Inflációs konferencia 1ea

Hubble-tv. → (Gamow) → termikus egyensúly → fotongáz

→ FRW-metrika → Friedmann-egyenlet $a(t)$ gömbszimmetrikus tágulás

1948–50 atommag → primordiális nukleoszintézis (BBN) $k_B T \sim MeV$

Alpher, Gamow, Hoffmann

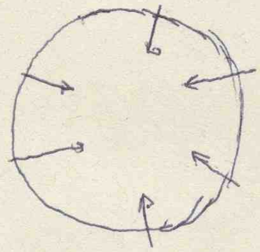
atom → Rekombináció $k_B T \sim eV$

$\cancel{\langle E_\gamma \rangle \ge \Delta E_{hiperfinom}}$ $\langle E_\gamma \rangle \ll \Delta E_{hiperfinom}$ lecsatolódás → CMB

2.725 K

$a(t) \cdot T =$ állandó

spektrális sűrűségeloszlás



- nemrelativisztikus Newtoni grav.-ot produkáló részecskék (e^-, p^+) töltött 5 + sötét 23
- sugárzás N_ν
- sötét energia 72%

$\frac{\Delta T}{T} \sim 10^{-5}$

COBE 1992
WMAP 2001–2007 $\langle T(\Theta_1) T(\Theta_2) \rangle$
PLANCK $\Delta\Theta < 1'$

Einstein egyenletek

lineáris perturbáció

Jeans-instabilitás

$H_0^{-1} \sim t_{univ}$

$\rho(\underline{x}) = \rho(1 + \delta(\underline{x}))$

ősgalaxisok

csillagok reionizáció

$z_r \sim 8-10$

$a_{ma} = z_0 \to z = 0$

polarizáció

$\langle T(\Theta_1) pol(\Theta_2) \rangle$

$\langle pol(\Theta_1) pol(\Theta_2) \rangle$

Newton kozmológiája | lineáris perturbációja

Einstein — " — | metrika — $10 = 5 + 3 + 2$

anyag + sugárzás fejlődési
Einstein-egy.

$T(\underline{x}, t) = T_{átlag}(t) \left(1 + \Theta(\underline{x}, t)\right)$

kezdőfeltétel – paradoxonok

$t = 10^{-32}$ s inflaton lebomlása

? eredete ?

2ea

2. ea

Newton kozmológiája folyadékcsepp saját grav. terében

Euler: $\frac{\partial v(\underline{x},t)}{\partial t} + (\underline{v}\,\mathrm{grad})\,\underline{v} + \mathrm{grad}\,\phi_{Newton} + \frac{1}{\varrho}\,\mathrm{grad}\,p = 0$

Newton: $\Delta\phi_N = 4\pi G_N \varrho$ $\qquad p = p(\varrho, s)$

entrópi

Megmaradási: $\frac{\partial\varrho}{\partial t} + \mathrm{div}(\varrho\underline{v}) = 0$ $\qquad$ adiabatikus $\qquad \frac{\partial s}{\partial t} + (\underline{v}\,\mathrm{grad})\,s = 0$

radiális, homogén $\qquad \underline{v} = H\underline{x} \qquad \varrho = \varrho_0(t) \qquad H = H(t)$

$s_0(t)$

anyagmegm: $\dot{\varrho}_0 + 3H\varrho_0 = 0$ $\quad \frac{\dot{\varrho}_0}{\varrho_0} + 3\frac{\dot{r}}{r} = 0 \qquad \varrho_0 r^3 = \text{áll} = \varrho_0(0)\, r_0^3$

Euler-div $\quad 3\dot{H} + 3H^2 + 4\pi G_N \varrho_0 = 0 \qquad H = \frac{\dot{r}}{r} \qquad \dot{H} = \frac{\ddot{r}}{r} - \frac{\dot{r}^2}{r^2} \qquad \frac{\ddot{r}}{r} = \dot{H} + H^2$

$\frac{\ddot{r}}{r} + \frac{4\pi}{3} G_N \varrho_0 = 0 \qquad \ddot{r} = -\frac{4\pi}{3} G_N \varrho_0(0)\, r_0^3 \frac{1}{r^2} = -\frac{G_N M_0}{r^2}$

$\frac{1}{2}\dot{r}^2 - \frac{G_N M_0}{r} = E \qquad H^2 = 2E\frac{1}{r^2} + \frac{2G_N M_0}{r^3}$ $\qquad$ „Friedman"

$E > 0$ nyitott $\qquad E < 0$ kötött $\qquad E = 0$

Sűrűségingadozások - Jeans instabilitás

megperturbáljuk

$\varrho = \varrho_0 + \delta\varrho \qquad \underline{v} = \delta\underline{v} \qquad s = s_0 + \delta s \qquad \delta p = \underbrace{\frac{\partial p}{\partial \varrho}}_{c_s^2}\delta\varrho + \underbrace{\frac{\partial p}{\partial s}}_{\sigma}\delta s$

$\delta s(\cancel{\underline{x}}, \cancel{t})$

időfüggetlen

$\frac{\partial\delta\underline{v}}{\partial t} + \underline{\nabla}\delta\phi_N + \frac{1}{\varrho_0}\left(c_s^2\nabla\delta\varrho + \sigma\nabla\delta s\right) = 0 \qquad \Big| \qquad \frac{\partial\delta\varrho}{\partial t} + \varrho_0\,\mathrm{div}\,\delta\underline{v} = 0$

$\Delta\delta\phi_N = 4\pi G_N \delta\varrho$ $\qquad$ adiabatikus, ~~[illegible]~~

$-\frac{1}{\varrho_0}\frac{\partial^2\delta\varrho}{\partial t^2} + \left(4\pi G_N + \frac{c_s^2}{\varrho_0}\Delta\right)\delta\varrho = -\frac{\sigma}{\varrho_0}\Delta\delta s$

$-\ddot{\delta\varrho}_{\underline{k}} + \left(4\pi G_N \varrho_0 - c_s^2 k^2\right)\delta_{\varrho_k} = \sigma k^2 \delta_{s_k}$ $\qquad 4\pi G_N\varrho_0 > c_s^2 k^2$

$4\pi G_N \varrho_0 = c_s^2 \frac{4\pi^2}{\lambda_J^2} \qquad \lambda > \lambda_J$ instabilitás

Hozzáadott tágulással

együtt mozgó koord rsz.

$\underline{x}(t) = a(t)\,\underline{q} \qquad a(0) = 1$

$\underline{v}(t) = \dot{a}(t)\,\underline{q} = \frac{\dot{a}}{a}\underline{x} \qquad H(t) = \frac{\dot{a}}{a} \qquad f(\underline{x}, t) = f(a(t)\underline{q}, t)$

kontinuitás:

$\delta(\underline{x}, t) = \frac{\delta\varrho}{\varrho_0} \qquad \frac{\partial\delta(a\underline{q}, t)}{\partial t}\Big|_{\underline{q}} + \frac{1}{a}\,\mathrm{div}_{\underline{q}}\,\delta\underline{v} = 0 \qquad \frac{\partial f}{\partial t}\Big|_{\underline{x}} = \frac{\partial f}{\partial t}\Big|_{\underline{q}} - H(\underline{q}\nabla_{\underline{q}})f$

$\frac{\partial\delta\underline{v}}{\partial t}\Big|_{\underline{q}} + \frac{1}{a}\nabla_q\delta\phi_N + \frac{c_s^2}{a}\nabla_q\delta = 0$ (adiabatikus) $\qquad \frac{1}{a^2}\Delta_q\phi_N = 4\pi G\varrho_0\delta$

Inf. kozm

$$\mathrm{div}_q \frac{\partial \delta v}{\partial t}\Big|_q \neq \frac{\partial}{\partial t}\left(\mathrm{div}\,\delta \underline{v}\right)\Big|_q = -Ha\dot{\delta} - a\ddot{\delta}$$

$\Downarrow$

$-a\ddot{\delta} - 2aH\dot{\delta}$

Fourier trafók: δ-nak

$$\ddot{\delta}_{\underline{k}} + 2H\dot{\delta}_{\underline{k}} + \left(\frac{c_s^2}{a^2}k^2 - 4\pi G_N \varrho_0\right)\delta_{\underline{k}} = 0 \qquad \delta(q,t) = \int \frac{d^3k}{(2\pi)^3} e^{i\underline{k}\underline{q}}\,\delta_{\underline{k}}(t)$$

$\lambda_{fiz} = a\lambda \qquad k = \frac{2\pi}{\lambda} \qquad k_{fiz} = \frac{2\pi}{a\lambda} = \frac{k}{a}$

Instabilitás $4\pi G_N \varrho_0 > c_s^2 \frac{k^2}{a^2}$

Vissza az átlagegyenlethez

$$\dot{H} + H^2 + \frac{4\pi}{3}G_N\varrho_0 = 0 \qquad \ddot{H} + 2H\dot{H} + \frac{4\pi}{3}G_N\left(-3H\varrho_0\right) = 0$$

$\dot{\varrho}_0 + 3H\varrho_0 = 0$

$k \sim 0$

$\delta_1 \sim H(t)$ — 1. partikuláris megoldás

$\delta_2 = f\cdot\delta_1$ — 2. független ilyen alakban keressük

Wronski

$W = \dot{\delta}_2\delta_1 - \dot{\delta}_1\delta_2 = \delta_1^2\dot{f}$

$\dot{W} = \ddot{\delta}_2\delta_1 - \ddot{\delta}_1\delta_2 = -2H\left(\dot{\delta}_2\delta_1 - \dot{\delta}_1\delta_2\right) = 2HW$

$Wa^2 = C$

$$\delta_2 = \int^t dt' \frac{C}{a^2(t')H^2(t')}\cdot H$$

ez időben csökken

Einstein kozmológia

$G^\mu{}_\nu = 8\pi G_N T^\mu{}_\nu \qquad g_{\mu\nu}X^\nu = X_\mu$

g

$\Gamma^\mu_{\alpha\beta} \to R_{\mu\nu}$

Christoffel Ricci

$R = R_{\mu\nu}g^{\mu\nu}$

$$G^\mu{}_\nu = g^{\mu\alpha}\left(R_{\nu\alpha} - \frac{1}{2}g_{\nu\alpha}R\right)$$

FRW $(ds)^2 = (dt)^2 - a(t)^2(d\underline{x})^2 = g_{\mu\nu}dx^\mu dx^\nu$

sík

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -a^2 & & \\ & & -a^2 & \\ & & & -a^2 \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1/a^2 & & \\ & & -1/a^2 & \\ & & & -1/a^2 \end{pmatrix}$$

$g^{\mu\alpha}g_{\alpha\nu} = \delta^\mu{}_\nu$

$$R_{00} = -3\left(\dot{H} + H^2\right) \qquad R_{ij} = \delta_{ij}\left(2\dot{a}^2 + a\ddot{a}\right) \qquad R = -3\left(2\dot{H} + 4H^2\right)$$

$$G^0{}_0 = 3H^2 = 8\pi G_N\varrho \;\Big|\; G_i{}^j = \delta_i{}^j\left(2\dot{H} + 3H^2\right) = \delta_{ij}\left(2\frac{\ddot{a}}{a} + H^2\right) = -8\pi G_N p \to \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\left(\varrho + 3p\right)$$

$$T^\mu{}_\nu = \begin{pmatrix} \varrho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$

$\varrho + 3p < 0 \quad p < -\frac{\varrho}{3}$ ha ilyen az anyag akkor gyorsulva tágul

$p = w\varrho \qquad w = 0$ nem-rel.

$w = 1/3$ sugárzás

$w = -1$ kozmológiai áll.

$$T^\mu{}_{\nu;\mu} = \frac{\partial T^\mu{}_\nu}{\partial x^\mu} + \Gamma^\mu_{\alpha\mu}T^\alpha{}_\nu - \Gamma^\mu_{\nu\alpha}T^\alpha{}_\mu \to \nu = 0 \qquad \dot{\varrho} + 3H(\varrho + p) = 0$$

$\dot{\varrho} + 3H(1+w)\varrho = 0$

$\varrho a^{3(1+w)} = $ áll.

$$G^0{}_0 = 3H^2 = 8\pi G_N\varrho \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \underbrace{\frac{8\pi G_N}{3}\varrho_0}_{H_0^2}\frac{1}{a^{3(1+w)}}$$

$a(t_0) = 1$

$w = -1 \quad a(t) = a(0)e^{H(t-t_0)}$

$w = 1/3 \quad a(t) \sim \sqrt{t} \qquad w = 0 \quad a(t) \sim t^{1/3}$

ma anyag+ észm állandó

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho_m + \rho_\Lambda - 3\rho_\Lambda) = \frac{4\pi G_N}{3}(2\rho_\Lambda - \rho_m)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0\left(\frac{\rho_m}{\rho_0} + \frac{\rho_\Lambda}{\rho_0}\right) = H_0^2\left(\Omega_{m0}\frac{1}{a^3} + \Omega_\Lambda\right)$$

$$\frac{\rho_{m0}}{\rho_0}\frac{1}{a^3} + \frac{\rho_\Lambda}{\rho_0}$$

$$\Omega_{m0} \qquad \Omega_\Lambda$$

Görbület esetén:

$$(ds)^2 = (dt)^2 - a^2(t)\left[\frac{(dr)^2}{1-kr^2} + r^2 d\Omega^2\right]$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} = \qquad \boxed{-\frac{k}{a^2} = \frac{8\pi G}{3}\rho_K}$$

↳ „görbületi anyagsűrűség"

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\rho_0}{a^3} + \frac{\rho_{K(0)}}{a^2}\right)$$

$$\frac{8\pi G}{3}\cancel{\frac{\rho_0}{a}}$$

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\frac{\rho_0}{a} = \frac{4\pi G}{3}\rho_{K0}$$

$k = -1$ nyitott

$k = +1$ zárt

$k = 0$ sík

$k \neq 0$