ab} \cdot V_b - G_{ba} \cdot V_a) = \sum_b G_{ab} V_b - V_a \sum_b G_{ba} =$$

$G_{ab} = \frac{2e^2}{h} T_{a \leftarrow b}$

$$= \sum_b G_{ab} V_b - V_a \sum_b G_{ab} =$$

$$= \sum_b G_{ab} (V_b - V_a) \quad \square$$

Rd.  $V_a - v_a$   $V_a$   $\rightarrow$   $v_a$   $\rightarrow$   $v_a$   
elektra

$\Rightarrow I_a = 0$   
well legen  $\Rightarrow \sum_b G_{ab} = \sum_b G_{ba}$   $\rightarrow$   $\sum_b G_{ab} (V_b - V_a)$   $\rightarrow$   $I_a = 0$   $\rightarrow$   $I_a = 0$

isregzalby

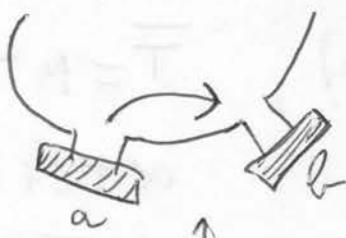
$\Rightarrow I_a = \sum_b G_{ab} (V_b - V_a) \rightarrow$  (es lehrgaben  
~~aus programm.~~  $\rightarrow$   $I_a = 0$ )

und  $G_{ab} = \frac{2e^2}{h} T_{a \leftarrow b}$

Upl. magn. Umlen

$$G_{ba}(\underline{B}) \neq G_{ab}(\underline{B})$$

reciprocitas:



magn. Umlen a

polymer nem megfordithats!

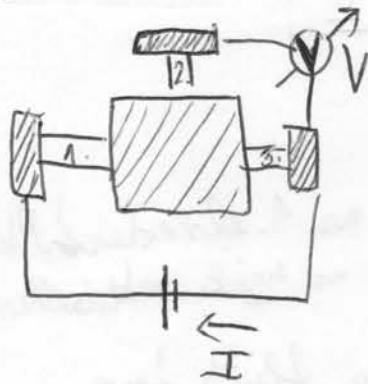
$$\text{fde } G_{ba}(\underline{B}) = G_{ab}(-\underline{B})$$

igaz lenne

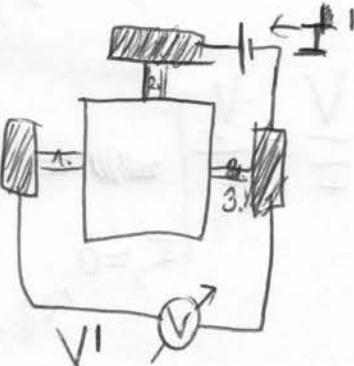
beléjtük késle  
kohesiv vezeték  
( $L_d \gg L$ )

3) nelda: 3 terminal

a) 3-terminal



reciprocal m. (Megcsereljük az  $\rightarrow$   
 $\leftarrow$   $V$  mellett)



$$R_{36} = \frac{V}{I}$$

↑  
3-terminal

$$R'_{36} = \frac{V'}{I}$$

$$I_1 = \sum_k G_{1k} (V_0 - V_1) = G_{12} (V_0 - V_1) + G_{13} (V_0 - V_3)$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -G_{12} - G_{13} & G_{12} & G_{13} \\ G_{21} & -G_{21} - G_{23} & G_{23} \\ G_{31} & G_{32} & -G_{31} - G_{32} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$\Rightarrow I_1 + I_2 + I_3 = 0$$

↓

$$\text{legen } V_3 = 0$$

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -G_{12} - G_{13} & G_{12} \\ G_{21} & -G_{21} - G_{23} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}, \quad R = \begin{pmatrix} G \end{pmatrix}^{-1}$$

$$R'_{36} = \frac{V}{I^1} = \left. \frac{V_1}{I_1} \right|_{\substack{I_2=0}} = R_{21} \quad \rightarrow \text{az 1. elrendszelőkkel}$$

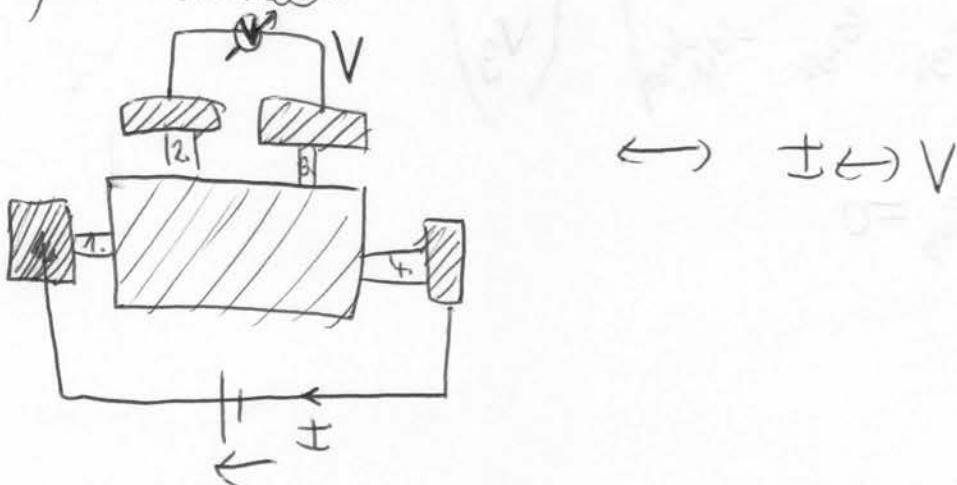
az eggyik matrixelemet megjukkanak

$$R'_{36} = \frac{V}{I^1} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = R_{12} \quad \text{az 2. művelet nem folyik át annak}$$

ha fordítva megyük az áramot  
és az 2.-et, az indexek  
megcsereklődnek

(~~aztán~~ (azben) ez nem illeti  
többi, mint van egy invertálás  
is)

b) 4 terminal:



$$V_4 = 0$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -G_{12}-G_{13}-G_{14} & G_{12} & G_{13} \\ G_{21} & -G_{21}-G_{23}-G_{24} & G_{23} \\ G_{31} & G_{32}-G_{34} & -G_{31}-G_{32}-G_{34} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

reciprocate m.

$$\Rightarrow \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = R \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$$R_{46} = \frac{V}{I} = \frac{V_2 - V_3}{I_1} \quad \left| \begin{array}{l} I_2 = I_3 = 0 \\ I_1 \neq 0 \end{array} \right. = R_{21} - R_{31}$$

$$R_{46}^1 = \frac{V}{I} = \frac{V_1}{I_2} \quad \left| \begin{array}{l} I_1 = 0 \\ I_2 = -I_3 \end{array} \right. = R_{12} - R_{13} \quad \xrightarrow{\text{reciprocitas}}$$

4. brau

X.07.

rechts:  $R_{36} = R_{21}$

$R_{36}^1 = R_{12}$

3 term.

$R_{46} = R_{21} - R_{31}$

$R_{46}^1 = R_{12} - R_{13}$  4 term.

reciprocitätstetes:

1) Makroskop. m.-re: stronger

Makroskopikus m.-re:

$\downarrow R_{46}(\underline{B})$  fluktuiert

eine is igas:  $G_{ab}(\underline{B}) = G_{ba}(-\underline{B})$  (ebel kov. a reciprocitas)

$$\begin{cases} R_{46}(\underline{B}) = R_{46}^1(-\underline{B}) \\ R_{36}(\underline{B}) = R_{36}^1(-\underline{B}) \end{cases}$$

$$R \stackrel{\text{h\"older bel\'atjuk}}{=} \underline{\underline{G}}^{-1} \underline{\underline{G}}$$

reciprocity m\'eth

$$\underline{\underline{R}}^{-1} \Big|_{\underline{\underline{B}}} = (\underline{\underline{R}}^{-1})^T \Big|_{-\underline{\underline{B}}} \quad (\underline{\underline{R}}^{-1})^T = (\underline{\underline{R}}^+)^{-1}$$

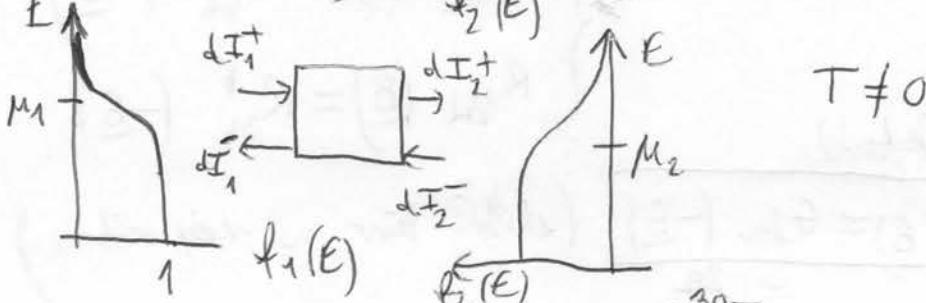
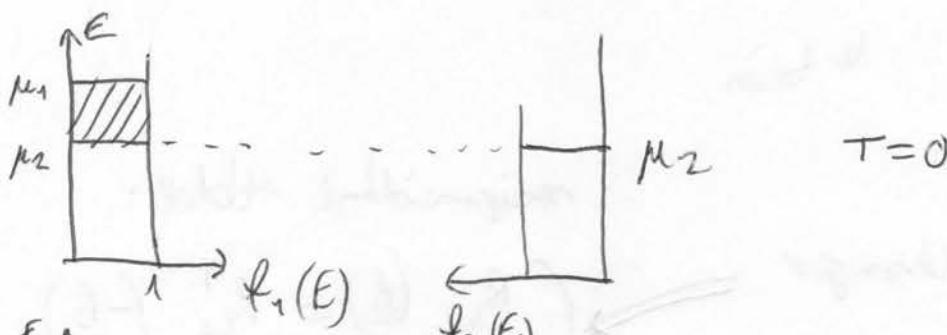
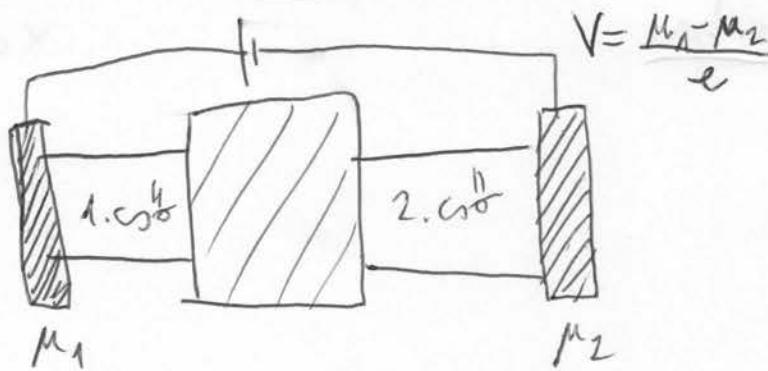
$$= (\underline{\underline{R}}^{-1})^{-1} \Big|_{\underline{\underline{B}}} = ((\underline{\underline{R}}^{-1})^T)^{-1} \Big|_{-\underline{\underline{B}}} = \underline{\underline{R}}^+ \Big|_{-\underline{\underline{B}}} \Rightarrow \boxed{\underline{\underline{R}}(\underline{\underline{B}}) = \underline{\underline{R}}^T(-\underline{\underline{B}})}$$

$$R_{31} \Big|_{\underline{\underline{B}}} = R_{21} \Big|_{\underline{\underline{B}}} = R_{12} \Big|_{-\underline{\underline{B}}} = R_{31}^+ (-\underline{\underline{B}});$$

(az 1. k\'epesegh\'el ez  
k\"ov. az elterjed\l as\'er)

$$R_{41} \Big|_{\underline{\underline{B}}} = R_{21} - R_{31} \Big|_{\underline{\underline{B}}} = R_{12} - R_{13} \Big|_{-\underline{\underline{B}}} = R_{41}^+ \Big|_{-\underline{\underline{B}}} \quad \checkmark \text{ (2. k\'aptuk  
m\'akr.-ra)}$$

2)  $T \neq 0$  h\"om\"erkelhet\l



$$dI_1^+ = \frac{2e}{h} \cdot M \cdot f_1(E) dE$$

kontaktszabály  
direct jövő  
áram

$$f_1(E) = \frac{1}{e^{B(E-\mu_1)} + 1}$$

$$dI_2^- = \frac{2e}{h} M' \cdot f_2(E) dE$$

$$dI_2^+ = T \cdot dI_1^+ + (1-T') \cdot dI_2^-$$

$$dI_1^- = (1-T) dI_1^+ + T' dI_2^-$$

net áram:

$$\begin{aligned} dI &= dI_1^+ - dI_1^- = dI_2^+ - dI_2^- = T \cdot dI_1^+ + (1-T') dI_2^- - dI_2^- = \\ &= T \cdot dI_1^+ - T' dI_2^- = \frac{2e}{h} M(E) \cdot T(E) \cdot f_1(E) dE - \frac{2e}{h} M'(E) T'(E) \cdot \\ &\quad \cdot f_2(E) dE \end{aligned}$$

$$\text{jelölés: } \overline{T}(E) = M(E) \cdot T(E)$$

$$\text{Ha: } \overline{T}(E) = \overline{T}'(E) \Rightarrow \boxed{I = \frac{2e}{h} \int \overline{T}(E) \cdot [f_1(E) - f_2(E)] dE}$$

belátjuk, hogy kábeleket vezetőkre igaz

$\downarrow$   
ez nem lin. függ.  
 $(\mu_1 - \mu_2) \sim V$ -nek

$-3t$  Mikor lehetsően?

• ha  $\mu_1 = \mu_2 \Rightarrow I=0 \Rightarrow f_1(E) = f_2(E)$

• ha  $\mu_1 - \mu_2$ : "kicsi"

$$\delta I = \frac{2e}{\hbar} \int [F(E)|_{eq} \cdot \delta(f_1 - f_2) + (f_1 - f_2)|_{eq} \cdot \delta F(E)] dE$$

eq: egyenlő lenne  $\mu_1 = \mu_2$

$0 \leftarrow \mu_1 = \mu_2$  egyszerűbbé válik

$$\delta(f_1 - f_2) = (\mu_1 - \mu_2) \cdot \frac{\partial f_0}{\partial \mu}|_{eq} = -\frac{\partial f_0}{\partial E} \cdot \frac{(\mu_1 - \mu_2)}{\mu_1}$$

$$f_0(E) = \frac{1}{e^{\beta(E-\mu)} + 1} \quad \mu = E_F$$

$$G = \frac{\delta I}{(\mu_1 - \mu_2)/2} = \frac{2e^2}{\hbar} \int F(E) \cdot \left( -\frac{\partial f_0}{\partial E} \right) dE \quad T \neq 0$$

$$(ha T \rightarrow 0 \Rightarrow f_0(E) = \Theta(E_F - E))$$

$$\Rightarrow -\frac{\partial f_0}{\partial E} = \delta(E_F - E)$$

$$G = \frac{2e^2}{\hbar} \int F(E_F) \quad \leftarrow \text{ezt kaptuk mitte! } \checkmark$$

Kérdés: Mi az, hogy  $\mu_1 - \mu_2$  "kicsi"?

Mekkkor  $F(E) \rightarrow$  kv. mecha

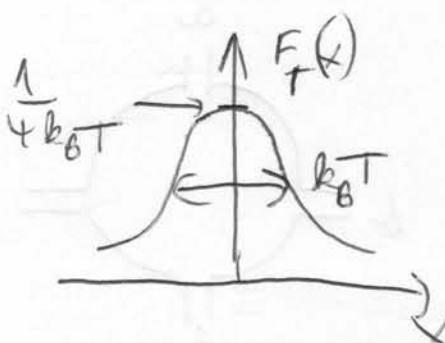
$$f_1(E) - f_2(E) = \int_{\mu_2}^{\mu_1} \left( \frac{d}{dE'} \frac{1}{e^{\beta(E-E')} + 1} \right) dE' = \int_{\mu_2}^{\mu_1} \left[ -\frac{d}{d(E-E')} \frac{1}{e^{\beta(E-E')} + 1} \right] dE'$$

$\underbrace{\hspace{10em}}$

$$\equiv F_+(E-E')$$

more..

$$F_+(x) = -\frac{d}{dx} \frac{1}{e^{\beta x} + 1} = \frac{1}{4k_B T} \cdot \frac{1}{\frac{\partial^2 \beta x}{2}}$$



beskonyg für.

$$f_1(E) - f_2(E) = \int_{\mu_2}^{\mu_1} F_+(E-E') dE'$$

$$I = \frac{2e}{h} \int \bar{F}(E) \left[ \int_{\mu_2}^{\mu_1} F_+(E-E') dE' \right] dE =$$

$$= \int_{\mu_2}^{\mu_1} \frac{2e}{h} \left[ \int_{-\infty}^{\infty} \bar{F}(E) F_+(E-E') dE \right] dE' =$$

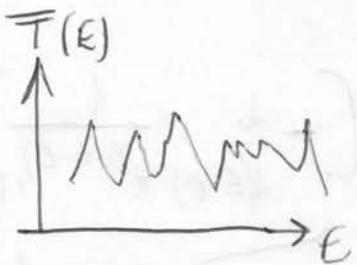
$\underbrace{\hspace{10em}}$

$$= \frac{1}{e} \int_{\mu_2}^{\mu_1} \hat{G}_+(E') dE'$$

$\hat{G}_+(E') = \frac{2e^2}{h} \int \bar{F}(E) F_+(E-E') dE$

$\rightarrow$  konvolució  $\Rightarrow$   $k_B T$  több. -ban  
lesz érválasztás

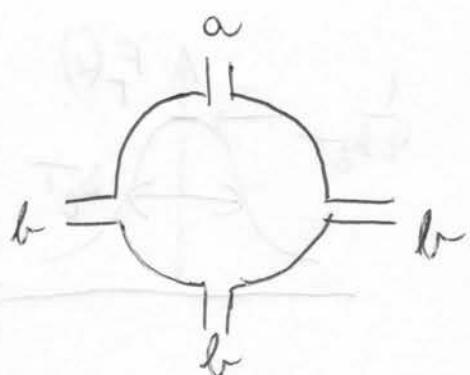
az így való: az  $\hat{G}_+(E)$  nem (gyengén) függ  $E$ -től



bei  $M_1 - M_2 \ll k_B T$ , also a so-called  
bias  
eleverstet

### 3) $T \neq 0$ Romers Regel, fiktiver terminal

$$I_a = \frac{2e}{h} \sum_b \left[ \bar{T}_{ab}(E) f_b(E) - T_{ba}(E) f_a(E) \right]$$



isergesäßlich:  $\sum_b \bar{T}_{ab}(E) = \sum_b \bar{T}_{ba}(E)$  nem függ b-tól ]

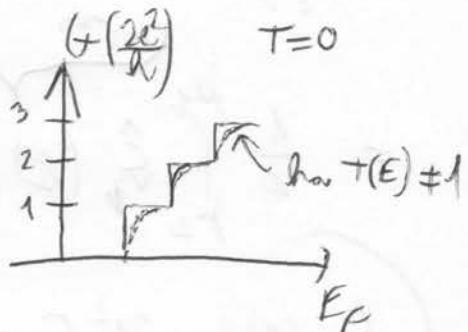
$$\Rightarrow I_a = \frac{2e}{h} \int \sum_b \bar{T}_{ab}(E) \left( f_b(E) - f_a(E) \right) dE$$

$$I_a = \sum_b G_{ab} (N_b - V_a)$$

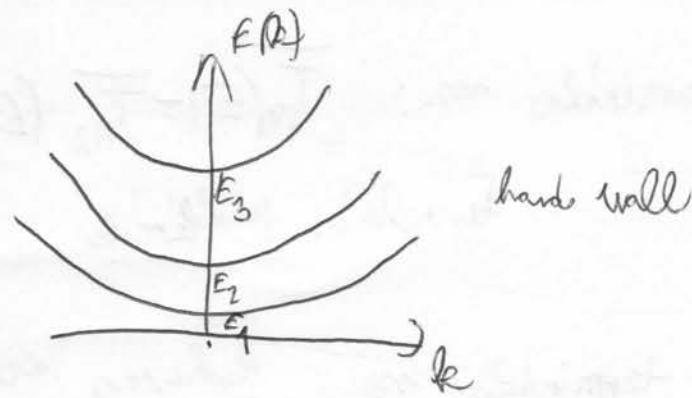
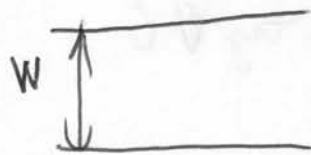
ahol

$$G_{ab} = \frac{2e^2}{h} \int \bar{T}_{ab}(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

$$(T=0 : G_{ab} = \frac{2e^2}{h} \bar{T}_{ab}(E_F))$$



4)



| $W$    | $E_1$ |
|--------|-------|
| 250 nm | 64 mK |
| 10 nm  | 40 K  |
| 1 nm   | 400 K |

$$E_n = \hbar^2 \cdot E_1$$

$$E_1 = \frac{\hbar^2 \pi^2}{2 m_e \cdot W^2}$$

minél vékonyabb  $\Leftarrow$

~ szabag, szabahom-en

annál jobban átellen ~~ágyatolt~~ en. szinteket

dekor jól, ha külön áltam letek a  
szinteket  $\Rightarrow k_B T \ll E_1$

5) Kell-e használni a Pauli-elvét?

$$I_a = \frac{2e}{h} \sum_b \left[ f_b(E) f_a(E) - \cancel{f_a(E) f_b(E)} \right] dE$$

$$f_b \cdot (1-f_a)$$

Pauli-elv minden e- mehet csak  $e^-$   $\rightarrow$  ebb is figyelembe kell venni

$$f_a \cdot (1-f_b)$$

$$I_a = \frac{2e}{h} \left\{ \sum_b \overline{f}_b f_a - \overline{f}_a f_a \right\} dE$$

- 2 terminalis m.:  $\bar{T}_{21}(E) = \bar{T}_{12}(E) \Rightarrow$  az extra tag <sup>mindig</sup> 0  
tunelési probléma
  - több terminalis m.: köhözvetítés az extra tag = 0  
(decschel belátás)
- $\Rightarrow$  a torábbírában ~ Pauli-elvvel nem kell figy. vennünk,  
mert csak ~~több~~ (minimál) köhözvetítés nélunk

Fótbírákkal:

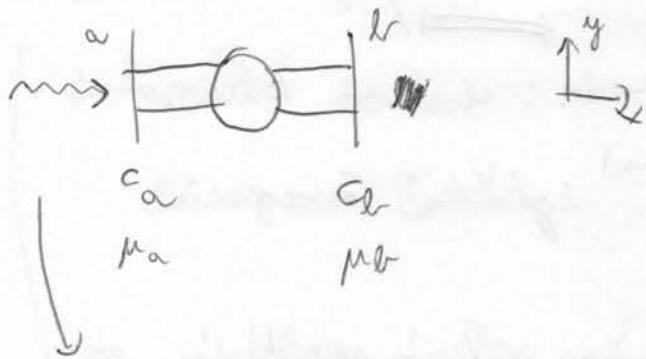
$$\bar{F}(E) \leftarrow \underline{\underline{S}}(E) \leftarrow \text{Green-f}: \hat{G}(E) = (E - \hat{A})^{-1}$$

szorosmatrix,  
 $S$  matrix  $\leftarrow$  (node matching  
... )



5. óra

## Kohérens veretű



$$\psi_p = x^{(a)}(y_a) \cdot \frac{1}{\sqrt{L}} \cdot e^{ik_p^{(a)} p} \cdot x_a$$

↓  
haladt ( $x_a$  irányban)

Kohérens modulus  $\leftarrow$  adott berakás pot. -lól határozandó meg

pl. hard wall;  $B=0$

$$\frac{\pi^2}{2m} \psi = E \psi$$

$$\frac{\pi^2}{2m} \left( i k_p^{(a)} \right)^2 + \frac{B^2}{2m} \left( \frac{p \pi}{w_a} \right)^2 = E$$

$$\frac{\pi^2}{2m} \left( k_p^{(a)} \right)^2 + \left( \frac{p \pi}{w_a} \right)^2 = E$$

$$-\frac{\hbar^2}{2m} \left( i k_p^{(a)} \right)^2 + \frac{B^2}{2m} \left( \frac{p \pi}{w_a} \right)^2 = E$$

$$\frac{\hbar^2}{2m} \left[ \left( k_p^{(a)} \right)^2 + \left( \frac{p \pi}{w_a} \right)^2 \right] = E = \frac{\hbar^2}{2m} k_F^2$$

$$k_p^{(a)} = \sqrt{k_F^2 - \frac{p^2 \pi^2}{w_a^2}}$$

$$p \leq \left( \frac{k_F \cdot w_a}{\pi} \right) = M_a \text{ vezetőmodul}$$

meljük minden végünk  
 $\Rightarrow$  ha adott, hogy ~~ha~~  $k_F$  ~~adott~~  $\Rightarrow$   $p$ ,  
 az definíciója a haladó nullam  
 impulsusát (az en. miatt)

"a" dd.:  $\Psi_p^{(a)}(x_a, y_a) = \underbrace{x_p^{(a)}(y_a)}_{\text{befindt sich}} \cdot \frac{1}{\sqrt{L}} e^{ik_p^{(a)} x_a} + \sum_{m \neq a} x_m^{(a)}(y_a) \hat{r}_{mp}^{(aa)} \frac{1}{\sqrt{L}} e^{-ik_m^{(a)} x_a}$

"b" dd.:  $\Psi_p^{(b)}(x_b, y_b) = \sum_{l \in b} \underbrace{\hat{f}_l^{(ba)}}_{\text{a. d. l. l.}} \cdot \underbrace{x_l^{(b)}(y_b)}_{\text{nyitott csatornákra}} \cdot \frac{1}{\sqrt{L}} e^{ik_l^{(b)} x_b}$

Linearkomb.: rögzített hullámok  
 $\hat{r}_{mp}^{(aa)}$  egymással összegződik  
 p. modulus reflexus amplitudója az  
 m. módon a ddalon

$m \neq a \rightarrow m \leq M_a = \sqrt{\frac{B \cdot N_a}{\pi}}$   
 hard wall

$\hat{f}_a^{(a)} = \frac{e^{i\theta}}{2m} \left( \Psi_x^* - \Psi_x^* \right) = \dots$

$I_p^{(a)} = \int_{C_a} \hat{f}_a \cdot dy_a \hat{x}_a = \dots = \frac{e\hbar}{m} \cdot \frac{1}{L} k_p^{(a)} - \frac{e\hbar}{m} \frac{1}{L} \sum_n \hat{r}_{np}^{(aa)} \left[ \hat{r}_{np}^{(aa)} \right]^*$

$\hat{r}_{np}^{(aa)}$  lehetséges komplex is  
 $(\text{pl. } B \neq 0)$

m2.  
 $\int x_m^{(a)}(y_a) \left[ \hat{x}_m(y_a) \right]^* dy_a = \delta_{nm}$

ortogonálisak  
 a rezonan.-k

$r_{np}^{(aa)} = \hat{r}_{np}^{(aa)} \cdot \sqrt{\frac{k_n^{(a)}}{k_p^{(a)}}}$

$I_p^{(a)} = \frac{e\hbar}{m} \frac{k_p^{(a)}}{L} \left( 1 - \sum_{m \neq a} r_{np}^{(aa)} r_{np}^{(aa)*} \right)$

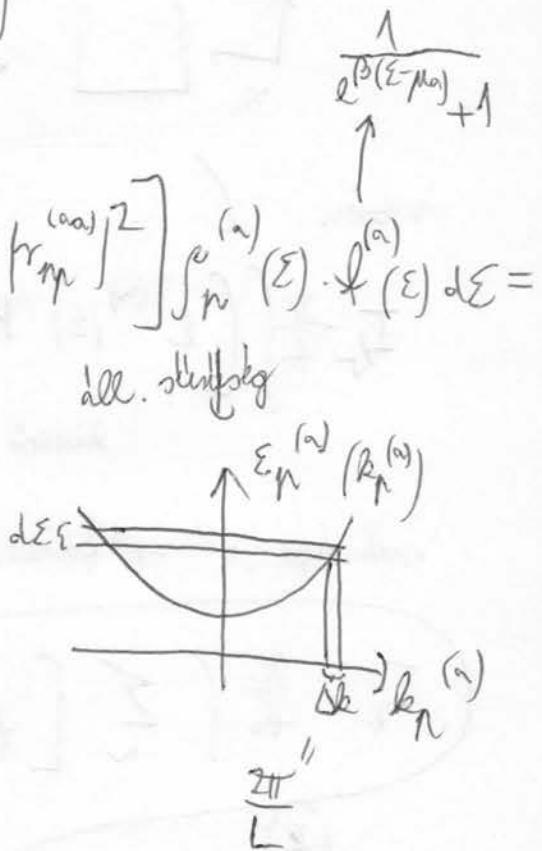
$$I_n^{(a)} = T_n^{(a)} \rightarrow 1 = \sum_{n \in a} \left( |r_{np}|^2 + |t_{np}|^2 \right)$$

$$I_c = \sum_n I_n^{(a)} = \frac{1}{L} \int \sum_n \frac{e^{\beta(\varepsilon - E_n)}}{m} f_n^{(a)} \left[ 1 - \sum_n |r_{np}^{(aa)}|^2 \right] f_n^{(a)}(\varepsilon) \cdot f^{(a)}(\varepsilon) d\varepsilon =$$

merk:

$$\Delta E = dE \cdot \frac{1}{\frac{\partial \varepsilon_n^{(a)}(E_n^{(a)})}{\partial E}} = dE \cdot \frac{1}{\frac{\partial^2 E_n^{(a)}}{\partial E^2} m}$$

||



$$f_n^{(a)}(\varepsilon) = 2 \frac{\Delta E}{2\pi} = \frac{L}{T} \frac{m}{\hbar^2} \frac{1}{E_n^{(a)}}$$

$$= \frac{1}{L} \sum_m \frac{1}{m} \sum_n f_n^{(a)} \left( \frac{m}{2\pi} \frac{1}{E_n^{(a)}} \right) f^{(a)}(\varepsilon) d\varepsilon \left( \underbrace{\sum_n 1}_{M_a} - \sum_{n \in a} |r_{nn}^{(aa)}|^2 \right) =$$

$$= \frac{2e}{h} \int f^{(a)}(\varepsilon) \left( M_a - \sum_{n \in a} r_{nn}^{(aa)} |r_{nn}^{(aa)}|^* \right) d\varepsilon$$

$$I_c = \frac{2e}{h} \int f^{(a)}(\varepsilon) \sum_{\substack{n \in a \\ m \in b}} \left( \frac{f^{(ba)}}{m} \right)_{mn} \left( \frac{f^{(ba)}}{m} \right)^*_{mn} d\varepsilon$$

↓

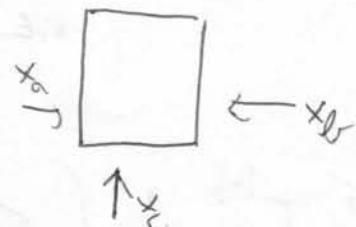
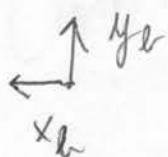
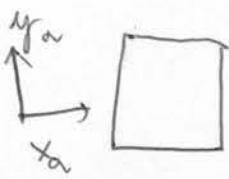
$$I_{c_a} = I_c - \text{loss} :$$

$$\left( \frac{f^{(ba)}}{m} \right)_{mn} = \left( \frac{f^{(ba)}}{m} \right)_{mn} \sqrt{\frac{k_e^{(ba)}}{E_n^{(a)}}}$$

$$\sum_{\substack{n \in a \\ m \in b}} \left( \frac{f^{(ba)}}{m} \right)_{mn} \left( \frac{f^{(ba)}}{m} \right)^*_{mn} + \sum_{\substack{n \in a \\ m \in b}} \left( \frac{f^{(ba)}}{m} \right)_{mn} \left( \frac{f^{(ba)}}{m} \right)^*_{mn} = M_a$$

innen  
megmaradás  
fejzi ki

lokális k. m.



2 term.

$$I_b = \frac{2e}{h} \int \left[ f^{(a)}(\varepsilon) M_a \delta_{ab} - \sum_{n \neq a} \left( \underline{\underline{f}}^{(n)} \right)_{mn} \left( \underline{\underline{f}}^{(n)} \right)_{mn}^* f^{(a)}(\varepsilon) \right] d\varepsilon$$

kimenő áram

bejáró áram

sok term.:  $\rightarrow$  felosz. a terminálakra

$$I_b = \frac{2e}{h} \int \sum_a \left[ f^{(a)}(\varepsilon) M_a \cdot \delta_{ab} - \sum_{n \neq a} \left( \underline{\underline{f}}^{(n)} \right)_{mn} \left( \underline{\underline{f}}^{(n)} \right)_{mn}^* f^{(a)}(\varepsilon) \right] d\varepsilon$$

$M_b \cdot f^{(b)}(\varepsilon)$

kiterj:

$$M_a \left\{ \begin{array}{ccc} \underline{\underline{f}}^{(aa)} & \underline{\underline{M}}_a & \underline{\underline{M}}_c \\ \underline{\underline{f}}^{(ab)} & \underline{\underline{f}}^{(ba)} & \underline{\underline{f}}^{(ac)} \\ \underline{\underline{f}}^{(ca)} & \underline{\underline{f}}^{(ac)} & \underline{\underline{f}}^{(cc)} \end{array} \right.$$

$$M_b \left\{ \begin{array}{ccc} \underline{\underline{f}}^{(ba)} & \underline{\underline{f}}^{(ab)} & \underline{\underline{f}}^{(bc)} \\ \underline{\underline{f}}^{(cb)} & \underline{\underline{f}}^{(bc)} & \underline{\underline{f}}^{(cc)} \end{array} \right.$$

$$M_c \left\{ \begin{array}{ccc} \underline{\underline{f}}^{(ca)} & \underline{\underline{f}}^{(cb)} & \underline{\underline{f}}^{(cc)} \end{array} \right.$$

ha  $b=a$ , ez  $r^{(aa)}$

(Reflexív)  $\Rightarrow$  atállítható

$$\| r^{(aa)} = f^{(aa)}$$

$$= M_a \left[ \begin{array}{cccc} M_a & S_{11} & S_{12} & \dots & S_{1M_a} \\ S_{21} & & & & \\ \vdots & & & & \\ S_{M_a 1} & & & \dots & S_{M_a M_a} \end{array} \right] = S$$

S matrix

$$M_t = \sum_a M_a$$

kezsmásítás a terminálak

azaz „rintet” (nem jellegük külön,  
hogy melyik terminálból származott, is megijedt)

anamnegmaradas

$$\sum_m |S_{mn}|^2 = 1 \Rightarrow S \text{ matrix unit}$$

$$S_{\underline{\underline{r}}\underline{\underline{r}}} + S_{\underline{\underline{t}}\underline{\underline{t}}} = S_{\underline{\underline{r}}\underline{\underline{t}}} + S_{\underline{\underline{t}}\underline{\underline{r}}} = 1$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$\begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \left( S_{11}^* S_{11} + S_{21}^* S_{21} = 1 \right) \quad \text{anamnegm. miatt}$$

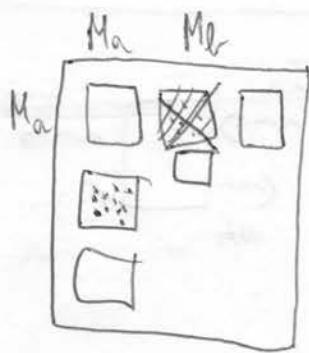
$$\Rightarrow |\text{out}\rangle = S |\text{in}\rangle$$

↑  
visszatérő áram  
(két ellenes terminalia visszatérő áram is)

↑  
az erőközpontba bejáró áram

$$\bar{T}_{ba} = \sum_{\substack{\text{near} \\ \text{meh}}} |S_{mn}|^2 =$$

$$= \sum_{\substack{\text{near} \\ \text{meh}}} \left( \underline{\underline{t}}^{(ba)} \right)_{mn} \cdot \left( \underline{\underline{t}}^{*(ba)} \right)_{mn}$$



első osztály  
klönegysége

~~a - b~~ ~~b~~ a közöcenrum

~~la menü~~ ds. orszáknak  
áram

$$\sum_b \bar{T}_{ba} = M_a \quad \leftarrow S \text{ unitábol}$$

index  
lex

$$\sum_a T_{ab} = M_b$$

$$\sum_a \bar{T}_{ab} = \sum_b M_b = M_+$$

-44-

(\*) -elk (40. dd)

$$I_b = \frac{2e}{\hbar} \int \left( M_b f^{(b)}(\varepsilon) - \sum_a \bar{T}_{ba} f^{(a)}(\varepsilon) \right) d\varepsilon = \cancel{\frac{2e}{\hbar} \int}$$

$$= \frac{2e}{\hbar} \int \left[ \sum_a \bar{T}_{ab} \cdot f^{(b)}(\varepsilon) - \sum_a \bar{T}_{ba} f^{(a)}(\varepsilon) \right] d\varepsilon$$

- elgel a idealis

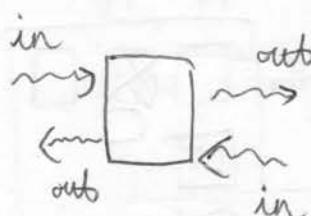
Landauer-  
-Büttiker  
-formula

k.m. megvalasztva miatt van

$$I_b = \sum G_{ba} (U_b - U_a)$$

$$G_{ba} = \frac{2e^2}{\hbar} \int \bar{T}_{ba}(\varepsilon) \left( -\frac{d\phi}{d\varepsilon} \right) d\varepsilon \equiv \frac{2e^2}{\hbar} \bar{T}_{ba}(\varepsilon_K)$$

ahol  $\bar{T}_{ba} = \sum_{nba} |S_{mn}|^2$



az S matrix igy ~~lehet~~ definíálva

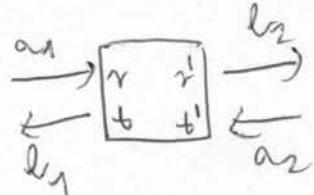
$\Rightarrow$  -t  
a hőv. árak  
fogék  
kizáni)

$\equiv$  (igazolható)  
nem csináltunk, csak  
Überpostotthattuk

a lejáró és kiinduló állapotokat igy, ~~jól keveredik a hő~~

ha a terminal nemcsigból készül, hanem a szövencentrum  
nemcsigból)

pl. 2 terminal



$$S = \begin{bmatrix} r_1 & \frac{r_1}{r_1 + r_2} \\ \frac{r_1}{r_1 + r_2} & r_2 \end{bmatrix} \rightarrow \begin{array}{l} \text{máximas} \\ \text{particionables} \\ 2 \text{ terminales} \end{array}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

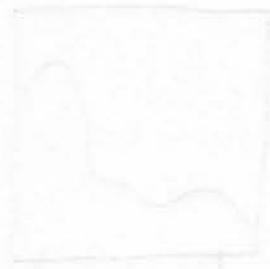
\*  $\begin{array}{l} \rightarrow \text{right} \\ \rightarrow \text{left} \end{array}$

$$b_1 = r_1 a_1 + \frac{r_1}{r_1 + r_2} \cdot a_2$$

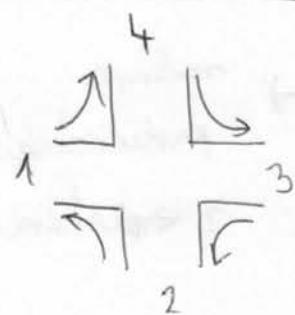
$$b_2 = \frac{r_1}{r_1 + r_2} a_1 + r_2 a_2$$

6. brav

o) 3m.



1) PRB (Phys. Rev. B) 46, 9648 (1992)



$$\bar{T}_{13} = \bar{T}_{31} = \bar{T}_{42} = \bar{T}_{24} = T_F$$

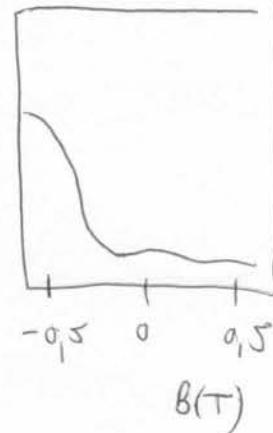
↑  
forward

$$\bar{T}_{21} = \bar{T}_{32} = \bar{T}_{43} = \bar{T}_{14} = T_R$$

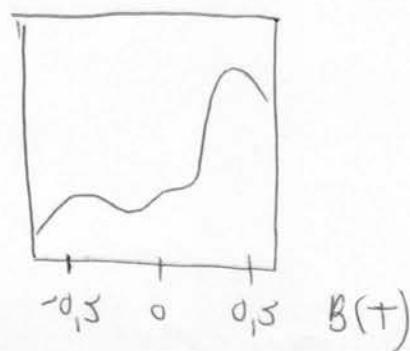
$$\bar{T}_{41} = \bar{T}_{12} = \bar{T}_{23} = \bar{T}_{34} = T_L$$

(symmetria  
misto)

$$\bar{T}_F = \bar{T}_{31}$$



$$T_R = T_{14}$$

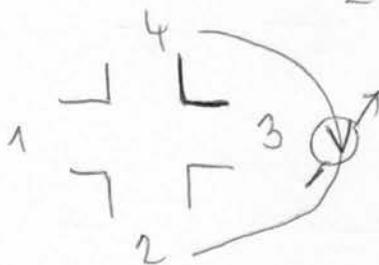


$$V_4 = 0$$

1. Hall-mérés

$$\frac{2e^2}{h} \overline{T}_{\text{ea}}(E_F)$$

$$\left. \begin{array}{l} 1-3: \text{dram} \\ 2-4: \text{voltmérő} \end{array} \right\} R_H = ?$$



$$I_1 = -I_3$$

$$I_2 = I_4 = 0$$

$$I_{\text{br}} = \sum_a G_{\text{ea}} (U_a - U_{\text{br}})$$

$$\begin{aligned} I_{11} &= G_{11}(V_1 - V_1) + G_{12}(V_1 - V_2) \\ &\quad + G_{13}(V_1 - V_3) + G_{14}(V_1 - V_4) \end{aligned}$$

$$\begin{aligned} &= \left( G_{12} + G_{13} \right) V_1 - G_{12} V_2 - G_{13} V_3 \\ &\quad - G_{14} V_4 \end{aligned}$$

$$T_o = T_F + T_L + T_R$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \frac{2e^2}{h} \begin{pmatrix} T_o & -T_L & -T_F \\ -T_R & T_o & -T_L \\ -T_F & -T_R & T_o \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

// inverz

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$$R_{21} = \frac{h}{2e^2} \cdot \frac{T_L T_o + T_F T_R}{\Delta}$$

$$R_{23} = \frac{h}{2e^2} \cdot \frac{T_R T_o + T_F T_L}{\Delta}$$

$$\Delta = (T_L + T_F) [T_L^2 + T_F^2 + 2T_F^2 + 2T_F T_L + 2T_F T_R]$$

$$I_2 = 0, I_3 = -I_1$$

$$V_2 = (R_{21} - R_{23}) I_1$$

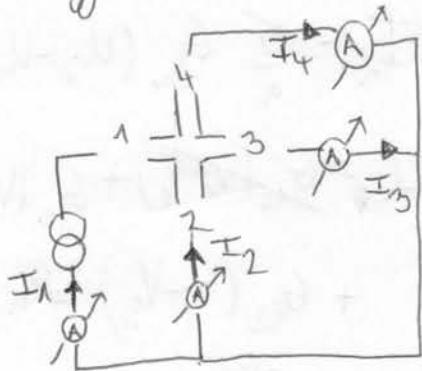
$$R_H = \frac{V_2 - V_4}{I_1} = R_{21} - R_{23} = \frac{h}{2e} \frac{(T_L - T_R)(T_L + T_F)}{\Delta}$$

$$I_1 = -I_3, I_2 = I_4 = 0$$



$\leftarrow$  ha  $T_F, T_L, T_R$  -t ismétlik, ezzel a Hall-mérésrel ellenőrizhetjük a képletek helyességet

Hogyan mérjük ki  $T_F, T_L, T_R$ -et?



$V_2 = V_3 = 0 \rightarrow$  eredető tömörzések zajnak

$$I_1 = \frac{2e^2}{R} T_0 \cdot V_1$$

$$I_2 = \frac{2e^2}{R} (-T_R) \cdot V_1$$

$$I_3 = \frac{2e^2}{R} \cdot (-T_F) \cdot V_1$$

$\rightarrow$  általánosságban  $I_1 = V_1 / R$

$T_F, T_L, T_R$  mérhető

## $\underline{\underline{S}}$ - tulajdonságai

$$1) \sum_m |S_{mn}|^2 = 1 \quad \forall m \text{ re } \sum_{m,n} |S_{mn}|^2 = 1 \Rightarrow S \text{ unitárius} \\ (\text{használva } \sum_{m,n} |S_{mn}|^2 = 1)$$

$$\underline{\underline{S}}^\dagger \underline{\underline{S}} = 1 = \underline{\underline{S}} \underline{\underline{S}}^\dagger$$

$$S_{kn} = \left( \underline{\underline{f}}^\dagger \underline{\underline{f}} \right)_{km} S_{mn} = \sum_m S_{mk}^* S_{mn}$$

$$\text{Ha } b=n \Rightarrow 1 = \sum_m S_{mn}^* S_{mn} = \sum_m |S_{mn}|^2$$

2) örzegorodály

$$\sum_a \bar{T}_{ab} = \sum_a \sum_{n \in b} \sum_{m \in a} |S_{mn}|^2 = \sum_{n \in b} 1 = M_b$$

$$\boxed{\sum_a \bar{T}_{ab} = \sum_a \bar{T}_{ba} = M_b}$$

3) Reciprocitásról feltétek, hogy

$$G_{ab}|_B = G_{ba}|_{-B}$$

kohézív vezetőre

$$\underline{\underline{S}}|_B = \underline{\underline{S}}|_{-B}$$

Ezt kell belátni!

Bis:  $H\psi = E\psi$ ,  $H = \left( \frac{\hbar^2}{2m} \text{grad} - eA \right)^2 + U(x,y)$

$$H^* \psi^* = E\psi^* \quad \text{es } B \mapsto -B$$

↑  
Euler

$$\left[ \left( \frac{\hbar^2}{2m} \text{grad} + eA \right)^2 + U \right] \psi^* = E\psi^*$$

$$\Rightarrow \psi^*(x,y) \Big|_{-B} = \psi(x,y) \Big|_B$$

↑

→ (da ist  $\psi$  jik  
a mo. → B then  
ist  $\psi$  jik  $-B$  - it is)

Konjugat:  $\text{leimend} \leftrightarrow \text{bemend}$

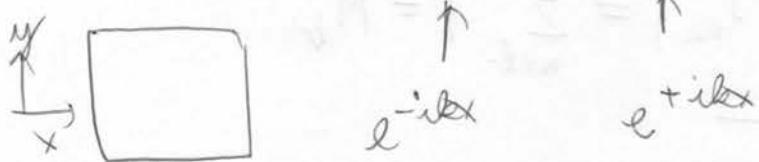


Diagram illustrating the relationship between the momentum representation and the position representation:

Left side (Momentum Representation):

- States are represented by arrows labeled  $\Gamma_1, \Gamma_2, \dots$ .
- Transitions are represented by arrows labeled  $\Gamma_i \rightarrow \Gamma_j$ .
- A bracket indicates a transformation:  $\Gamma = \sum_{i,j} a_i \Gamma_i \rightarrow \Gamma = \sum_{i,j} b_i \Gamma_i$ .

Right side (Position Representation):

- States are represented by arrows labeled  $\Gamma_1, \Gamma_2, \dots$ .
- Transitions are represented by arrows labeled  $\Gamma_i \rightarrow \Gamma_j$ .
- A bracket indicates a transformation:  $\Gamma = \sum_{i,j} b_i \Gamma_i \rightarrow \Gamma = \sum_{i,j} a_i \Gamma_i$ .

Relationship between representations:

- $b_i = S | a_i \rangle_{+B} \Rightarrow b_i^* = S^* | a_i^* \rangle_{+B}$
- $a_i^* = S | b_i^* \rangle_{-B} \Rightarrow a_i^* = S^{-1} | b_i^* \rangle_{-B}$
- $S^* | B \rangle = S^{-1} | -B \rangle = S^+ | -B \rangle$

Annotations:

- (idiotikoresi invariantiell)  $\rightarrow$  (jezi ki)
- 48

4) a) konvergencia

$$\underline{S} = M_2 \left\{ \begin{array}{c} M_1 \\ \frac{t_{ba}}{t_{aa}} \\ \frac{t_{bb}}{t_{aa}} \\ \vdots \end{array} \right\}$$



$\underline{S} = \begin{bmatrix} \underline{\underline{r}} & \underline{\underline{t}} \\ \underline{\underline{t}} & \underline{\underline{r}} \end{bmatrix}$  → ez a konvergencia

bal. jobb.

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \underline{S} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \underline{\underline{r}} a_1 + \underline{\underline{t}} a_2 \\ \underline{\underline{t}} a_1 + \underline{\underline{r}} a_2 \end{pmatrix}$$

dysz harmonikus

$b_1, b_2$  amplitúdái

nyitott csatornák

szabály a csatornán

b) zárta kapcsolás:



$$\begin{pmatrix} b_1 \\ u \end{pmatrix} = \underline{S}_1 \begin{pmatrix} a_1 \\ v \end{pmatrix}, \quad \begin{pmatrix} v \\ b_2 \end{pmatrix} = \underline{S}_2 \begin{pmatrix} u \\ a_2 \end{pmatrix}$$

$$\begin{pmatrix} \underline{\underline{r}}_1 & \underline{\underline{t}}_1 \\ \underline{\underline{t}}_1 & \underline{\underline{r}}_1 \end{pmatrix}$$

$$\begin{pmatrix} \underline{\underline{r}}_1 \\ \underline{\underline{r}}_2 \end{pmatrix} = \underline{\underline{S}} \begin{pmatrix} \underline{\underline{a}}_1 \\ \underline{\underline{a}}_2 \end{pmatrix} \quad \xrightarrow{\#} \underline{\underline{S}} = ?$$

elimináljuk

$\underline{\underline{y}} - \underline{\underline{t}}$  és  $\underline{\underline{v}} - \underline{\underline{b}}$

a fenti egyszerűsítésről

$$\underline{\underline{t}} = \underline{\underline{t}}_2 \left( \underline{\underline{I}} - \underline{\underline{r}}_1^T \underline{\underline{r}}_2 \right)^{-1} \underline{\underline{r}}_2$$

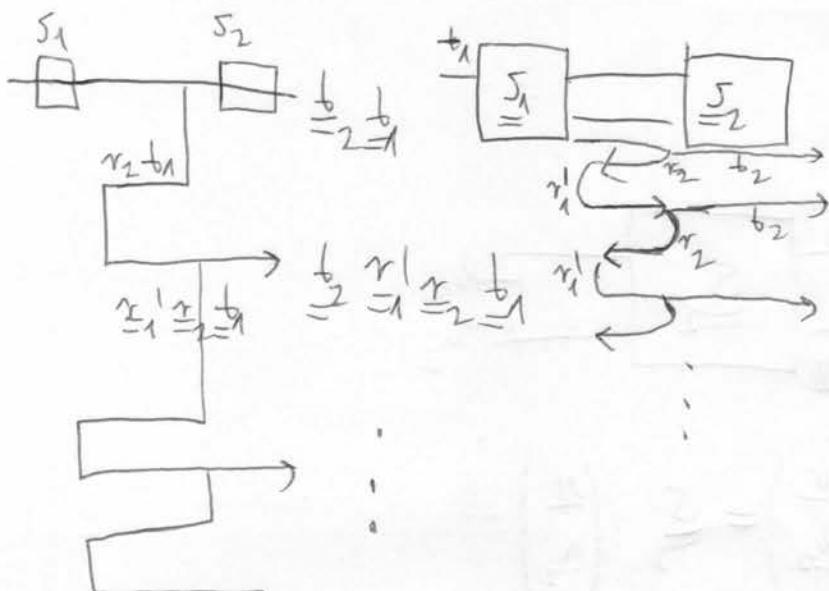
$$\underline{\underline{v}} = \underline{\underline{r}}_1 + \underline{\underline{t}}_1^T \underline{\underline{r}}_2 \left( \underline{\underline{I}} - \underline{\underline{r}}_1^T \underline{\underline{r}}_2 \right)^{-1} \underline{\underline{t}}_1$$

$$\underline{\underline{r}} = \underline{\underline{r}}_1^T \left( \underline{\underline{I}} - \underline{\underline{r}}_2 \underline{\underline{r}}_1^T \right)^{-1} \underline{\underline{t}}_1$$

$$\underline{\underline{r}} = \underline{\underline{r}}_2^T + \underline{\underline{t}}_2 \left( \underline{\underline{I}} - \underline{\underline{r}}_2^T \underline{\underline{r}}_2 \right)^{-1} \underline{\underline{r}}_2^T \underline{\underline{t}}_2$$

fürkészésben: ~~szimmetria~~  
vegezzen metani sorba! fogjuk fel

$$\underline{\underline{t}} = \underline{\underline{t}}_2 \left( \underline{\underline{I}} + \underline{\underline{r}}_1^T \underline{\underline{r}}_2 + \underline{\underline{r}}_1^T \underline{\underline{r}}_2 \underline{\underline{r}}_1^T \underline{\underline{r}}_2 + \dots \right) \underline{\underline{t}}_1$$

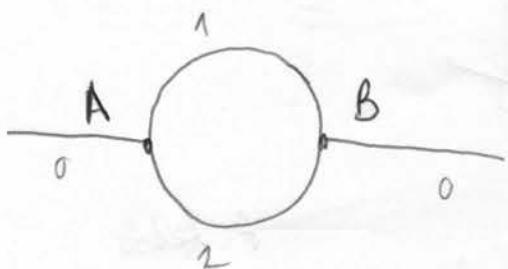


(használható többi példa is megírható)

c) paklusamoji bapesdas

pėdė: egymodusių režimo (tai mada  
is atbalanoto  
būd)

tai. A & B simetriški, 3 elagėsas



$$\underline{S} := \begin{bmatrix} c & \sqrt{\varepsilon} & \sqrt{\varepsilon} \\ \sqrt{\varepsilon} & a & b \\ \sqrt{\varepsilon} & b & a \end{bmatrix}, \quad a, b, c, \varepsilon \in \mathbb{R}$$

$\underline{S}$  uniter  $\Rightarrow$

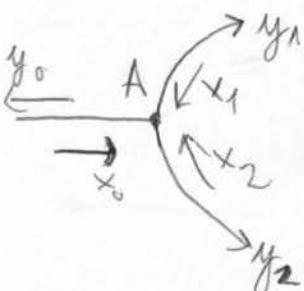
$$c = \sqrt{1-2\varepsilon} \Rightarrow 1 \text{ parameterinė (pl. } \varepsilon)$$

$$a = \frac{1-c}{2}$$

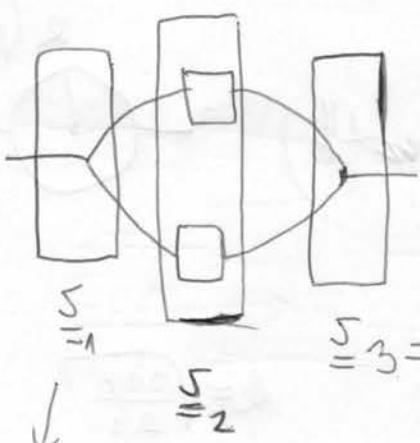
$$b = \frac{1+c}{2}$$

□: patricijalisk  $\begin{bmatrix} \frac{r}{\varepsilon} & \frac{t}{\varepsilon} \\ \frac{t}{\varepsilon} & \frac{z}{\varepsilon} \end{bmatrix} = \underline{S}$

$\uparrow$  a komponent  
betėjyja arba net



$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} c & \sqrt{\varepsilon} & \sqrt{\varepsilon} \\ \sqrt{\varepsilon} & a & b \\ \sqrt{\varepsilon} & b & a \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} =$$

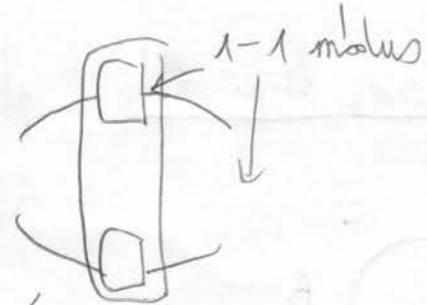
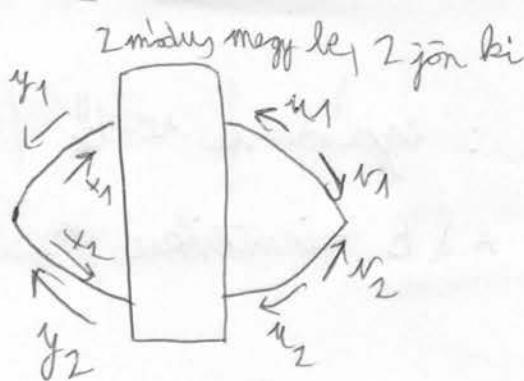


← visavietinių sarašų koper.-ra

lotus  
lattice

$$\underline{S}_1 = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \quad \underline{S}_2 = \begin{pmatrix} \sqrt{\varepsilon} & \sqrt{\varepsilon} \\ \sqrt{\varepsilon} & \sqrt{\varepsilon} \end{pmatrix}$$

2-lėl 1-modus  
megy



f: felső

A: alsó

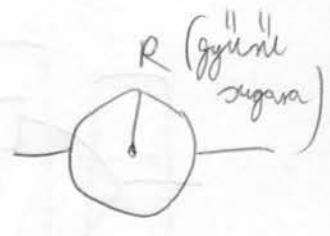
$$\begin{pmatrix} y_1 \\ y_2 \\ u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} r_F & 0 & b_F' & 0 \\ 0 & r_A & 0 & b_A' \\ b_F & 0 & r_F' & 0 \\ 0 & b_A & 0 & r_A' \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \end{pmatrix}$$

$$\underline{\underline{S}}_2 = \begin{bmatrix} \underline{\underline{r}}_2 & \underline{\underline{b}}_2' \\ \underline{\underline{b}}_2 & \underline{\underline{r}}_2' \end{bmatrix}$$

$\Rightarrow \underline{\underline{S}}_1, \underline{\underline{S}}_2, \underline{\underline{S}}_3$  -os szabályt kaphatunk

speciális

- egyszerűsítés  $r_A = r_F = 0$  (szima rezgés)



- $U = e^{ikx}$  L hosszú rezgés hőmegy.



$$e^{ik(x+L)} = e^{ikL} \cdot e^{ikx}$$

$$k = \sqrt{\frac{2mE}{\sigma L^2}}$$

szabály  
leap

$$b_F' = e^{i\theta}$$

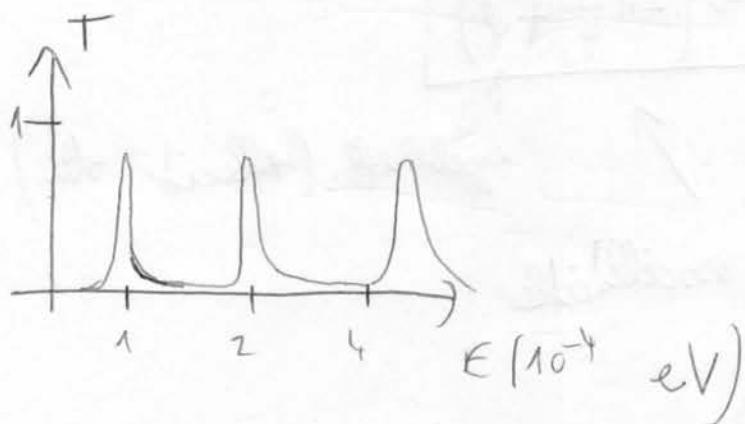
vegyesd megfeszítés

$$\theta = k \cdot R \cdot \pi = \sqrt{\frac{2mE}{\sigma L^2}} R \cdot \pi$$

$$\dots \Rightarrow f = \frac{2\epsilon \cdot e^{i\theta}}{1 - c^2 e^{2i\theta}}$$

interferencia a 2 ág között,  
amikor találkoznak

$$T = H^2 = \frac{4\epsilon^2}{1 - 2c^2 \cos 2\theta + c^4}$$



$$R = 100 \text{ nm}$$

$$\epsilon = 0,025$$

"Kharlamov - Bohm - lík."

d) (B → t → a gyűrűben): ha  $R \cdot T \ll L_F$  fájós. hosszú

$$t_f \rightarrow t_f \cdot e^{i \frac{e}{\hbar} \int_{\text{felv.}}^{\text{ki}}} A ds = t_f \cdot e^{i k_F}$$

$$(\Psi(r, B=0) \stackrel{?}{=} e^{i \frac{e}{\hbar} \int_0^r A ds} (r) ds \leftarrow \text{Elv. elegendő } A \neq 0 \text{ mellel})$$

$$t_f \rightarrow t_f \cdot e^{i \frac{e}{\hbar} \int_{\text{felv.}}^{\text{ki}}} A ds = t_f \cdot e^{i k_F}$$

$$t_1 = e^{i k_1} |t_1| = \sum_{\text{all paths}} A_p \quad t_2 = e^{i k_2} |t_2| = \sum_{\text{all paths or also}} A_p$$



$$(k_1, k_2 \neq k_h, k_f)$$

$$T = (t_1 + t_2)^2 = |t_1|^2 + |t_2|^2 + t_1 t_2 e^{i k_1 - i k_2} + t_1 t_2 e^{-i k_1 - i k_2} =$$

$$= \boxed{T_1 + T_2 + 2\sqrt{T_1 T_2} \cos(\ell_1 - \ell_2)}$$

$$\ell_1 - \ell_2 = \oint A dz = \frac{e}{h} \phi = \frac{e}{h} \cdot 2\pi \cdot \phi = \boxed{2\pi \cdot \frac{\phi}{\phi_0}}$$

$$\phi = B \cdot A = B R^2 \pi \quad \phi_0 = \frac{h}{e} \quad (\text{Josephson eff.})$$

(nah  $e \rightarrow 2e$ )

alle Rechtecke

$$T = T_1 + T_2 + 2\sqrt{T_1 T_2} \cos\left(2\pi \frac{\phi}{\phi_0} + \ell\right)$$

magn. tr.

für. Ben

oscillate  $\rightarrow \frac{h}{e}$  oscillate

↑  
egyel eff. (reflexion, att.)

Földön reflexion  $\rightarrow \frac{h}{Ne}$  osc.

erklärt nem szokottak

látom, mert a kohärencia  
eddig elhasznált!

szabadon



$A$  a folyás / pl.

akkor is, ha  $B=0$  a  
mág. -plan!

$\rightarrow$  kapunk röth  
nullamokat

Endető Aharonov-Bohm - eff.:



diff. szárai hatás.

$$\frac{d\phi(\ell)}{d\ell}$$

$$\Phi$$

$$H = \frac{(e \cdot A)^2}{2m}$$

$$A(r) = A(r) \cdot \frac{e}{r}$$

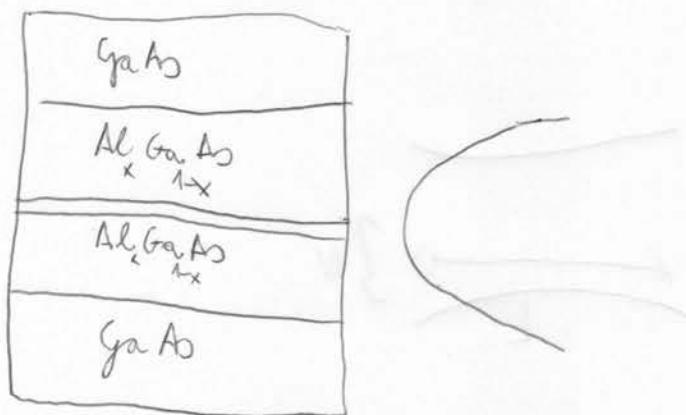
$$A \cdot 2\pi r = \Phi \quad (\text{Fluxus})$$

$$A(r) = \frac{\Phi}{2\pi r} \rightarrow e \text{ a Disc-fluxus}$$

7. drav

1) 2DEG

a)



b) typiskus paraméterek:

$$m_{\text{eff}} = 0,067 \cdot m_e$$

$$n_s = 4 \cdot 10^{11} \frac{1}{\text{cm}^3} \quad (\text{e-}^{\parallel\parallel\perp\perp})$$

$$\epsilon_F = 14 \text{ meV} \rightarrow \text{kicsi}$$

$$\lambda_F = 40 \text{ nm}$$

$$(\text{zabahom (300K)} \approx 25 \text{ meV})$$

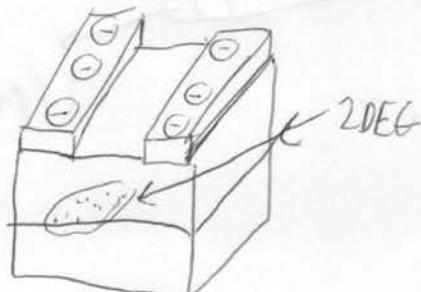
$$\text{zabahom időben } l = k \cdot T = 10^2 - 10^4 \text{ nm} \quad l \gg \lambda_F$$

$$L_d = 200 \text{ nm} \quad \text{fesz. hossz}$$

$$T \sim \text{néhány nál } \text{mK}$$

$$(\text{fémekre } \lambda_F = 0,1 \text{ nm} \quad \epsilon_F \sim 1 \text{ eV})$$

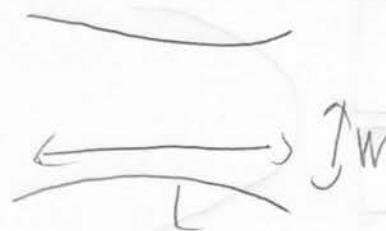
c) kapuk



## d) Transport - kontinuierlich

- diffusivs kontinuierl.
- ballistisches tast.
- kriech ball.

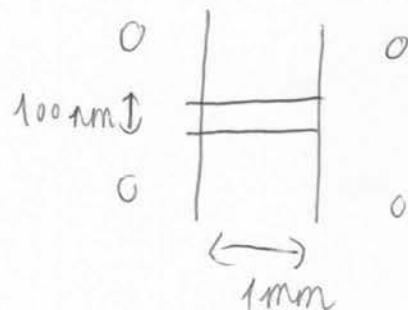
$$W < l < L$$



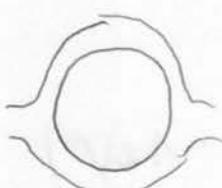
• farokohens brantung erlaubt beruhige  
nem jo a uniklassikus diff. transport lehre

- geng. lokalisiert
- universalis verloren - flukt
- Aharonov-Bohm-eff.
- Quantum-Hall-eff.

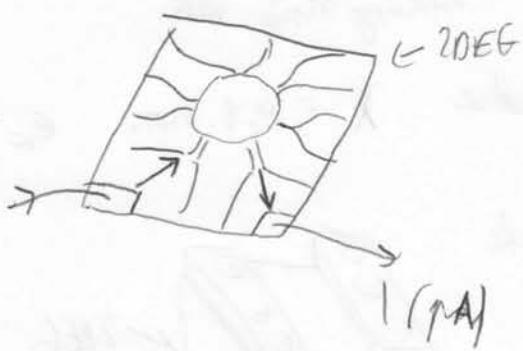
## e) Nanodroste



## kr. gyűrűk



## kr. jötyök



split gates:



## 2) Quantumsz.

a) diskret v. folytonos en. minták

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ | \\ E \\ | \\ \text{---} \\ | \\ E_n \\ | \\ \text{---} \end{array} \quad N(E) = \sum_{n=1} \Theta(E - E_n)$$

$$\frac{dN}{dE} = \rho(E) \leftarrow \text{minimális az energiaszintek}$$

$$dN = \rho(E) dE$$

3D  $\rho_{3D} \sim \sqrt{E}$

2D  $\rho_{2D}(E) = \frac{m\omega}{\pi\hbar^2} = \text{const.}$

1D  $\rho_{1D}(E) \sim \frac{1}{\sqrt{E - E_n}}$

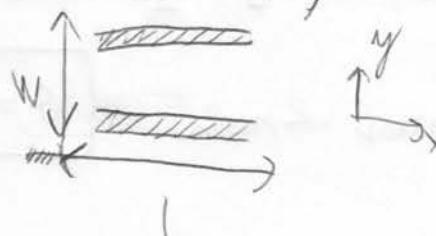
0D  $S_{00}(E) \sim \delta(E - E_n)$

f) ideális hullámvezetők  $(\hat{H} = \frac{\hbar^2}{i} \frac{\partial}{\partial x}, \hat{p}_x^2 = -\hbar^2 \frac{\partial^2}{\partial x^2})$

$$\hat{H}\Psi = E\Psi$$

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m_{\text{eff}}}$$

merő fel:

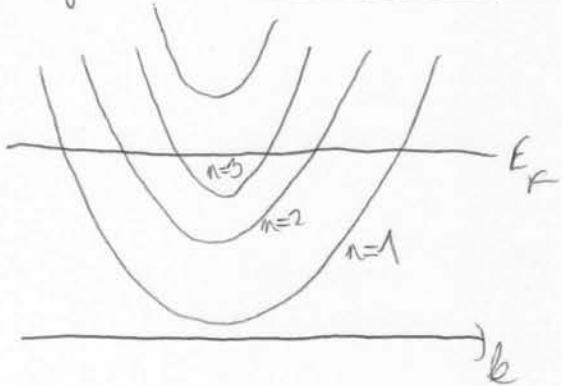


$$E_n(k) = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + E_n, n=1, 2, 3, \dots$$

$$E_n = \frac{\hbar^2 \pi^2}{2m_{\text{eff}} W^2} n^2$$

$$\Psi(x, y) = e^{ikx} \cdot \underline{\sin \frac{n\pi y}{W}} \quad X(y)$$

c) - Nyitott atomhálózat



besztrukturált szab = nyitott atomhálózat

$$M = \left[ \frac{k_F W}{\pi} \right]$$

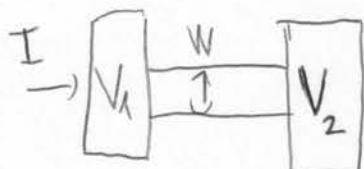
↑  
egységes

fémek:  $M \sim 1$  millió

nanonasz.:  $M < 20$

↑  
 $k_F, W$  kicsi !!!

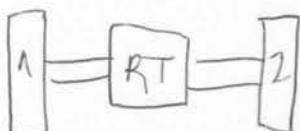
- 2 terminal, tökélés nélk:



$$G = \frac{2e^2}{h} M$$

$$G = \frac{1}{V} = \frac{1}{R}$$

- 2 terminal, nem tökélés nélk

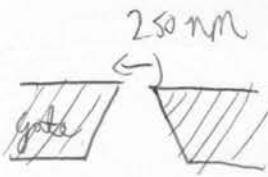


$$R = 1 - T \quad 0 \leq T \leq 1 \quad + \text{szabonára}$$

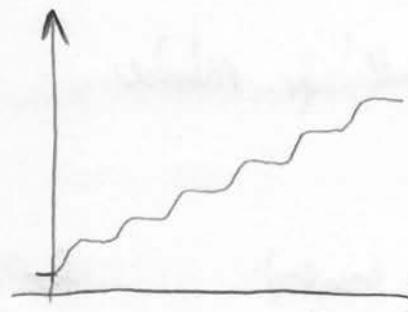
Fondaúer-formula:

$$G = \frac{2e^2}{h} \frac{T}{T}$$

$$= \sum_{n=1}^{\infty} T_n$$



$$\epsilon(2e^2/h)$$



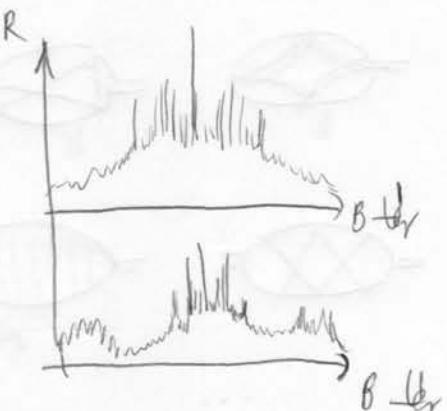
$V$  (gate voltage)

voltárok a nyitott csat. működés

$$\frac{2e^2}{h}$$

$$\frac{h}{2e^2} = 12,9064 \text{ kV} \cdot \text{n} \rightarrow \text{emitt tudás jól kiérni}$$

kv. Baromz  
~~Néhány rész~~

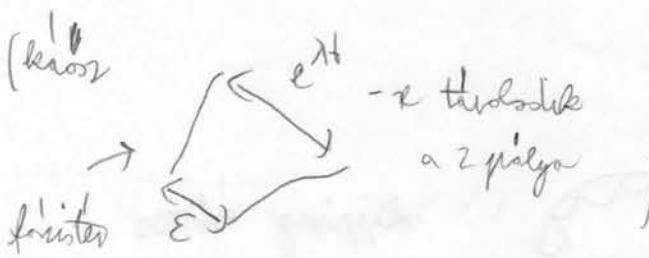


Klasszikusan

← Kvantum

← integrálható

(nem-kvantum)



-> threshold  
a 2 pólus

a szabadalakos

igazolás

a Mész

kv. káro

fogl. eset

Hol van az elülső?

mindeket m., ben díszítjük

en. mintek művek

energiavissza

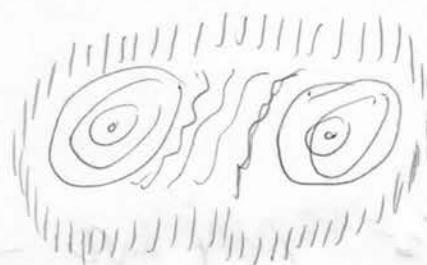
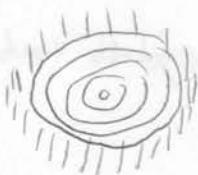
elosztásban van bár!

3) STEM (pastasso oligotomiker.) (mérés)



Hullámlás mérése

a) ker. körben (gyo. corals) ker. stadiion



b) →

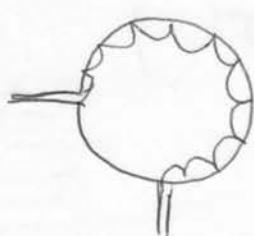
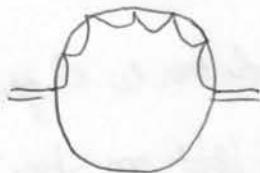
Ab. riandás



...

"bouncing ball"

c) biliárdokról és magnes terén



skipping orbits  
(edge states)



Kardiyofriggels : gas varázslat (Gondolat zelkonyról)

"There's plenty of room at the bottom" (Richard Feynman, 1959)

100 atom  $\rightarrow$  1 lit

0,1 mm kocka  $\rightarrow$  világ összes hőt vagy

János

XI. 11.

1) gyenge lokalizáció

~~diffraktív tartomány, nincs,~~

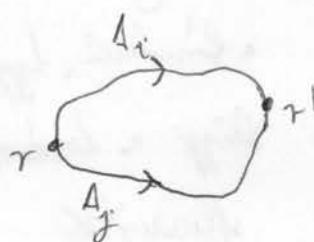


diffraktív tartomány : valamit változik ( $e$ )  $\Rightarrow$  ~~termikus hullámosság~~

$E_d \gg 1$

annak a mér-e, hogy az  $e$  + idő alatt  $r$ -hez  $r'$ -hez jut

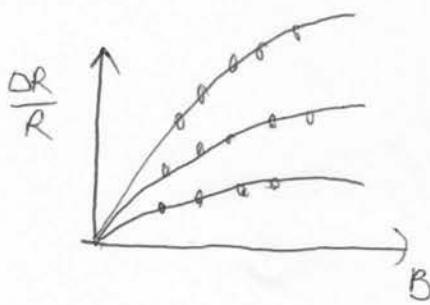
$$P(r, r', t) = \left| \sum_i A_i e^{i k r_i} \right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j^* e^{i k (r_i - r_j)}$$



(dia)

ioltsükörési szin. szűl mágnes törjelentetben  $\rightarrow$  csökken a  
gyenge lokalizáció

$$\Delta R/R \equiv \frac{R(0) - R(B)}{R(0)}$$



$$\text{kanonikus imp.: } p = \underline{mv} - e\underline{A}$$

föriskülönség:

↑  
kinetikus imp.

$$\psi = \frac{1}{\hbar} \int p + d\ell - \frac{1}{\hbar} \int p - d\ell = \frac{1}{\hbar} \int p + d\ell + \frac{1}{\hbar} \int p - d\ell =$$

$\int k(x) dx$  föris

$L_\phi \gg L_W$

$\frac{L}{IW}$

• mérséki pontok

- elmélet → 1 libparaméter:  $L_\phi$



$$= \int \frac{1}{\hbar} p d\ell$$

$$\psi = \frac{2e}{\hbar} \int \varphi A dS = \frac{2eBS}{\hbar} = 4\pi \frac{\phi}{\hbar}$$

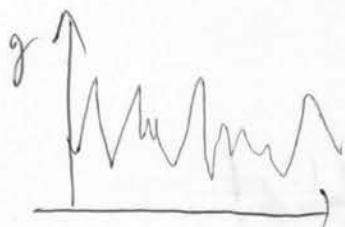
$$\left(\phi_0 = \frac{\hbar}{e}\right)$$

||  
a förisküli, legy  
leírja a hárás  
fluensét

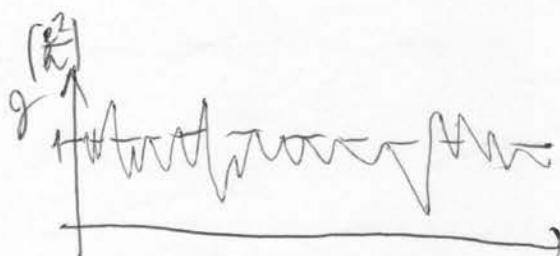
## 2) VCF (Universalis veretl. Sep. Flukt.)

$$G = \frac{I}{V} = g \frac{e^2}{h}$$

Kaputon. vibratotársa  $\rightarrow$  körülbelül 1 millió alakja



$V(mV)$   
Gate



t

$$\delta G \sim \frac{e^2}{h}$$

egyen á minta  
mehedől, konfig.

$$G \text{ fluk. Var}(G) = \langle (G - \langle G \rangle)^2 \rangle$$

## 3) Aharanor - Bohm - effektus

Young -干涉es kísérlete

(b=0)

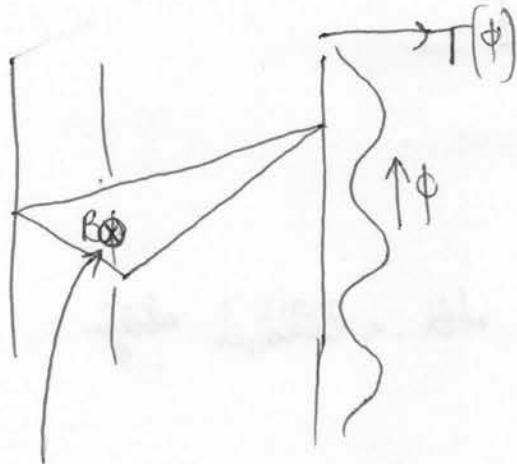


$$T^{\text{total}} = |t_1 + t_2|^2 = |[t_1] e^{i\phi_1} + [t_2] e^{i\phi_2}|^2 =$$

$$= |t_1|^2 + |t_2|^2 + |t_1||t_2| \cdot e^{i\phi_1 - i\phi_2} + |t_1||t_2| e^{-i\phi_1 + i\phi_2} =$$

$$= |t_1|^2 + |t_2|^2 + |t_1||t_2| \cos \phi$$

$B \neq 0$



kis szögű

kiül  $\vec{B} = 0$ , de  $\vec{A} \neq 0 \Rightarrow$  az e- körn egy fázis!

$$\theta_i(\phi) = \phi_i(0) - \frac{e}{h_0} \int A ds$$

zi

$$\delta(\phi) = \delta(0) - \frac{e}{h_0} \int A ds = \delta(0) - 2\pi \frac{\phi}{\phi_0} \quad (\phi_0 = \frac{h}{e})$$

$$T(\phi) = a_1^2 + a_2^2 + 2a_1 a_2 \cos \left[ \delta(0) - 2\pi \frac{\phi}{\phi_0} \right]$$

$$\hookrightarrow T(\phi + n\phi_0) = T(\phi)$$

a transzmisszió (periodikusan) függ

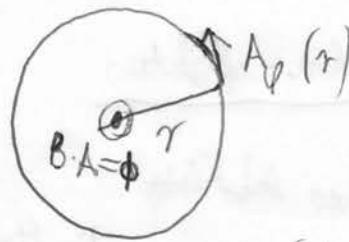
~ hozzájárulási

Aharanov, Bohm, Phys. Rev. Lett. (1959)

eredeti problema  $\rightarrow \sigma(\phi)$

$$\text{Dirac-fluxus} - 6\pi$$

$\sigma(\phi)$  negy!  $\Rightarrow$  kv. mech.-ban  $A$  az elszigetelés, nem  $B$



$$A_\phi(r) \cdot 2\pi r = \phi$$

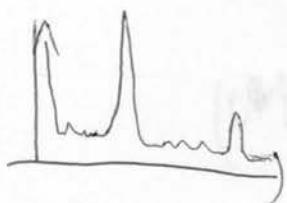
$$A_\phi(r) = \frac{\phi}{2\pi r}$$



## Első zártorsa részletek (Wehr)

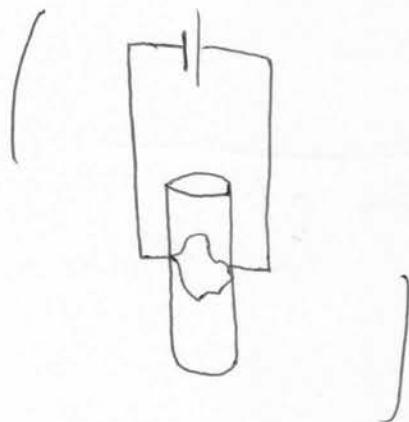


merőszögletes aránygyűrű



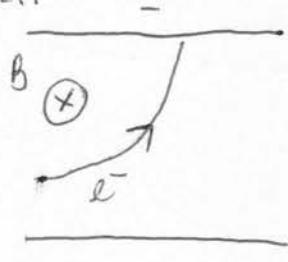
← vezetékkörbeneg Fourier-tr.-ja

$$\frac{h}{e} \rightarrow \frac{h}{2e}$$



## 4) Kvantum Hall-effektus

a) klass.:



$$B = \begin{pmatrix} 0 & \\ 0 & \\ B & \end{pmatrix} \quad E = \begin{pmatrix} E_x \\ E_y \\ \vdots \end{pmatrix} \quad \vec{J} = e n_j \vec{v}$$

$$\frac{mv}{l} = e(E + v \times B)$$

keresztirányú  
klass. általános kör

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} \quad \vec{J} = \underline{\underline{E}}$$

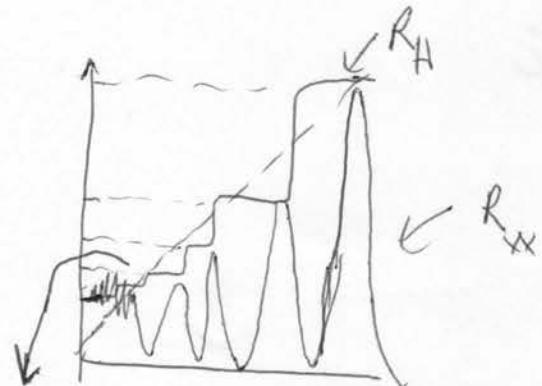
Konzeptfeldwerk mittlere  $\sigma$   $\equiv$  eige Matrix (Tensor)

$$\rho = \sigma^{-1} = \begin{pmatrix} \dots & -\frac{B}{en_y} \\ \frac{B}{en_y} & \dots \end{pmatrix}$$

$$R_H = \frac{V_y}{I} = \frac{E_y}{j_x} = \rho_{yx} = \frac{B}{en_y} \sim B \quad \text{da } j_y = 0 !$$

b) Eigene Quanten-Hall-Eff. (Klitzing, 1981)

Wieliczka 1985



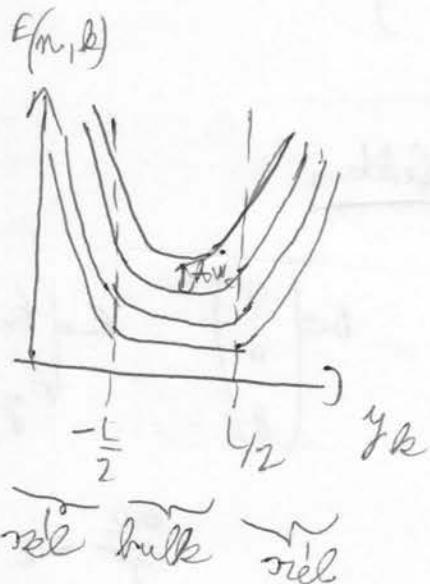
Quantensprung an der Kontaktstelle

$$w_c = \frac{eB}{m}$$

Kohler:

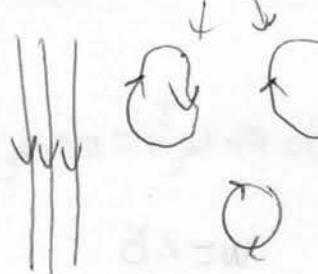
- 
- 
- 
- $j_{\text{bulk}} w_c$
- $j_{\text{bulk}} w_c$
- $j_{\text{bulk}} w_c$

bulk Landau-Nive

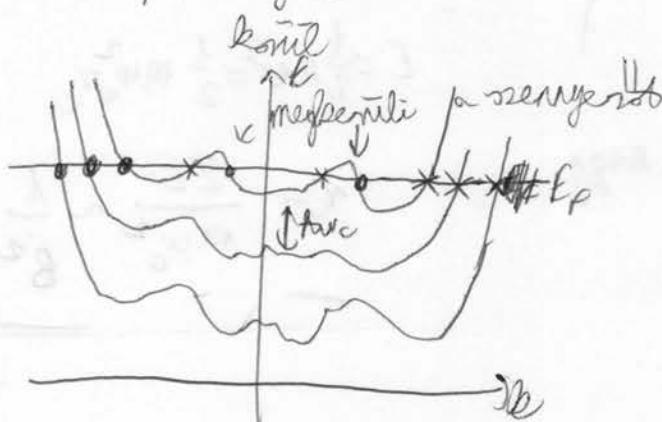


$$y_B = \frac{\hbar k}{eB}$$

lokálizált elláptató → csak nem vezető

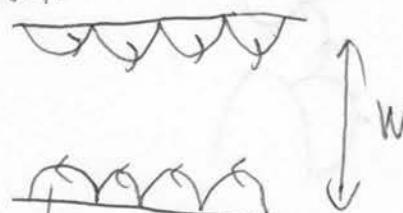


pl. rengeteg



elällaptozás (edge state) (skipping orbit)

mass. ben



hatás (skipping) pályák

álló, hosszú legyen  
~~állandó~~ attól

az elällaptozás bőrétt

$(\Psi_1 | V | \Psi_2)$

→ nincs átforás!

ami elindít az egységek  
elällaptozás, az meg is  
elindít a másik oldalra.

ha  $B \neq 0$ ,

de  $\omega_c \neq 0$

↓

attól az

elällaptozás száma

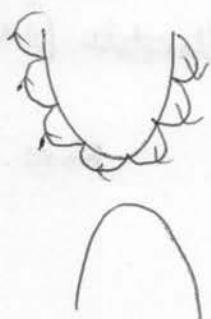
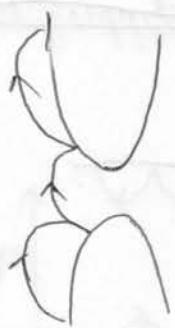
$$I = \frac{2e}{L_x} \sum_{n, k} \frac{1}{h} \frac{\partial E}{\partial k} = \frac{2e}{L_x} \sum_n \int_{\mu_2}^{\mu_1} \frac{1}{2\pi} \frac{1}{h} \frac{\partial E}{\partial k} dk = \frac{2e}{\pi} \sum_n \int_{\mu_2}^{\mu_1} dE =$$

$$\frac{e}{\pi} = \frac{\pm}{V} = \frac{2e^2}{h} M \quad \leftarrow = \frac{2e}{\pi} M \cdot \frac{eV}{(\mu_1 - \mu_2)}$$

M ahol monolán voltakat

nyitott csat. száma

# Konstrukció 20EG mintára

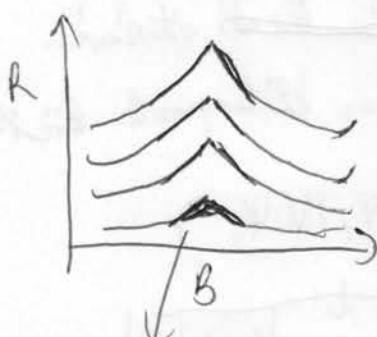


$$evB = m \omega_c^2 r = mv \omega_c$$

$$w = \frac{eB}{m}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega_c^2 r_c^2$$

$$r_c = \frac{2E}{m\omega_c^2} \sim \frac{1}{B^2}$$



lökés módosítás

(C) több kvantum Hall-eff

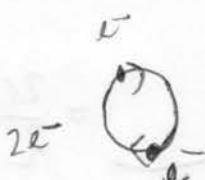
M több rész

(Störmer, Ioni, ...)

+ Laughlin (elmelet)

Nobel-díj (1998)

$e^-$ -k részben válik  $\rightarrow$  ab-Br. leágazás

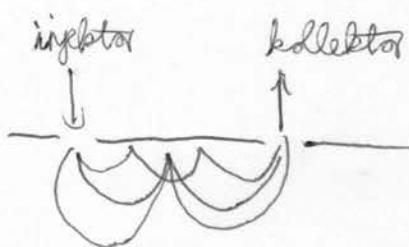


Laughlin - állapot (kötött, lezonikus  
állapot alakul ki)

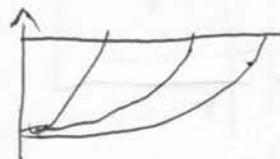
$\downarrow$   
széle: minden alapállapot alakul ki

(nincs szabadon  
a felső állapot)

## Elektronisk fokussialas



Kvantika = klass. patykla  
buskojo



fokussialas

+ kvantikak leantumos  
magyarato

## 5) Kvantum dotok



kv. dot virág.

$$R \sim 1\text{ }\mu\text{m} - 1\text{ nm}$$

- IR spekt.
- vezetőkör

elektrostatis. energia ( $2e^-$  a doton)

$$E_{cb} = \frac{e^2}{2C} \quad C = 4\pi\epsilon_0 \epsilon \cdot R$$

$V_{SD}$   
(source, drain)

### VEDEK • pl. lateralis dotok



source drain



kayukat  
is alkalmazhat



### • Mollekula's dotok

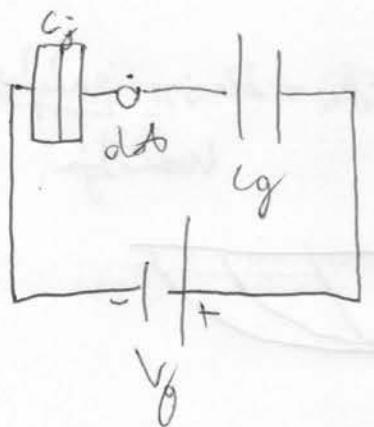
dramtli torulval

a mol. en. növekedés

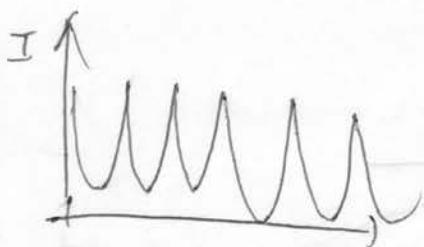
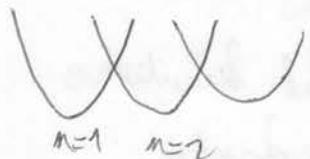
↳ fokusszertes szigetelés



cb-alkal (helyt. modell)



$U \approx f(t)$   
( $n, V_g$ )



felvez. dot

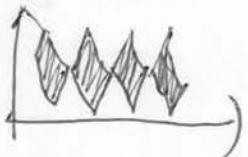
$V_{bias}$

$Q_0$

$\leftarrow$  (Cb-diamond)

(Cb-alkal miatt  
alakult ki)

$\sim$  min. hangerő



működés atomok (artificiell atom): több e- a doboz

energiarizálás

9.6a

S matrix kishindaya

$$0) \quad \underline{\underline{S}} \underline{\underline{S}}^+ = \underline{\underline{S}}^+ \underline{\underline{S}} = \mathbb{1}$$

Ket-terminal  $\rightarrow S = \begin{pmatrix} \underline{\underline{\gamma}} & \underline{\underline{\beta}} \\ \underline{\underline{\beta}}^\dagger & \underline{\underline{\gamma}}^\dagger \end{pmatrix}$

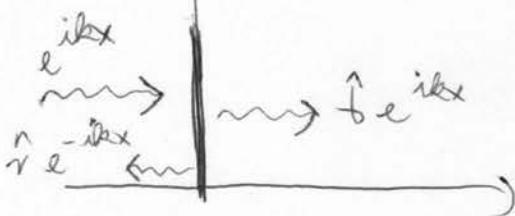
a) mode matching  $\rightarrow$  transformatrix

b) Green-matrices

1) Mode-matching

a) Dirac-delta Ordnatrix, 1D

$$\uparrow U(x) = U_0 \cdot \delta(x)$$



$$\frac{\hbar^2 k^2}{2m} = E$$

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi + U_0 \delta(x) \Psi(x) = E \Psi(x)$$

$$\Psi(x) = \begin{cases} e^{ikx} + \hat{r} e^{-ikx}, & x \leq 0 \\ \hat{t} \cdot e^{ikx}, & x \geq 0 \end{cases}$$

wake 1 modes:  $\hat{r} = r$ ,  $\hat{t} = t$

$$\left[ \frac{-\hbar^2}{2m} \Psi \right]_{-\varepsilon}^{+\varepsilon} + U_0 \cdot \Psi(0) = 0$$

hat. feld.  $|\Psi_+| = |\Psi_-|$

$$|\Psi'_+| - |\Psi'_-| = \frac{2mU}{\hbar^2} \Psi(0)$$

$$1+r = b$$

$$b \cdot ik - (ik - ik \cdot r) = \frac{2mU_0}{\hbar^2} (1+r)$$

~~$$\lambda = \frac{2mU_0}{\hbar^2}$$~~

$$\lambda = \frac{2mU_0}{\hbar^2}$$

$$(\lambda) = \frac{1}{m^2} \cdot \cancel{\lambda} = \frac{1}{m}$$

$$\int U(x) dx = U_0$$

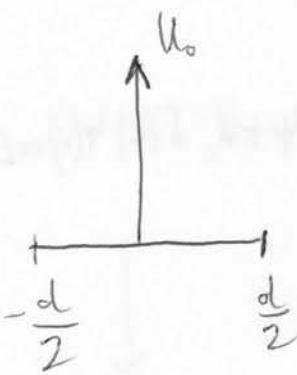
$$r = \frac{-i \frac{\lambda}{2}}{k + i \frac{\lambda}{2}}$$

$$b = \frac{k}{k + i \frac{\lambda}{2}}$$

$$\text{minim}: r = r \quad b = b$$

$$\Rightarrow \underline{\underline{S}} = \frac{1}{k + i \frac{\lambda}{2}} \begin{pmatrix} -i \frac{\lambda}{2} & k \\ k & -i \frac{\lambda}{2} \end{pmatrix}$$

b)



$$\underline{\underline{S}}_{\text{res}} = \begin{pmatrix} 0 & e^{ik\frac{d}{2}} \\ e^{-ik\frac{d}{2}} & 0 \end{pmatrix}$$

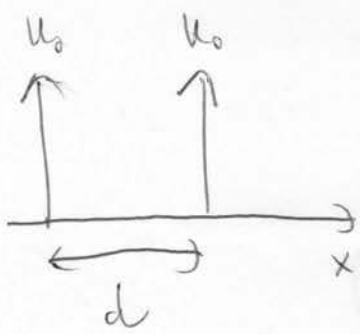
$$\Psi = e^{ikx} \rightarrow e^{ik(x+d/2)} = e^{ikx} \cdot e^{ik\frac{d}{2}}$$

$$\underline{\underline{S}} = \frac{e^{ikd}}{k + i \frac{\lambda}{2}} \begin{pmatrix} -i \frac{\lambda}{2} & k \\ k & -i \frac{\lambda}{2} \end{pmatrix}$$

→ reiter:

$$\begin{pmatrix} i \frac{\lambda}{2} & k \\ k & i \frac{\lambda}{2} \end{pmatrix}$$

$$\frac{1}{k^2 + (\frac{\lambda}{2})^2} \begin{pmatrix} -i \frac{\lambda}{2} & k \\ k & -i \frac{\lambda}{2} \end{pmatrix} \left( \dots \right)$$



$$\Rightarrow T(E) = \frac{T_1^2}{1 - 2R_{\text{co}}\theta + R_1^2} \quad T = |H|^2$$

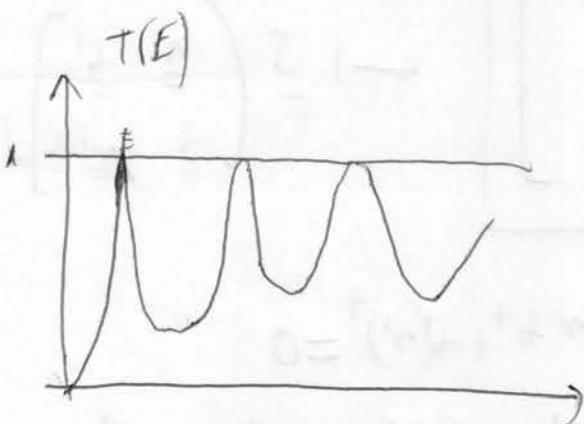
$$T_1 = \frac{\omega^2 v^2}{\omega^2 v^2 + k_0^2} \quad R_1 = \frac{U_0^2}{\omega^2 v^2 + k_0^2}$$

(HF)

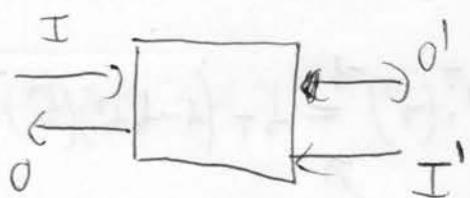
veliz.

$$\text{schl} \quad n = \frac{\hbar k}{m}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\theta = 2 \left( k d + \arctg \frac{kv}{U_0} \right)$$



Transfer-matrix ( $M$ )  $\rightarrow$  Mi a kapcsolat az  $S$  és  $M$  között?



$$\begin{pmatrix} 0 \\ 0' \end{pmatrix} = S \begin{pmatrix} I \\ I' \end{pmatrix} \quad \rightarrow \quad S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \quad \rightarrow \quad 0 = rI + t'I' \quad I = M_{21}I + M_{12}0$$

$$\begin{pmatrix} 0' \\ I' \end{pmatrix} = M \begin{pmatrix} I \\ 0 \end{pmatrix} \quad \rightarrow \quad 0' = M_{11}I + M_{12}0 \quad I' = M_{21}I + M_{22}0$$

$$0 = \underline{\underline{r}} \underline{\underline{I}} + \underline{\underline{t}}' \underline{\underline{I}} \rightarrow \underline{\underline{t}}' = \underline{\underline{t}}'^{-1} (0 - \underline{\underline{r}} \underline{\underline{I}})$$

$$\underline{\underline{r}}' = \underline{\underline{t}} \underline{\underline{I}} + \underline{\underline{r}}' \underline{\underline{I}}$$

$$0 = \underline{\underline{t}} \underline{\underline{I}} + \underline{\underline{r}}' \underline{\underline{t}}^{-1} (0 - \underline{\underline{r}} \underline{\underline{I}})$$

$M_{11}$

$M_{12}$

$$\underline{\underline{M}} = \begin{bmatrix} t - r' (t')^{-1} r & r (t')^{-1} \\ - (t')^{-1} r & (t')^{-1} \end{bmatrix} \quad \leftarrow \quad \underline{\underline{S}} = \begin{bmatrix} \underline{\underline{r}} & \underline{\underline{t}}' \\ \underline{\underline{t}} & \underline{\underline{r}}' \end{bmatrix}$$

$$\text{mivel } \underline{\underline{S}} \text{ unitär } (S S^+ = 1) \Rightarrow r b^+ + t (r')^+ = 0$$

$$t' = \cancel{t} - r b^+ (r')^{+1}$$

$$(t')^{-1} = - (r')^+ (b^+)^{-1} \cdot r^{-1}$$



$$\boxed{M_{11} = t + r' (r')^+ (b^+)^{-1} \cancel{r} = t + r' r^+ (b^+)^{-1} = t + (1 - t b^+) (b^+)^{-1} = (b^+)^{-1}}$$

orthog.

$t = b b^+ + r' (r')^+$

hasonlóan:

$$\underline{\underline{M}} \rightarrow \underline{\underline{S}} = \begin{pmatrix} \underline{\underline{r}} & \underline{\underline{t}}' \\ \underline{\underline{t}} & \underline{\underline{r}}' \end{pmatrix}$$

nemekkéven est  
szokkal jobb  
számolni

$$\underline{\underline{r}} = - M_{22}^{-1} M_{21}$$

$$\underline{\underline{r}}' = M_{12} M_{22}^{-1}$$

$$\underline{\underline{t}} = M_{11} - M_{12} M_{22}^{-1} M_{21} = (M_{11}^+)^{-1}$$

$$\underline{\underline{t}}' = M_{22}^{-1}$$

1 modusus

$$M = \begin{pmatrix} 1 & \frac{r}{b} & b \\ \frac{r}{b} & 1 & -\frac{r}{b} \\ -\frac{r}{b} & -\frac{r}{b} & 1 \end{pmatrix} \quad r, b \in C$$

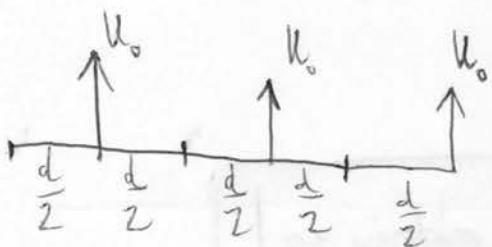
$b = b^*$

$$b r^* = -r b^*$$

$$= \begin{pmatrix} 1 & -\frac{r^*}{b^*} \\ -\frac{r}{b} & 1 \end{pmatrix}$$

$$\frac{r^*}{b^*} = -\frac{r}{b}$$

$$\frac{r}{b} = -\frac{r^*}{b^*}$$



$$\begin{array}{c} u_0 \\ \hline \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} \end{array} \rightarrow S = \frac{e^{ikd}}{k+i\frac{\lambda}{2}} \begin{bmatrix} -i\frac{\lambda}{2} & k \\ k & -i\frac{\lambda}{2} \end{bmatrix} =$$

$$\Rightarrow M = \begin{pmatrix} (1-i\frac{\lambda}{2k})e^{ikd} & -i\frac{\lambda}{2k} \\ i\frac{\lambda}{2k} & (1+i\frac{\lambda}{2k})e^{-ikd} \end{pmatrix}$$

$N=17$  : elijas

$$M^N \Rightarrow S \rightarrow I \rightarrow T = \boxed{T = \frac{1}{17} \left( \frac{b}{b^*} \right)}$$

Tribk:  $\underline{\underline{M}}^N \rightarrow$  hogy számoljuk ki?

$$\begin{vmatrix} M_{11}-\lambda & M_{12} \\ M_{21} & M_{22}-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - (\text{Tr } \underline{\underline{M}}) \lambda + \det \underline{\underline{M}} = 0$$

$$\det \underline{\underline{M}} = \frac{1}{f^*} \frac{1}{f} - \frac{rr^*}{f+f^*} = \frac{1-rr^*}{f+f^*} = \frac{ff^*}{f+f^*} = 1$$

$$\lambda^2 - \lambda \cdot \text{Tr } \underline{\underline{M}} + 1 = 0$$

A kompl. egyenlötet kielégíti a matrix is!

$$\underline{\underline{M}}^2 - \underline{\underline{M}} \cdot \text{Tr } \underline{\underline{M}} + 1 = 0$$

$$\underline{\underline{M}}^2 = \underline{\underline{M}} \text{Tr } \underline{\underline{M}} - 1$$

$$\begin{aligned} \underline{\underline{M}}^3 &= \underline{\underline{M}} \underline{\underline{M}}^2 = \underline{\underline{M}} (\underline{\underline{M}} \text{Tr } \underline{\underline{M}} - 1) = \underline{\underline{M}}^2 \text{Tr } \underline{\underline{M}} - \underline{\underline{M}} = (\underline{\underline{M}} \text{Tr } \underline{\underline{M}} - 1) \text{Tr } \underline{\underline{M}} - \underline{\underline{M}} = \\ &= \underline{\underline{M}} (\text{Tr } \underline{\underline{M}})^2 - \text{Tr } \underline{\underline{M}} \cdot \underline{\underline{M}} \end{aligned}$$

...

$$\underline{\underline{M}}^N = \frac{1}{\sin \phi} \cdot \left( N \sin N\phi - \frac{1}{2} \sin(N-1)\phi \right), \quad \cos \phi = \text{Re } M_{11} = \frac{1}{2} \text{Tr } \underline{\underline{M}}$$

$$\cos \phi = \frac{1}{2} \left( 2 \cos kd - \frac{\lambda}{2k} \cdot 2 \cdot \sin kd \right) = \cos(kd) + \frac{\lambda}{2k} \sin(kd)$$

↓

...



ha  $N \rightarrow \infty$ , ergo egy  $\rightarrow$  periodiken mcs  $\rightarrow$  savenkerend

Als los 0  $\sim T(E)$ , akol a vinnende gapje van

Mit van több módsz eredmény?

$$\det M = 1$$

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad \downarrow \quad \lambda_1 \lambda_2 = 1 \Rightarrow \lambda_2 = \frac{1}{\lambda_1}$$

$$M^N \rightarrow \begin{pmatrix} \lambda_1^N & 0 \\ 0 & \lambda_2^N \end{pmatrix} \quad \text{ha } \lambda_1 > 1 \Rightarrow \lambda_2 < 1$$

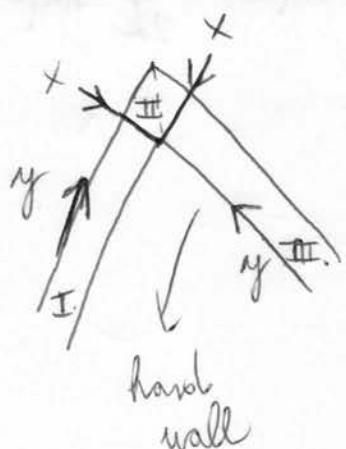
$1 > \lambda_2^N$  kicsi lesz

numerikusan



elvonalik az S matrix! (faktorizál)

$\Rightarrow$  megvalósítható S matricával minden, de stabilabb numerikusan



Mekkora lesz a viszonytosság?

$$\Psi_{\pm}(x,y) = e^{\pm ik_p^{(1)} y} X_p^{(1)}(x) + \sum_j r_j^{(1)} e^{\pm ik_j^{(1)} y} X_j^{(1)}(x)$$

$$\Psi_{\pm}(x,y) = \sum_j c_j \psi_j(x,y)$$

$$\underline{\Psi_{\pm} = \sum_j \frac{f^{(1)}_j}{f^{(2)}_j} e^{\pm ik_j^{(2)} y} X_j^{(2)}(x)}$$

I.  $(k_j^{(1)})^2 + \left(\frac{j\pi}{a}\right)^2 = k_p^2$ ,  $X_j^{(1)} = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{j\pi}{a} x\right)$

longitud. keretmátrix

III.  $(k_j^{(2)})^2 + \left(\frac{j\pi}{a}\right)^2 = k_F^2$ ,  $X_j^{(2)}(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{j\pi}{a} x\right)$

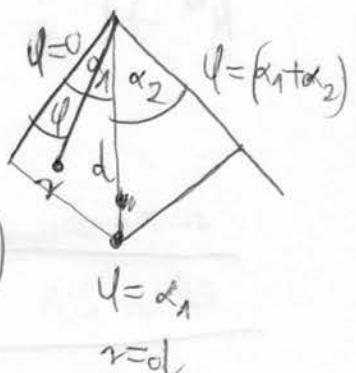
II. Milyen legyen  $\phi_j$ ? Olyat érdemes válni, ami a határon 0

$$\phi_j(x,y) := \uparrow_{V_j} f_r(r) \cdot \sin(V_j \varphi) - \theta$$



Bessel-fn.

$$-\frac{\uparrow_{V_j} (k_d) \sin(V_j \alpha_1)}{\uparrow_{V_{j+1}} (k_d) \sin(V_{j+1} \alpha_1)} \uparrow_{V_{j+1}} (k_r) \sin(V_{j+1} \varphi)$$



↓ Tisztja-e a határakat?

- $\psi = 0 \rightarrow \phi = 0$
- $\psi = \alpha_1 + \alpha_2 \rightarrow \phi = 0 \leftarrow \sin \nu_j (\alpha_1 + \alpha_2) = 0$
- $\nu_j = \frac{\pi}{\alpha_1 + \alpha_2} \cdot j$
- $\psi = \alpha_1 \text{ is red } \rightarrow \phi = 0$

$$-\frac{\hbar^2}{2m} \Delta \Psi = E \Psi$$

$$\Psi(r, \vartheta) = J_m(kr) e^{im\vartheta}$$

$$J_m(kr) \sin(m\vartheta)$$

ugyanaz!

most  $m = \nu_j$  több

$\Rightarrow J_{\nu_j}(kr) \cdot \sin(\nu_j \vartheta)$  minden megoldás (csak most  $\nu_j$  több)

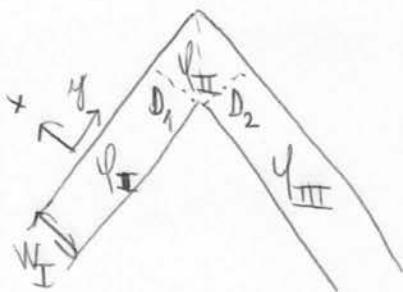
↓

több.

Kön. öre: illusztráció a határánkon

XII.2.

10. drá



$$N_I (\text{nyitott csat. } \alpha_1 \pm \text{tart. lán}) = \left[ \frac{k_F \cdot N_I}{\pi} \right]$$

$$\Psi_I = e^{ik_p^{(I)} y} \cdot X_p^{(I)}(x) + \sum_j \boxed{r_j^{(I)}} e^{-ik_j^{(I)} y} \cdot X_j^{(I)}(x)$$

↑ autonómikus lán

$$\Psi_{II} = \sum_j \boxed{b_j^{(II)}} e^{ik_j^{(II)} y} \cdot X_j^{(II)}(x)$$

$$\Psi_{III} = \sum_j c_j \phi_j(x, y) \rightarrow \sum_j g_j \phi_j(r, \varphi)$$

Matching:

$$\Psi_{\pm} \Big|_{D_1} = \Psi_{\mp} \Big|_{D_{III}} ; \quad \Psi_{\pm} \Big|_{D_2} = \Psi_{\mp} \Big|_{D_2}$$

$$\frac{\partial}{\partial y} \Psi_{\pm} \Big|_{D_1} = \frac{\partial}{\partial y} \Psi_{\mp} \Big|_{D_{III}} ; \quad \frac{\partial \Psi_{\pm}}{\partial y} \Big|_{D_2} = \frac{\partial \Psi_{\mp}}{\partial y} \Big|_{D_2}$$

↳ Rány eggyel tömök van?

$$\langle X_e | \Psi_{\pm} \rangle = \langle X_e | \Psi_{\mp} \rangle$$

$$\langle X_e^{(y)} | \phi_j(r, \varphi) \rangle \leftarrow \text{ez az integrálba el kell}\rightleftharpoons \text{végzni, mert } X_e \text{ nem ortogonális!}$$

$$M = \begin{pmatrix} r_j^{(N)} \\ b_j^{(N)} \end{pmatrix} \}^{M_I}$$

$\downarrow M_{III}$

inhomogen, mert  $e^{ik_p^{(I)} y} X_p^{(I)}(x)$  tag  
nem fog kiesni

ilyen típusú egyenletekről beszél, ha leírunk  $\langle X_e \rangle$ -el

pl. numerikusan el lehet végezni

$$\text{ell.: } \sum_j \left( |r_j^{(N)}|^2 + |b_j^{(N)}|^2 \right) = 1 \quad \text{addig p-e}$$

Megj. 1:

Akkor, hogy ez numerikusan teljesüljön, be kell venni a zárt csatolmányt is!  $\rightarrow$  itt  $k_p$  imaginárius

$\downarrow$

ezek a veretlenkép.-hei nem járulnak hozzá, DE  
megmaradásukhoz kellende

Megj. 2:

$$\frac{\partial}{\partial y} Y_{\frac{1}{2}} \Big|_{0,1} \rightarrow r = \sqrt{x^2 + y^2}$$

$$\tan \varphi = \frac{y}{x}$$

Bessel-fv.-ek

$$\frac{1}{\cos \varphi} \cdot \frac{\partial \varphi}{\partial y} = \frac{1}{x}$$

$$\frac{\partial \varphi}{\partial y} = \frac{\cos^2 \varphi}{r \cos \varphi} = \frac{\cos \varphi}{r}$$

$$\begin{aligned} \frac{\partial}{\partial y} &= \frac{\partial}{\partial r} \cdot \frac{\partial r}{\partial y} + \frac{\partial}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial y} = \frac{\partial}{\partial r} \cdot \frac{2xy}{2\sqrt{r}} + \frac{\partial}{\partial \varphi} \cdot \frac{\cos \varphi}{r} = \\ &= \frac{\partial}{\partial r} \cdot \sin \varphi + \frac{\partial}{\partial \varphi} \cdot \frac{\cos \varphi}{r} \end{aligned}$$

megj. 3:

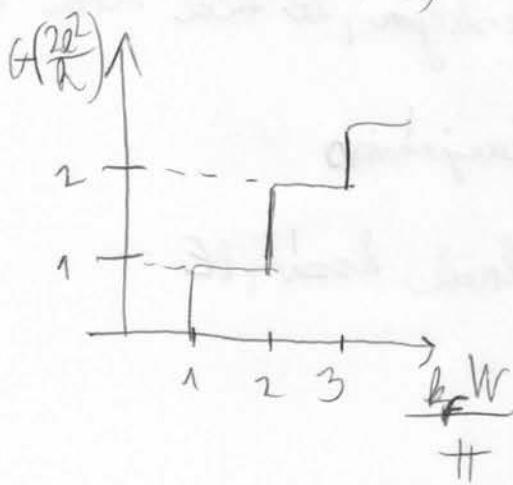
Mi van, ha máshogy illesztjük?



→ ~~elsődleges rezonans a határon~~  
illesztési, mégis az általános módszer könnyebb a M-est kihozni, DE numerikusan instabilabb

megj. 4:

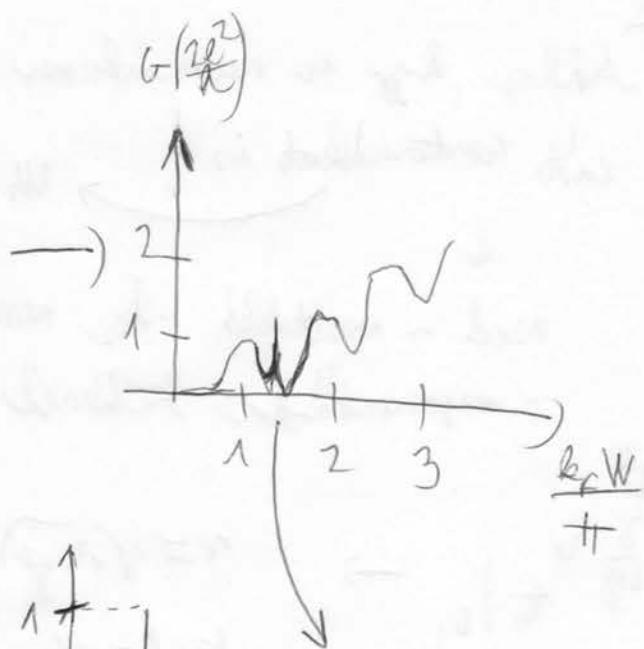
$$G = \frac{2e^2}{\hbar} \text{Tr} \left( \begin{smallmatrix} b^\dagger & b \\ b & b^\dagger \end{smallmatrix} \right)$$



tökéletes vésett

szűkített sp. m.  
(leképzési szabály)

színes spektrum m.



Fano - rezonancia



Wigner - rezonancia

→ ha a 2 találkozik, akkor

szokott bivalens

Fano - rezonancia

⇒ Mode-matching nem til a általánosan alkalmazható módszer (geometrikus erősen függ)

# Green - fr. - os módszer

1)  $[E - H(r)] G(\underline{r}, \underline{r}'; E) = \delta(\underline{r} - \underline{r}') + \text{h.o. felt. (pl. Dirichlet)}$

főbbes  $\underline{r}$ -ben  $\rightarrow$  mi lesz a  
megoldás  $\underline{r}'$ -ben

a határon  $G=0$ )  
 $\underline{r}=0, \underline{r}'=0$

$$H(r) \# \phi_n(r) = E_n \phi_n(r)$$

$$\int \phi_n(r) \phi_m(r) dr = \delta_{nm}$$

$$\sum_n \phi_n(r) \phi_n^*(r) = \delta(r - r')$$

$$\phi_n(r) = \langle r | \phi_n \rangle \quad (\text{helyexpr.})$$

$$\delta(r - r') H(r) = \langle r | H | r' \rangle \quad \text{egress. op. matrica}$$

$$G(\underline{r}, \underline{r}'; E) = \langle r | G(E) | r' \rangle$$

helyexpr. -ban

$$\langle r | r' \rangle \langle r | = 1$$

$$\int dr |r\rangle \langle r| = 1$$

$$(E - H) G = 1$$

vis:

$$\langle r | (E - H) G | r' \rangle = \langle r | \underbrace{1}_{?} | r' \rangle = \delta(r - r')$$

b.o.:  $E G(\underline{r}, \underline{r}') - \int d\underline{r}'' \underbrace{\langle r | H | r'' \rangle}_{\delta(r - r'')} \underbrace{\langle r'' | G | r' \rangle}_{-\delta(r'' - r')} \rightarrow E G(r, r') - H(r) G(r, r') = \delta(r - r')$

↓

$$\hat{G} = \frac{1}{E - H} \quad \text{operatoros repr.}$$

$$\boxed{\hat{G}(E) = (E - \hat{H})^{-1} = (E - H)^{-1} \cdot \sum_n |\phi_n\rangle \langle \phi_n| = \sum_n (E - E_n)^{-1} |\phi_n\rangle \langle \phi_n|}$$

$$= \sum_n \frac{|\phi_n\rangle \langle \phi_n|}{E - E_n}$$

~~operator~~  
spektralrepräsentacio (projektor-  
felder)

$$\boxed{G(r, r') = \langle r | \hat{G}(r') = \sum_n \frac{\langle r | \phi_n \rangle \langle \phi_n | r' \rangle}{E - E_n} = \sum_n \frac{\phi_n(r) \phi_n^*(r')}{E - E_n}}$$

felxible representacio

2) perturbacio:

$$H = H_0 + H_1$$

$$G_0 = (E - H_0)^{-1}$$

$$G = (E - H)^{-1} = (E - H_0 - H_1)^{-1}$$

$$\underbrace{(E - H_0 - H_1)}_{G_0^{-1}} G = 1$$

$$G_0 \cdot / \quad G_0^{-1} G - H_1 G = 1$$

$$G - G_0^{-1} H_1 G = G_0$$

$$\boxed{G = G_0 + G_0^{-1} H_1 G}$$

Dyson-egyenlet (l. Soltész ±)  
-P4

J mo. iteracional

$$G^{(1)} = G_0 + G_0 H_1 G_0$$

$$G^{(2)} = G_0 + G_0 H_1 G_0 + G_0 H_1 G_0 H_1 G_0 + \dots$$

⋮

retarded  $G = (E + i\gamma - H)^{-1}$

3) elipses:

$$\hat{G} \leftrightarrow \underline{\underline{S}} \rightarrow \underline{\underline{\underline{P}}} \rightarrow T \rightarrow G \text{ (verteilspur)}$$

$\hat{G}$  es  $\underline{\underline{S}}$  konservativ:

a) gekoppelte or singulär

$$\begin{array}{c} A_m^- \leftarrow \xrightarrow{\text{singulär}} A_m^+ \\ \hline \end{array}$$

$$(E-H) G(x, y, x', y') = \delta(x-x') \delta(y-y')$$

$$G(x, y, x', y') = \sum_m A_m^\pm (y') \chi_m(y) e^{ik_m|x-x'|}$$

$$k_m = \sqrt{\frac{2m(E-\varepsilon_m)}{\hbar^2}}$$

$$\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + U(y) \right] \chi_m(y) = \varepsilon_m \chi_m(y)$$

illenstellen or singulär:

$$\text{I. } G(x, x') \Big|_{x=x'+0} = G(x, x') \Big|_{x=x'-0} \quad \text{if } S$$

$$\text{II. } \frac{\partial G(x, x')}{\partial x} \Big|_{x=x_0} - \frac{\partial G(x, x')}{\partial x'} \Big|_{x=x'_0} = \frac{2m}{\hbar^2} \delta(y - y') \quad (\text{Feldintegrale in Green-Fkt.-s. feld' eignen})$$

$$\text{I. } \rightarrow \sum_m A_m^+ X_m(y) = \sum_m A_m^- X_m(y) \quad / \cdot \int dy X_n^*(y)$$

$$\text{II. } \rightarrow \sum_m i k_m X_m(y) A_m^+ = - \sum_m A_m^- (-ik_m) X_m(y) = \frac{2m}{\hbar^2} \delta(y - y') \\ / \int dy X_n^*(y)$$

$$\text{III. } \rightarrow A_n^+ = A_n^-$$

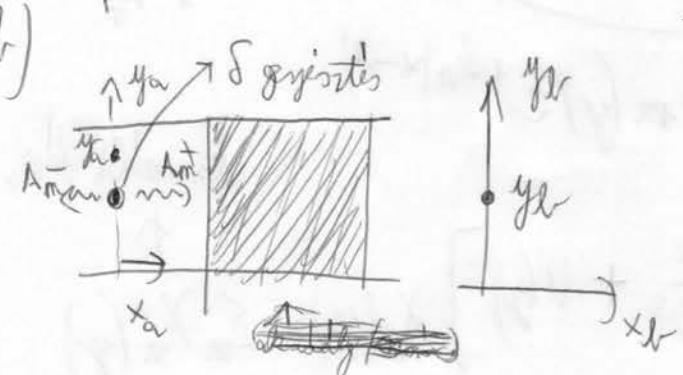
$$\underline{\text{IV. } ik_n \cdot (A_n^+ + A_n^-) = \frac{2m}{\hbar^2} X_n(y')}$$

$$A_n^+ = A_n^- = -\frac{i}{\hbar v_n} X_n^*(y') \quad \text{und} \quad v_n = \frac{\hbar k_n}{m}$$

$$\Rightarrow \boxed{G(x, y, x', y') = \sum_n -\frac{i}{\hbar v_n} e^{ik_n(x-x')} \cdot X_n(y) \cdot X_n^*(y')}$$

(X-Ber + rotierendes Kell)

aus w. Green-Fkt.-e



$$G^{(R)}(y_b, y_a) \equiv G^{(R)}(y_b, x_r=0, y_a, x_a=0)$$

mechan. lass. a. attenz  
Nulllin. y\_r-ber., ha  
y\_a-ber. van ~~a~~ gesp. s

$$G_{ba}(y_b | y_a) = \sum_{\substack{m \neq n \\ n \neq b}} \left[ \delta_{nm} \delta_{ab} A_m^- + \hat{\delta}_{nm} A_n^+ \right] X_n^{(b)}(y_b)$$

Ugyen akkoruk bell lenne a Green-fn. -nak

$$\cancel{\text{ha}} \quad a+b=0$$

sok hullám van, melyek  $A_m$  van viszavez. (reflex)

$$\text{lyuk innen: } A_m^+ = A_m^- = -\frac{i}{\hbar v_n} X_n^*(y') \rightarrow$$

$$G_{ba}(y_b | y_a) = \sum_{\substack{m \neq n \\ n \neq b}} -\frac{i}{\hbar \sqrt{v_m^{(a)} v_n^{(b)}}} \cdot X_m^{*(a)}(y_a) X_n^{(b)}(y_b) [\delta_{nm} \delta_{ab} + \hat{\delta}_{nm}]$$

összefüggés S és G között:

$$/ \cdot \delta_{ya} \delta_{yb} X_n^{(a)}(y_a) \cdot X_n^{(b)}(y_b)$$

$$S_{nm} = -\delta_{nm} \delta_{ab} + i \hbar \sqrt{v_m^{(a)} v_n^{(b)}} \left[ \int X_n^{(a)*}(y_b) G_{ba}(y_b | y_a) X_m^{(a)}(y_a) dy_a dy_b \right]$$

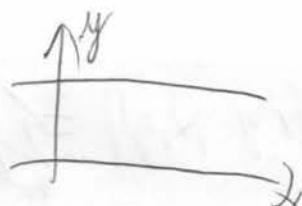
↳ Fischer-Lee-relatív: ~~PRB~~ Phys. Rev. B 23, 6851 (1981)

• B-tér: Barranger-Sone: ~~PRB~~ Phys. Rev. B 40, 8169 (1989) ← Sandauer

c) masik levezetés (a)-ra

$$\hat{G}^{(R)}(E) = (E - H + i\eta)^{-1}$$

$$G^{(R)}(\underline{r}, \underline{r}' | E) = \sum_m \frac{4\pi r_m^{(a)} 4\pi r_m^{*(b)}}{E - E_m + i\eta}$$



Kubo-formula

$$\{ H \Psi_m(x) = E_m \Psi_m(x) \}$$

$$H = \frac{p_x^2 + p_y^2}{2m} + \text{har. fkt.}$$

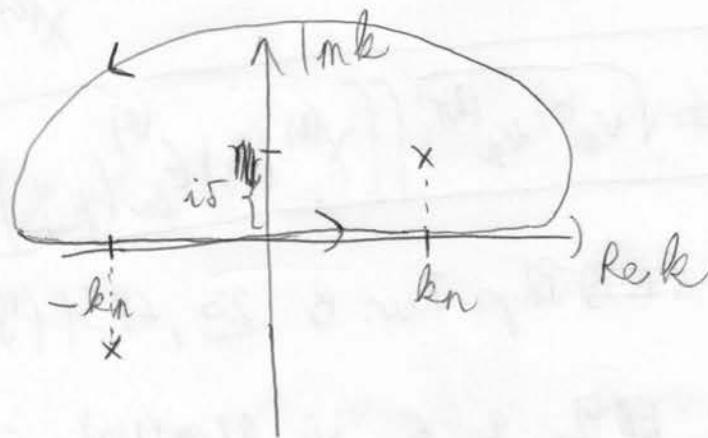
ansatz:  $\Psi_{n,k}(x,y) = \frac{1}{\sqrt{L}} e^{ikx} \cdot X_n(y) \leftarrow \sqrt{\frac{2}{W}} \sin\left(\frac{n\pi}{W}y\right)$

$\uparrow$  direkt fkt.       $\downarrow$  nicht mehr  $X_n(y)$

$$G^{(R)}(x_1, y_1 | x_1, y_1) = \frac{1}{L} \sum_{n,k} \frac{e^{ik(x-x_1)} X_n(y_1) \cdot X_n(y)}{E - \frac{k^2 \hbar^2}{2m} - \frac{\hbar^2}{2m\left(\frac{n\pi}{W}\right)^2} + i\eta} =$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int dk \frac{e^{ik(x-x_1)} X_n(y_1) X_n(y)}{E + i\eta - \frac{\hbar^2}{2m} \left[ k^2 + \left(\frac{n\pi}{W}\right)^2 \right]}$$

$\int dk \rightarrow \sqrt{\frac{dk}{2\pi}} \cdot L$



- 1) da  $x > x_1 \Rightarrow \text{Im } k > 0$ , abkorcseng le az -ben  $\rightarrow$
- 2) da  $x < x_1 \Rightarrow \text{Im } k < 0$ ,  $-11 \rightarrow$

$$1) G^{(R)}(x_1, y_1 | x_1, y_1) = \frac{1}{2\pi} \int dk \frac{e^{ik_n(x-x_1)} X_n(y_1) X_n(y)}{-\frac{\hbar^2}{2m} + k_n^2}$$

- ff

mer.: plus  
 Rest  $\left( \frac{f(z)}{g(z)} \right) = \frac{f(z^*)}{g'(z^*)}$

$$= \sum_n -\frac{i}{\hbar v_n} e^{ik_n(x-x')} X_n(y) X_n(y')$$

$\times \times \times$

2) polus  $\rightarrow -k_n$ -ben  
 $\leftarrow \rightarrow \ominus$  irány  $\Rightarrow G(x,y, x', y') = \sum_n \frac{-i}{\hbar v_n} e^{-ik_n(x-x')} X_n(y) X_n(y')$

$\Rightarrow$  ugyanazt kapunk, mint a teljesítmény módszerek!

XII.

11. lira

0) újm.:

$$(E - \hat{H}) \hat{G} = 1$$

$$\hat{G}(E) = \sum_n \frac{\langle n | \rangle}{E - E_n}$$

ahol  $\hat{H}|n\rangle = E_n|n\rangle$

$$\underbrace{\phi_n(r)}_{\sim} \quad \underbrace{\phi_n^*(r')}_{\sim}$$

$$G(r, r'; E) = \langle r | \hat{G} | r' \rangle = \sum_n \frac{\langle r | n \rangle \langle n | r' \rangle}{E - E_n} = \sum_n \frac{\phi_n(r) \phi_n(r')}{E - E_n}$$

$$f(r-r') H(r) = \langle r | \hat{H} | r' \rangle$$

1) Dirac-delta mennyiséges:

re parton mennyiséges

2)  $H = H_0 + H_1$

$r \rightarrow \infty \rightarrow H = \lambda \delta(r - r_0)$

Dyron-egyenlet:  $G = G_0 + G_0 H_1 G$

$$(E - H_0 - H_1)G = 1 \quad \nearrow$$

$$G_0^{-1} \quad (G_0 = (E - H_0)^{-1})$$

$\equiv |r_e\rangle$

$$\text{Más: } \hat{H}_1 = \lambda |l\rangle\langle l|$$

$$\langle r | H_1 | r' \rangle = \lambda \underbrace{\langle r | l \rangle}_{\delta(r-r_e)} \underbrace{\langle l | r' \rangle}_{\delta(r'-r_e)} = \lambda \underbrace{\delta(r-r_e) \delta(r'-r_e)}_{\hat{H}_1} =$$

$$= \delta(r-r_e) H_1(r)$$

$H_1$  helyepr. -ja adja  
a  $\delta$  mennyiséget

$$G = G_0 + G_0 \cdot \lambda |l\rangle\langle l| G_0 + G_0 \lambda |l\rangle\langle l| G_0 \lambda |l\rangle\langle l| G_0 + \dots =$$

$$= G_0 + \lambda G_0 |l\rangle [1 + \lambda \langle l | G_0 | l \rangle + \dots] \langle l | G_0 =$$

$$= G_0 + \lambda G_0 |l\rangle \frac{1}{1 - \lambda \langle l | G_0 | l \rangle} (l | G_0 = G_0 + \lambda \frac{G_0(l) (l | G_0)}{1 - \lambda G_0(l, l; E)}$$

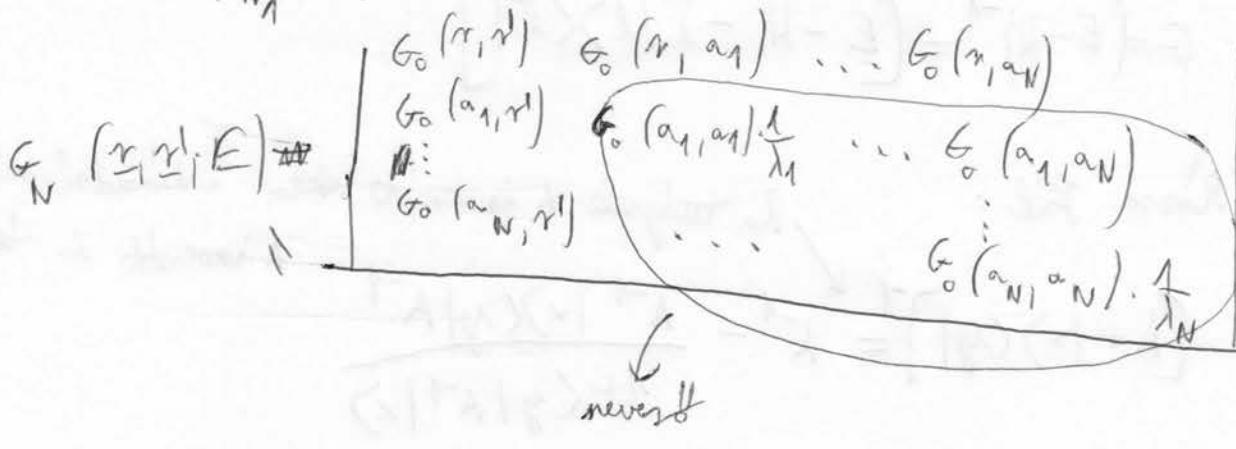
$$\Rightarrow G(r, r'; E) = \langle r | G | r' \rangle = G_0(r, r'; E) + \lambda \frac{G_0(r, r_e) G_0(r_e, r')}{1 - \lambda G_0(r_e, r; E)}$$

$\Rightarrow$  spec.  $H_1$ -re fel lehet összessíteni -  
• Dyron-egyenletet - egzaktf

2) több Dirac - deltaos mennyességek: (viszán nem kell)

$$G \rightarrow G_0$$

$$G + H_1 \rightarrow G$$



c) 1 mennyességek leveretés:

teljes hatás leírása (1)

$$G(r, r') = \langle r | G | r' \rangle = \langle r | G_0 | r' \rangle + \underbrace{\langle r | G_0 | r' \rangle}_{\text{...}} \langle r | H_1 | r' \rangle \langle r' | H_1 | r \rangle \underbrace{\langle r' | G | r \rangle}_{\text{...}} \cdot dr dr'$$

$$G(r, r') = G_0(r, r') + \iint dr_1 dr_2 G_0(r, r_1) \delta(r_1 - r_2) \lambda \cdot \delta(r_1 - r_2) G(r_2, r') = \\ = G_0(r, r_1) + \lambda G_0(r, r_1) \cdot G(r_1, r')$$

$$\text{Legyen } r = r_1 \Rightarrow G(r_e, r') = G_0(r_e, r') + \lambda G_0(r_e, r_1) \underline{G(r_1, r')}$$

$$G(r_e, r') = \frac{G_0(r_e, r')}{1 - \lambda G_0(r_e, r_1)}$$

viszánva  $G(r_e, r')$ -le:

$$G(r, r') = G_0(r, r') + \frac{G_0(r, r_e) G_0(r_e, r')}{1 - \lambda G_0(r_e, r_1)}$$

ellen nem összegünk  
mertani zárt! → akkor is  
igaz, ha a kibocs. > 1

d) 1. nemysk, 3. leveretet

$$H_1 = \lambda |e\rangle\langle e|$$

$$G = (E - H)^{-1} = [E - H_0 - \lambda |e\rangle\langle e|]^{-1}$$

Rössle Pál: Ha többük A inverz, akkor a dinamikai módszerrel  
A inverzét is kezeli

akk:  $[A + |x\rangle\langle y|]^{-1} = A^{-1} - \frac{A^{-1}|x\rangle\langle y|A^{-1}}{1 + \langle y|A^{-1}|x\rangle}$

Biz.:  $\left( A^{-1} - \frac{A^{-1}|x\rangle\langle y|A^{-1}}{1 + \langle y|A^{-1}|x\rangle} \right) (A + |x\rangle\langle y|) = 1 - \frac{A^{-1}|x\rangle\langle y|}{1 + \langle y|A^{-1}|x\rangle} +$

$$+ A^{-1}|x\rangle\langle y| - \frac{A^{-1}|x\rangle\langle y|A^{-1}|x\rangle\langle y|}{1 + \langle y|A^{-1}|x\rangle} =$$

$$= 1 - \frac{\cancel{A^{-1}|x\rangle\langle y|}}{1 + \langle y|A^{-1}|x\rangle} + \frac{\cancel{A^{-1}|x\rangle\langle y|} + \cancel{A^{-1}|x\rangle\langle y|A^{-1}|x\rangle\langle y|}}{1 + \langle y|A^{-1}|x\rangle} -$$

$$- \cancel{\frac{A^{-1}|x\rangle\langle y|A^{-1}|x\rangle\langle y|}{1 + \langle y|A^{-1}|x\rangle}} = 1 \quad /$$

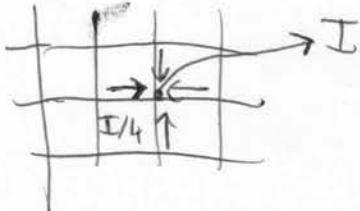
$\Rightarrow$  alk.:  $G = [E - H_0 - \lambda |e\rangle\langle e|]^{-1}$   
 ~~$\Rightarrow G = [E - H_0 - \lambda |e\rangle\langle e|]^{-1}$~~   
 $A \Rightarrow A^{-1} = G_0$

$$\Rightarrow G = G_0 - \frac{G_0(-\lambda)|e\rangle\langle e|G_0}{1 - \lambda \langle e|G_0|e\rangle}$$

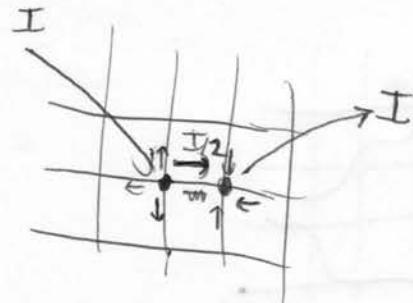
$$= G_0 + \lambda \frac{G_0|e\rangle\langle e|G_0}{1 - \lambda \langle e|G_0|e\rangle}$$

## 2) ellenállashálózat

a) 



szimmetria



$$I/2 \cdot R = U$$

↓

az egész

hálózat

ellenállása

$$\rightarrow \frac{U}{I} = R_e = \frac{R}{2}$$

de csak 2 normális rész

portra!

b)

$$R \cdot \Delta V(x) = -I(x)$$

$$\Delta V(x) = 4V(x) - \sum_n V(x+n)$$

↓

Replace -op. diskret kiszűtés

ezeket az Green-férfi-kt

kell meghat. → exkl. ellenállás tetsz. 2 pont között

9)



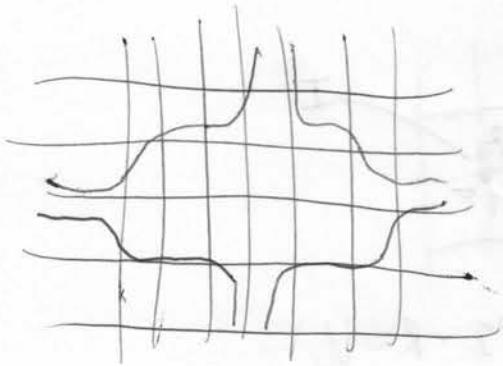
hányik 1 ellenállás → perturb.

belátható, hogy ez csak egy diszdropl

perturbált mű!

↳ használható az egészben összesen körök

## B) Schrödinger -egyenlet diskretizálása



1) dim.:  $H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + U(x)$

$\xrightarrow{x=j \cdot a}$

$$H \Psi \Big|_{x=ja} = -\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} \Big|_{x=ja} + U_j \Psi_j$$

$$\approx \frac{1}{a^2} (\Psi_{j+1} - 2\Psi_j + \Psi_{j-1})$$

$$H \Psi \Big|_{x=ja} = (U_j + 2\rho) \Psi_j - \rho \Psi_{j+1} - \rho \Psi_{j-1}$$

ahol

$$\rho \equiv \frac{\hbar^2}{2ma^2}$$

$$H = \begin{bmatrix} -\rho & & & & \\ -\rho & U_0 + 2\rho & -\rho & & 0 \\ & -\rho & U_1 + 2\rho & -\rho & \\ & & -\rho & U_2 + 2\rho & -\rho \\ 0 & & & & \ddots \end{bmatrix} \quad (\text{sorolag. mátrix})$$

Spec.  $U(x) = U_0$  (homogen m.)

$$\Psi_k(x) = e^{ikx} \rightarrow E(k) = U_0 + \frac{\hbar^2 k^2}{2m}$$

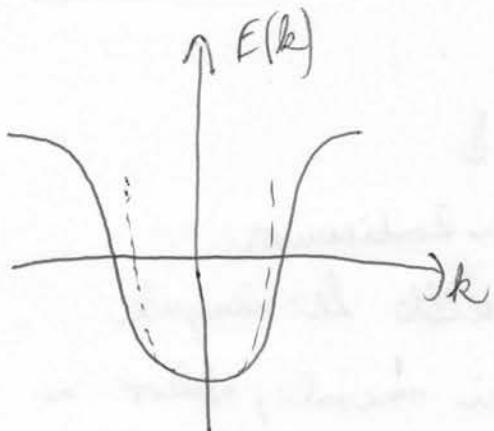
$$E\Psi_j = H\Psi_j \Rightarrow E\Psi_j = (U_0 + 2\gamma)\Psi_j - \gamma\Psi_{j+1} - \gamma\Psi_{j-1}$$

$$\Psi_j = e^{ikja}$$

tight-binding  
közelítés

$$E e^{ikja} = (U_0 + 2\gamma) e^{ikja} - \gamma e^{ik(j+1)a} - \gamma e^{ik(j-1)a}$$

$$E(k) = U_0 + 2\gamma - \gamma e^{ika} - \gamma e^{-ika} = U_0 + 2\gamma(1 - \cos ka)$$



$$\left(\gamma = \frac{\hbar^2}{2ma^2}\right)$$

$$\downarrow$$

$$a \rightarrow 0$$

$$1 - \frac{x^2}{2}$$

(malad  $\rightarrow$  disperziója)

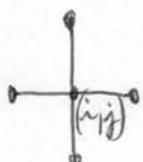


In folytonos m.-b akarunk leírni, finomítani kell a rezállásokat ( $a \rightarrow 0$ )

$\Downarrow$  egyszer több részről leír

egyre nagyobbról  $\rightarrow$  Hamiltoniás kell diagonalizálni

2.) 2 dim.:  $E(k_x, k_y) = U_0 + \frac{\hbar^2 k^2}{2m}$  műb. rezákkal



$$E\Psi_{ij} = (U_0 + 2\gamma)\Psi_{ij} - \gamma \cdot \Psi_{i+1,j} - \gamma \cdot \Psi_{i-1,j} - \gamma \cdot \Psi_{i,j+1} - \gamma \cdot \Psi_{i,j-1}$$

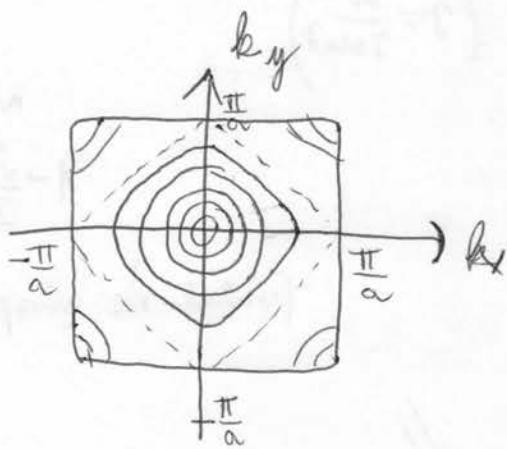
$$\psi_{ij} = e^{ik_x a + i k_y j a}$$

$$E(k) = U_0 + \frac{1}{2} \gamma - \gamma \cdot e^{-ik_x a} - \gamma e^{ik_x a} - \gamma e^{-ik_y a} - \gamma e^{ik_y a} =$$

$$= U_0 + 2\gamma \left( 1 - \cos k_x a - \cos k_y a \right)$$

$$\underbrace{\left( 1 - \frac{(k_x a)^2}{2} \right)}_{\frac{(k_x^2 + k_y^2)a^2}{2}} \quad \underbrace{\left( 1 - \frac{(k_y a)^2}{2} \right)}$$

$$\frac{(k_x^2 + k_y^2)a^2}{2}$$



ha a kontinuum  
 modellel le akarjuk  
 tenni nincsal, akkor a  
 $\gamma$  parameterek kell jöl  
 belüni (úgy, hogy az en.  
 a fizikai köreken essen)

$$\left( \sum_{j=1}^{M+N} H_{ij} + \frac{1}{E - H_{ll}} H_{lg} \right)$$

$$\sum_{\substack{j=1 \\ j \neq l}}^{M+N} \left( H_{ij} + \frac{H_{lj} H_{il}}{E - H_{ll}} \right) \psi_j = E \psi_i$$

$H_{ij}'$

$$\sum_{j=1}^{M+N-1} H_{ij}' \psi_j = E \psi_i$$

$$\text{ahol } H_{ij}' = H_{ij} + \frac{H_{il} \cdot H_{lj}}{E - H_{ll}}$$

egy + mű. fokkal csökkenetettsége!

laj:  $H_{ij}'$ -ben E lenne  
van

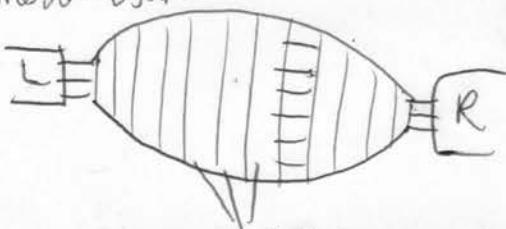
$H_{ij}$  →



Most es nem nagy gonol, most att. művesítésre számunkra

↳ E folytonos, addig E letezik nem csak a saját hálókban,  
nem pedig direkten. mintekéz besorítható

numerikusan



szabály → 1-1 Hamiltoni, mik kölcsönhatásnak

✓ - gf

$$\left( \sum_{j=1}^{M+N} H_{ij} + \frac{1}{E - H_{ll}} H_{lg} \right)$$

$$\sum_{\substack{j=1 \\ j \neq l}}^{M+N} \left( H_{ij} + \frac{H_{lj} H_{il}}{E - H_{ll}} \right) \psi_j = E \psi_i$$

$H'_{ij}$

$$\sum_{j=1}^{M+N-1} H'_{ij} \psi_j = E \psi_i$$

$$, \text{ ahol } H'_{ij} = H_{ij} + \frac{H_{il} \cdot H_{lj}}{E - H_{ll}}$$

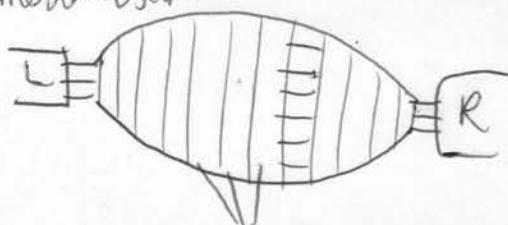
egy + műb. fokkal csökkenetettsége

laj:  $H'_{ij}$ -ben E lenne  
van

$H_{ij} \rightarrow$

Most ez nem nagy gonosz, most att. módszereket próbálunk  
 $\hookrightarrow$  E folytonos, addig E lehetséges csak a sajátball- skat,  
 nem pedig direkt en. minteket besoríthatunk

numerikusan

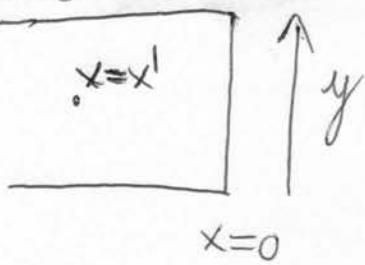


melettel  $\rightarrow$  1-1 Hamiltoni, amik kölcsönhatnak

Mit den  $g_L, g_R$ ?  $\rightarrow \omega \propto$  eines  $\omega_0$  Green-fv.-Lösung  
verfügbar leistet

## 2) Surface Green-fv.

a) <sup>(20)</sup> hard wall



$$\psi_{m,k}(x,y) = \sqrt{\frac{2}{L}} \cdot X_m(y) \cdot \sin(kx)$$

↓  
 diskret  
 (Anzahlen)  
 index

↓  
 fiktiv.  
 index

$\sqrt{\frac{2}{W}} \sin\left(\frac{m\pi y}{W}\right)$

$$G^R(x_1, y_1, x'_1, y') = G(x_1, y_1, x_1, y') = \frac{1}{L} \sum_m \sum_k \frac{X_m(y_1) X_m(y') \sin^2(kx)}{E - E_m - \frac{k^2 c^2}{2m} + i\eta}$$

$$G(r, r') = \sum_n \frac{\phi_n(r) \phi_n^*(r')}{E - E_n + i\eta}$$

mit:

$$\left( \frac{e^{ikx} - e^{-ikx}}{2i} \right)^2 = \frac{2 - e^{2ikx} - e^{-2ikx}}{4}, \quad \sum_k \rightarrow \frac{1}{2\pi} \int dk$$

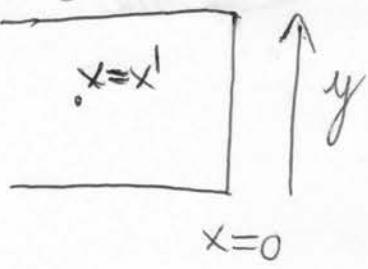
$$\textcircled{*} = \frac{1}{2\pi} \sum_m X_m(y) X_m(y') \cdot \int_{-\infty}^{\infty} dk \frac{1 - e^{2ikx}}{E - E_m - \frac{k^2 c^2}{2m} + i\eta} \rightarrow \text{residuum tabel} \rightarrow \dots$$

$\left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}, \text{da alle } e^{-2ikx} \rightarrow e^{2ikx} \text{ berne van} \right)$

Mit den gr<sub>L</sub>, gr<sub>R</sub>?  $\rightarrow \omega \propto$  ires  $\Leftrightarrow$  Green-fv.-L<sub>00</sub>  
verstöße le erkennt

## 2) Surface Green-fv.

a) <sup>(20)</sup> hard wall



$$\psi_{m,k}(x,y) = \sqrt{\frac{2}{L}} \cdot X_m(y) \cdot \sin(kx)$$

↓  
 diskret  
 (transzen.)  
 index

↑  
 foly.  
 index

$\sqrt{\frac{2}{W}} \sin\left(\frac{m\pi y}{W}\right)$

$$G^R(x_1, y_1 | x_1', y_1') = G(x_1, y_1 | x_1', y_1') = \frac{1}{L} \sum_m \sum_k \frac{X_m(y_1) X_m(y_1') \sin^2(kx)}{E - E_m - \frac{\hbar^2 k^2}{2m} + i\eta} \quad (*)$$

$$G(r, r') = \sum_n \frac{\phi_n(r) \phi_n^*(r')}{E - E_n + i\eta}$$

mit:

$$\left( \frac{e^{ikx} - e^{-ikx}}{2i} \right)^2 = \frac{2 - e^{2ikx} - e^{-2ikx}}{4}, \quad \sum_k \rightarrow \frac{L}{2\pi} \int dk$$

$$(*) = \frac{1}{2\pi} \sum_m X_m(y) X_m(y') \cdot \int_{-\infty}^{\infty} dk \frac{1 - e^{2ikx}}{E - E_m - \frac{\hbar^2 k^2}{2m} + i\eta} \rightarrow \text{residuum tabel} \rightarrow \dots$$

$\left( \int_{-\infty}^{\infty} \rightarrow \int_{-\infty}^{\infty}, \text{ da alle } e^{-2ikx} \rightarrow e^{2ikx} \text{ berne van} \right)$

$$\dots = \sum_m -\frac{2 \sin(k_m x)}{\hbar v_m} X_m(y) X_m(y') e^{ik_m x}$$

add (folgt. weiter)  $k_m = \sqrt{\frac{2m(E-E_m)}{\hbar^2}}$ ,  $v_m = \frac{\hbar k_m}{m}$

reisen:  $x=a$

~~( $x=a$ )~~

~~20 m/s~~ reisen  $E(k) = E_0 + 2\gamma - 2\gamma \cos(k_x a) - 2\gamma \cos(k_y a)$

$k_x$   ~~$\perp$~~   $k_m$   $\leftarrow$   $k_y$   $\rightarrow$    
 diskret  $\leftarrow$   $k_y = n$

$$g^R(y_i, y_j) = \sum_m -\frac{2 e^{ik_m a}}{\hbar v_m} \sin(k_m a) X_m(y) X_m(y')$$

$v_m = ?$  (reisen)

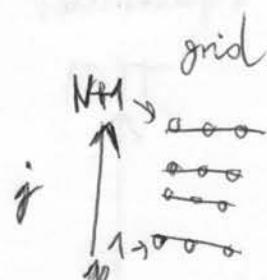
$$E(k_m) = E_0 + 2\gamma (1 - \cos k_m a) \quad (\text{d. dopp. Br.})$$

$$v_m = \frac{1}{\hbar} \frac{\partial E}{\partial k_m} = \frac{1}{\hbar} 2\gamma a \sin(k_m a)$$

$$g^R(y_i, y_j) = \sum_m -\frac{e^{ik_m a}}{2\pi a} X_m(y_i) X_m(y_j)$$

surface Green-fn.  
reisen

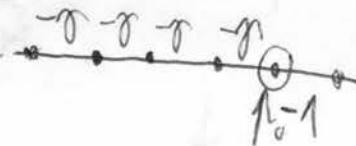
$$X_m(y_j) = \sqrt{\frac{2}{N\lambda}} \sin \frac{m\pi(j-1)}{N}$$



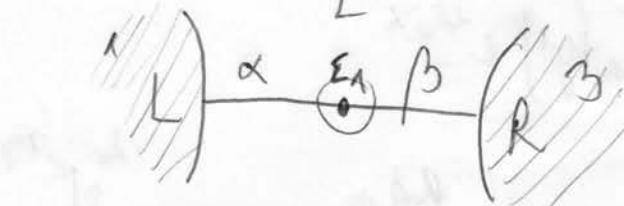
b) ⑩ surface Green-fn.

( $X_m=1$ ,  $m=1$  "Kerostromsader" van)

$$g_0 \equiv g^R(p_0-1, p_0-1) = -\frac{e^{ika}}{\pi a} \frac{-1}{1+i}$$



3) Kiszámíva a s. Green-fn.-t 1/f-be:



1D lencs eff. Hamiltoniája

$$\Rightarrow G = \begin{pmatrix} g_0^{-1} & & \\ E - H_L & -\alpha & 0 \\ -\alpha & E - \cancel{E_1} \xrightarrow{\text{zr cska}} & -\beta \\ & -\beta & E - H_R \\ & & g_0^{-1} \end{pmatrix}^{-1} = \begin{pmatrix} g_0^{-1} & -\alpha & 0 \\ -\alpha & E - \cancel{E_1} \xrightarrow{\text{zr cska}} -\beta \\ 0 & -\beta & g_0^{-1} \end{pmatrix}^{-1}$$

$E = -2z \cos(ka)$

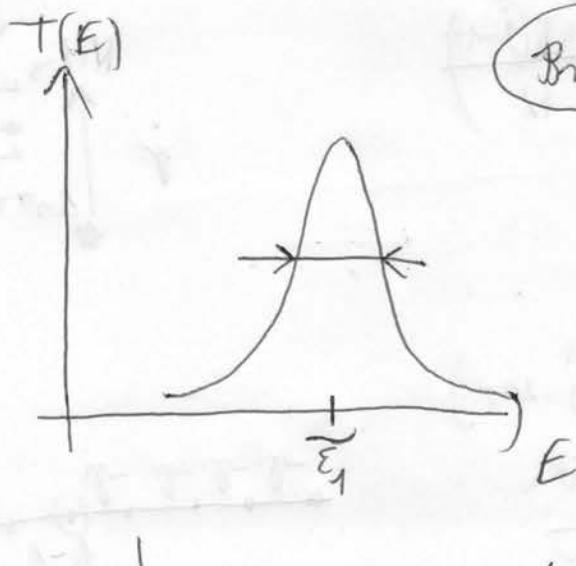
~~b(f)~~  $\downarrow$

$$b(k(E)) = G(1,3) \cdot v$$

transm.

$$T = |b|^2$$

4 parameter :  $E, \alpha, \beta, \eta$



Breit-Wigner-resonancia

$$T = \frac{4P_L P_R}{(E - \tilde{\epsilon}_1)^2 + (C_L + C_R)^2}$$

Lorentz-göbe  $\rightarrow$  felsőbb műszaki  
az  $\tilde{\epsilon}_1$  er. vissz. a leadékhez

ez adja meg,  
mennyire "csatolható"  
az  $\tilde{\epsilon}_1$  er. vissz. a leadékhez

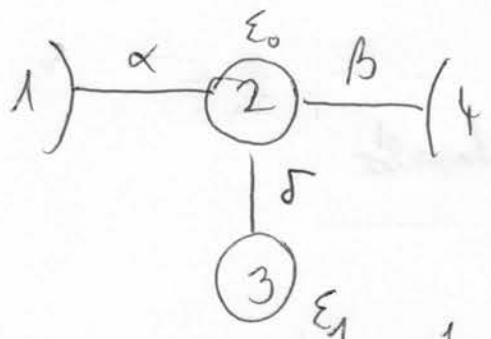
szemléletekben ha változtatjuk az energiát,

~~egy~~  $\tilde{\epsilon}_1$ -nél rezonancia fog történni a  
körepső részen az  $e^-$

1D mű-ekben tipikus az ilyen rész.

( $\tilde{\epsilon}_1$  a körepső rész  
sajtenergiájának  
jellemző)

#### 4) Fano-rezonancia



$$G = (E - H)^{-1} = \begin{pmatrix} g_0 & -\alpha & 0 & 0 \\ -\alpha & -2g_0\cos(ka) - \epsilon_0 - \gamma & 0 & -\beta \\ 0 & -\gamma & -2g_0\cos(ka) - \epsilon_1 & 0 \\ 0 & -\beta & 0 & g_0^{-1} \end{pmatrix}$$

$E = -2g_0 \cos(ka)$

körépen az

energiadaja

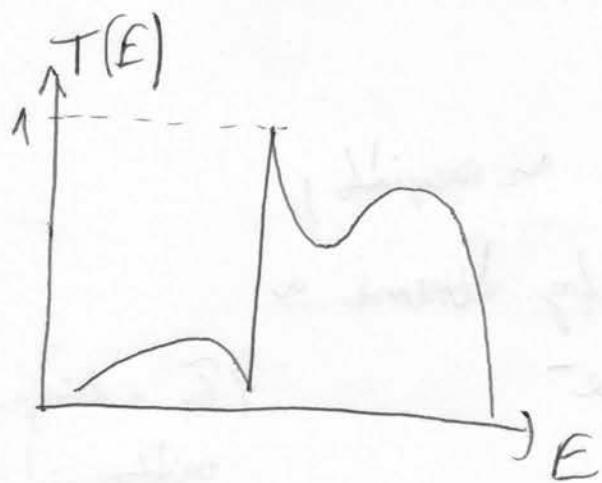
aholhol

nullámnak

$$\theta v = g_0 \sin(ka)$$

$$b = G(1,4)v$$

$$T = |b|^2$$



$$T(E) = \frac{(g + \varepsilon)^2}{1 + \varepsilon^2}$$

$$\varepsilon = \frac{E - E_1}{\delta}$$

↓  
ilyes látunk

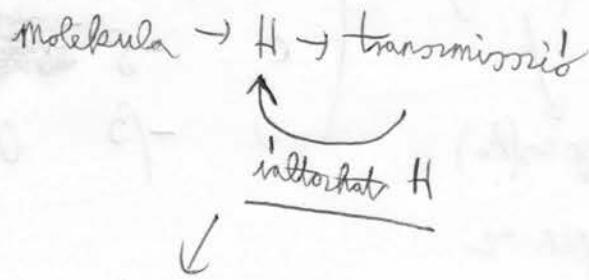
Fano-resonancia



konfigurációjának is → topológiai lag hasonló

→ mikroszkopikus standardekkal elég elterjedt módszer

↳ molecular electronics  
dynamics



iteráció

ab initio standardekkat ← mol. elhelyezkedése  
lehet közelíteni, reprodukálni