

Inflációs konferencia 20a

Hubble - tv. \rightarrow (Gamow) \rightarrow termikus egyensúly \rightarrow fotongáz

\rightarrow FRW-metrika \rightarrow Friedmann-egyenlet $a(t)$ gömszimmetrikus tágulás

1948-50

atommag \rightarrow Primordiális nukleoszintézis (BBN)

tágulás

Alpher, Gamow, Hoffmann

atom \rightarrow Rekombináció $k_B T \sim eV$

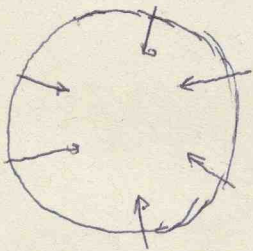
$k_B T \sim MeV$

~~$\langle E_\gamma \rangle \ll \Delta E_{\text{hiperfinom}}$~~ $\langle E_\gamma \rangle \ll \Delta E_{\text{hiperfinom}}$ lecsatolóda's \rightarrow CMB

2.725K

$a(t) \cdot T = \text{állandó}$

spektrális sűrűségelosztás



$\frac{\Delta T}{T} \sim 10^{-5}$

- nemrelativisztikus Newtoni grav. -ot produktív részecske
- sugárzás N_V foton + sötét 5 23
- sötét energia 72%

COBE 1992
WMAP 2001-2007
PLANCK $\Delta \Theta < 1^\circ$

$H_0^{-1} \sim t_{\text{univ}}$

$S(x) = S(1 + \delta(x))$

Einstein egyenletek
lineáris perturbáció
Jeans-instabilitás

csillagok

reionizáció

$z_r \sim 8-10$

polarizáció

$\langle T(\theta_1) \text{pol}(\theta_2) \rangle$

$\langle \text{pol}(\theta_1) \text{pol}(\theta_2) \rangle$

Newton kozmológiája | lineáris perturbációja
Einstein =||= | metrika $-10 = 5+3+2$

anyag + sugárzás fejlődési
Einstein-CM

$T(x,t) = T_{\text{allo}}(t) (1 + \Theta(x,t))$

kezdőfeltétel - paradoxonok

$t = 10^{-32} s$

inflaton
eredete?

lebegés

20a

Newton kozmologiaja folyadékosabb saját grav. térben

Euler: $\frac{\partial v(x,t)}{\partial t} + (v \text{ grad}) v + \text{grad} \phi_{\text{Newton}} + \frac{1}{\rho} \text{grad} p = 0$

Newton: $\Delta \phi_N = 4\pi G_N \rho$ $P = P(\rho, S)$

Megmaradási: $\frac{\partial \rho}{\partial t} + \text{div}(\rho v) = 0$ *adiabaticus* $\frac{\partial S}{\partial t} + (v \text{ grad}) S = 0$ *entropia*

radialis, homogén $v = Hx$ $\rho = \rho_0(t)$ $H = H(t)$

anyagmegn: $\dot{\rho}_0 + 3H\rho_0 = 0$ $\rho_0 r^3 = \text{all} = \rho_0(0) r_0^3$

Euler-div $3\dot{H} + 3H^2 + 4\pi G_N \rho_0 = 0$ $H = \frac{\dot{r}}{r}$ $\dot{H} = \frac{\ddot{r}}{r} - \frac{\dot{r}^2}{r^2}$ $\frac{\ddot{r}}{r} = \dot{H} + H^2$

$\frac{\ddot{r}}{r} + \frac{4\pi}{3} G_N \rho_0 = 0$ $\ddot{r} = -\frac{4\pi}{3} G_N \rho_0(0) r_0^3 \frac{1}{r^2} = -\frac{G_N M_0}{r^2}$

$\frac{1}{2} \dot{r}^2 - \frac{G_N M_0}{r} = E$ $H^2 = 2E \frac{1}{r^2} + \frac{2G_N M_0}{r^3}$ "Friedman"

$E > 0$ nyitott $E < 0$ kötött $E = 0$

Sűrűség ingadozások - Jeans instabilitás

megperturbált

$\rho = \rho_0 + \delta\rho$

$v = \delta v$

$S = S_0 + \delta S$

$\delta p = \frac{\partial p}{\partial \rho} \delta \rho + \frac{\partial p}{\partial S} \delta S$

δS (X, X)
 időfüggő

$\frac{\partial \delta v}{\partial t} + \nabla \delta \phi_N + \frac{1}{\rho_0} (c_s^2 \nabla \delta \rho + \rho_0 \nabla \delta S) = 0$ $\frac{\partial \delta S}{\partial t} + \rho_0 \text{div} \delta v = 0$

$\Delta \delta \phi_N = 4\pi G_N \delta \rho$ *adiabaticus*

$-\frac{1}{\rho_0} \frac{\partial^2 \delta \rho}{\partial t^2} + (4\pi G_N + \frac{c_s^2}{\rho_0} \Delta) \delta \rho = -\frac{\rho_0}{\rho_0} \Delta \delta S$

$-\ddot{\delta \rho} + (4\pi G_N \rho_0 - c_s^2 k^2) \delta \rho = 0$ $\sigma k^2 \delta \rho$

$4\pi G_N \rho_0 > c_s^2 k^2$

$4\pi G_N \rho_0 = c_s^2 \frac{4\pi^2}{\lambda^2}$

$\lambda > \lambda_J$ instabilitás

Központi tárgulással

egyenl. mozg. coord. r-sz.

$x(t) = a(t) q$ $a(0) = 1$

$v(t) = \dot{a}(t) q = \frac{\dot{a}}{a} x$ $H(t) = \frac{\dot{a}}{a}$

$f(x,t) = f(a(t)q, t)$

$\delta(x,t) = \frac{\delta \rho}{\rho_0}$ $\frac{\partial \delta(aq,t)}{\partial t} \Big|_q + \frac{1}{a} \text{div}_q \delta v = 0$ *kontinuitás*

$\frac{\partial f}{\partial t} \Big|_x = \frac{\partial f}{\partial t} \Big|_q - H(q \cdot \nabla_q) f$

$\frac{\partial \delta v}{\partial t} \Big|_q + \frac{1}{a} \nabla_q \delta \phi_N + \frac{c_s^2}{a} \nabla_q \delta \rho = 0$ (adiabaticus)

$\frac{1}{a^2} \Delta_q \phi_N = 4\pi G_N \rho_0 \delta$

$$\text{div}_q \left. \frac{\partial \delta v}{\partial t} \right|_q \neq \left. \frac{\partial}{\partial t} (\text{div} \delta v) \right|_q = -H a \dot{\delta} - a \ddot{\delta}$$

$$-a \ddot{\delta} - 2a H \dot{\delta}$$

Fourier transform: δ -null

$$\ddot{\delta}_k + 2H \dot{\delta}_k + \left(\frac{c_s^2}{a^2} k^2 - 4\pi G_N \rho_0 \right) \delta_k = 0$$

$$\delta(q, t) = \int \frac{d^3 k}{(2\pi)^3} e^{i \mathbf{k} \cdot \mathbf{q}} \delta_{\mathbf{k}}(t)$$

$$\lambda_{fiz} = a \lambda \quad k = \frac{2\pi}{\lambda} \quad k_{fiz} = \frac{2\pi}{a \lambda} = \frac{k}{a}$$

Instabilitás $4\pi G_N \rho_0 > c_s^2 \frac{k^2}{a^2}$

Vissza az dt-lageregenlethez

$$\dot{H} + H^2 + \frac{4\pi}{3} G_N \rho_0 = 0 \quad H + 2H^2 + \frac{4\pi}{3} G_N \rho_0 (-3H \rho_0) = 0$$

$$\dot{\rho}_0 + 3H \rho_0 = 0$$

Einstein 2. sz. egyenlete

$$G^M_{\nu} = 8\pi G_N T^M_{\nu}$$

$$g^{\mu\nu} X^{\nu} = X^{\mu}$$

$$g^{\mu\nu} \rightarrow R_{\mu\nu}$$

Christoffel Ricci
 $R = R_{\mu\nu} g^{\mu\nu}$

$$G^M_{\nu} = g^{\mu\alpha} (R^{\mu}_{\nu\alpha} - \frac{1}{2} g_{\nu\alpha} R)$$

FRW $(ds)^2 = (dt)^2 - a(t)^2 (dx)^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$

side $g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -a^2 & & \\ & & -a^2 & \\ & & & -a^2 \end{pmatrix} \quad g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1/a^2 & & \\ & & -1/a^2 & \\ & & & -1/a^2 \end{pmatrix}$

$$g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}_{\nu}$$

$$R_{00} = -3(\dot{H} + H^2) \quad R_{ij} = \delta_{ij} (2\dot{a}^2 + a\ddot{a}) \quad R = -3(2\dot{H} + 4H^2)$$

$$G^0_0 = 3H^2 = 8\pi G_N \rho \quad G^i_i = \delta_{ij} (2\dot{a}^2 + a\ddot{a}) = \delta_{ij} (2\frac{\ddot{a}}{a} + H^2) = -8\pi G_N p \rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} (\rho + 3p)$$

$$T^M_{\nu} = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$

$\rho = w \rho$
 $w=0$ nem-rel.
 $w=1/3$ Sugárzás
 $w=-1$ kozmologiai áll.

$\rho + 3p < 0 \quad p < -\frac{\rho}{3}$ ha ilyen
 az anyag akkor gyorsulást
 figyel

$$T^{\mu}_{\nu ; \mu} = \frac{\partial T^{\mu}_{\nu}}{\partial x^{\mu}} + \Gamma^{\mu}_{\alpha\mu} T^{\alpha}_{\nu} - \Gamma^{\mu}_{\nu\alpha} T^{\alpha}_{\mu} \rightarrow \nu = 0$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\dot{\rho} + 3H(1+w)\rho = 0$$

$$\rho a^{3(1+w)} = \text{const.}$$

$$G^0_0 = 3H^2 = 8\pi G_N \rho \rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho_0 \frac{1}{a^{3(1+w)}}$$

$$a(t_0) = 1/H_0$$

$w=-1 \quad a(t) = a(0) e^{H_0(t-t_0)}$
 $w=1/3 \quad a(t) \sim \sqrt{t} \quad w=0 \quad a(t) \sim t^{2/3}$

$k \sim 0$
 $\delta_1 \sim H(t)$
 $\delta_2 = f \cdot \delta_1$
 Wronskian $W = \delta_2 \dot{\delta}_1 - \dot{\delta}_2 \delta_1 = \delta_1^2 \dot{f}$
 $\dot{W} = \dot{\delta}_2 \dot{\delta}_1 - \dot{\delta}_1 \ddot{\delta}_2 = -2H(\dot{\delta}_2 \delta_1 - \dot{\delta}_1 \delta_2) = 2HW$
 $W a^2 = \text{const. } C$
 $\delta_2 = \int dt \frac{C}{a^2(t) H^2(t)} \cdot H$
 R ez idoben
 felhívás

