Húrelmélet

(2 hét múlva lesz óra)

1. ea

$c=1$
$\hbar=1$

Motiváció: elektromos $\underline{E}$, mágneses $\underline{B}$ $\Rightarrow F_{\mu\nu} = \begin{pmatrix} 0 & \underline{E} & & \\ & 0 & -B_z & B_y \\ -\underline{E} & B_z & 0 & -B_x \\ & -B_y & B_x & 0 \end{pmatrix}$

$x^\mu = (x^0, x^1, x^2, x^3)$
$(t, \vec{x})$

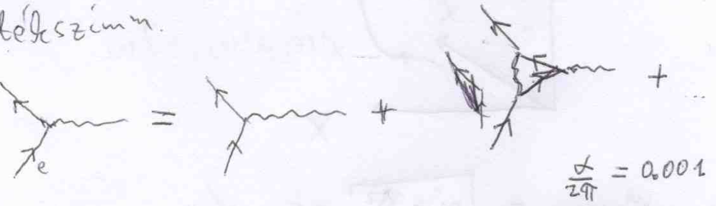
$$S = -\frac{1}{4g}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(\not{\partial} - m)\Psi\right)$$

$\eta^{\mu\nu} = (-1, 1, 1, 1)$

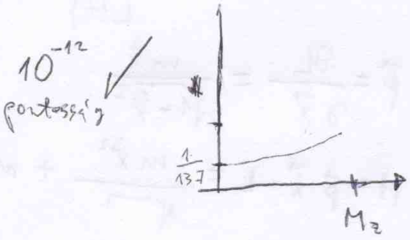
$x^\mu \to \Lambda^\mu{}_\nu x^\nu \qquad x^\mu x_\mu = x^\mu x^\nu \eta_{\mu\nu}$

$U(1)$ mértékszimm.

QED:



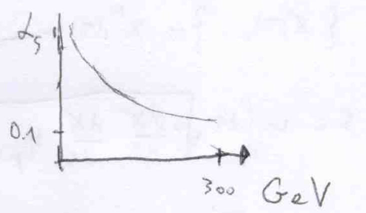
$\frac{\alpha}{2\pi} = 0.001$



gyenge kh: Fermi

$\Rightarrow SU(2)\otimes U(1)$

QCD: $SU(3)$ $\qquad \frac{\alpha_s}{4\pi} = O(1)$



$SU(N)$ $\quad N=4$ szuperszimmetrikus mértékelm. (ez a legszebb mértékelm.)

adj $\qquad F_{\mu\nu} \qquad \Phi_{1,2,\dots,6}$ $\qquad$ legnagyobb szimmetria

$\Psi_{1,\dots,4}$
$\bar{\Psi}_{1,\dots,4}$

$$S = \frac{1}{g^2}\int d^4x \, \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\bar{\Psi}\not{D}\Psi + \frac{1}{4}[\Phi,\Phi]^2 + \bar{\Psi}[\Phi,\Psi]\right]$$

g nem fut

szuperkonforminv.
$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{1}{|x|^{2\Delta_{\mathcal{O}}(g)}}$$

$\mathcal{O}$ dimenziója

$\Updownarrow$

$\mathrm{II}_B \qquad AdS_5 \times S^5$

$g_{\alpha\beta} \qquad g = \det g_{\alpha\beta}$

gravitációs kh.
$$S = \frac{1}{16\pi G_N}\int d^4x \sqrt{-g}\, R$$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \frac{1}{M_{pl}} h_{\alpha\beta}$$

$\hbar$
c
G_N

$8\pi G_N = \frac{\hbar c}{M_{pl}^2} \qquad M_{pl} = \sqrt{\frac{\hbar c}{8\pi G_N}} \approx 10^{18}\,\mathrm{GeV}$

(nálunk $\hbar = c = 1$)

$$S = \int d^4x \left[(\partial h)^2 + \frac{1}{M_{pl}} h(\partial h)^2 + \frac{1}{M_{pl}^2} h^2(\partial h)^2 + \dots\right]$$

nem jól kvantálható

húrelmélet a kvantumgrav. elmélete lehet

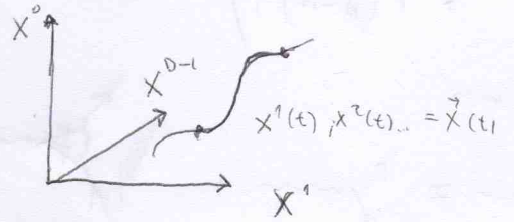
Irodalom pl: J. Polchinski: String theory

B. Zwiebach: A first course is string theory

David Tong: String theory (neten jegyzet) ← mi ezt követjük

Relativisztikus pontrészecske:

Minkowski tér: $X^0, X^1, \dots X^{D-1}$



$$S = -m\int dt \underbrace{\sqrt{1-\dot{\vec{x}}\cdot\dot{\vec{x}}}}_{L(t)}$$

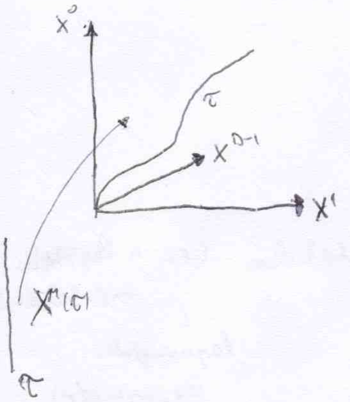
$$\vec{p} = \frac{\partial L}{\partial \dot{\vec{x}}} = \frac{m\dot{\vec{x}}}{\sqrt{1-\dot{\vec{x}}^2}}$$

$$H = \vec{p}\,\dot{\vec{x}} - L = \frac{m\dot{\vec{x}}^2}{\sqrt{\ \ }} + m\sqrt{1-\dot{\vec{x}}^2} = \frac{m}{\sqrt{\ \ }} = \sqrt{m^2+\vec{p}^2} = \varepsilon \quad \checkmark$$

mozg. egy.: $\dot{\vec{p}} = 0$

hátránya: - nem kovariáns

- hülye $\sqrt{\ }$-k



$$\{X^0(\tau), \dots\} = X^\mu(\tau)$$

$$S = -m\int d\tau \sqrt{-\frac{dX^\mu}{d\tau}\frac{dX^\lambda}{d\tau}\eta_{\mu\lambda}}$$

ha $\tau = t$, u.az mint a régi ✓

• Lorentz szimm. $X^\mu \to \Lambda^\mu{}_\lambda X^\lambda + c^\mu$

def.: $\Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\eta_{\mu_1\mu_2} = \eta_{\nu_1\nu_2}$

• újraparametrizálási szimmetria:

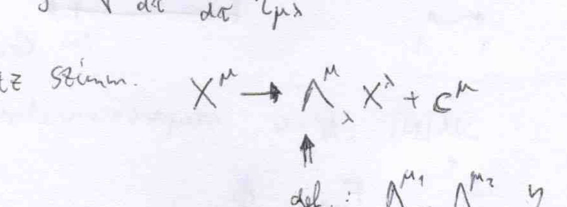
$$\tau \to \tilde{\tau} \quad \left|\frac{d\tilde{\tau}}{d\tau}\right| \neq 0 \qquad d\tau = d\tilde{\tau}\cdot\frac{d\tau}{d\tilde{\tau}} \qquad \frac{dX^\mu}{d\tau} = \frac{dX^\mu}{d\tilde{\tau}}\cdot\frac{d\tilde{\tau}}{d\tau}$$

$$S = -m\int d\tilde{\tau}\underbrace{\sqrt{-\frac{dX^\mu}{d\tilde{\tau}}\frac{dX^\nu}{d\tilde{\tau}}\eta_{\mu\nu}}}_{L} \implies X^0(\tau) = t(\tau) = \tilde{\tau}$$

ez igazából nem extra szab. fok

$$p_\mu = \frac{\partial L}{\partial \dot{X}^\mu} = \frac{m\frac{dX^\nu}{d\tau}\eta_{\mu\nu}}{\sqrt{\ \ }} \qquad p_\mu p^\mu = -m^2 \qquad p_0^2 > m^2$$

Nambu-Goto hatás



target space: ahol a húr terjed (most Minkowski)

világlepedő felszíne

$$S = -T\int d\sigma d\tau \sqrt{(\dot{X}\cdot X')^2 - (\dot{X}^2)(X'^2)}$$

diffgeoból lehet máshogy is kihozni (indukált metrika)

$$\underbrace{\left|\frac{dX^\mu}{d\sigma}\right|}_{X'}\underbrace{\left|\frac{dX^\mu}{d\tau}\right|}_{\dot{X}}\underbrace{\sin\alpha}_{\sqrt{1-\cos^2\alpha}} \qquad \cos\alpha = \frac{X'\cdot\dot{X}}{|X'||\dot{X}|}$$

$$\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\lambda \eta_{\mu\lambda} \qquad S = -T\int d^2\sigma\sqrt{-\gamma} \qquad \gamma = \det\gamma_{\alpha\beta}$$

$$\gamma = \begin{pmatrix} \dot X \dot X & \dot X X' \\ \dot X \cdot X' & X' \cdot X' \end{pmatrix}$$

fizikai TV jelentése

dimenziós paraméter, dimenziótlan

Legyen $X^0 = t = R\tau$

Amikor a húrnak épp nincs kin. energiája $\frac{\partial \vec X}{\partial \tau} = 0$

akkor a hatás $S = -T\int d\tau d\sigma R \left|\frac{\partial \vec X}{\partial\sigma}\right| = -T\int dt \cdot$ húr térbeli hossza

$X^\mu = (X^0, \vec X)$

$\dot X^\mu = (R, 0)$ $\quad X'^\mu = \left(0, \frac{\partial \vec X}{\partial \sigma}\right)$

T · húr hossza = potenciális energia

T = egységnyi hosszra eső energia

(nem olyan mint a radírgumi húr, Hooke-tv.-el)

hatás szimmetriái:

- Lorentz invariáns
- diffeomorfizmus inv. (újraparaméterezés) $\tau, \sigma \to \tilde\tau, \tilde\sigma$ $\quad X^\mu$ skalárként transzformál: $X^\mu(\sigma,\tau) = X^\mu(\tilde\sigma,\tilde\tau)$

olyan, mint egy 2D térelmélet (σ,τ a koordináta, X^μ skalártér)

mozg. egyenlet $\mathcal{L}(\partial_\alpha X^\mu)$ $\quad \alpha = 0, 1$ $\quad \partial_\tau\ \partial_\sigma$

$$\partial_\alpha \frac{\partial \mathcal L}{\partial(\partial_\alpha X^\mu)} = 0 \qquad \partial_\tau \frac{\partial\mathcal L}{\partial \dot X^\mu} = \partial_\tau \frac{\dot X\cdot X' X'_\mu - (X')^2 \dot X_\mu}{\sqrt{}}$$

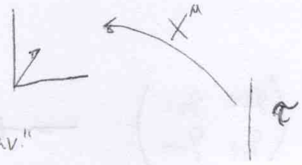
$$\delta\sqrt{-\gamma} = \frac12 \sqrt{-\gamma}\,\gamma^{\alpha\beta}\delta\gamma_{\alpha\beta}$$

$\det \gamma_{\alpha\beta} = \gamma$ $\quad$ mozgásegyenlet kicsit szebb alakban

$$\partial_\alpha\left(\sqrt{-\gamma}\,\gamma^{\alpha\beta}\partial_\beta X^\mu \eta_{\mu\nu}\right) = 0$$

Vissza a pontrészecskéhez!

$$S = -m\int d\tau \sqrt{-\dot X \dot X}$$



$$S' = -\frac12 \int d\tau \underbrace{\sqrt{-g_{\tau\tau}}}_{e}\left(m^2 + \underbrace{g^{\tau\tau}}_{-e^{-2}} \dot X^2\right)$$ „1D grav."

(bevezettünk egy új szab. fokot)

Állítás: S ekvivalens S'-vel

mozg. egy. e-re: $m^2 + \frac{1}{e^2}\dot X^2 = 0$ $\qquad e = \sqrt{\frac{-\dot X^2}{m^2}}$ visszaírva megkapjuk az eredetit

e-vel felírva már nincsen gyök a hatásban

szimmetriák:
- Lorentz szimmetria
- diffeomorf. inv $\quad X^\mu(\tau) = X^\mu(\tilde\tau)$ $\quad g_{\tau\tau}\, d\tau d\tau = \tilde g_{\tilde\tau\tilde\tau}\, d\tilde\tau d\tilde\tau$

$$e\, d\tau = \tilde e\, d\tilde\tau$$

Húrnál ugyanezt megcsináljuk:

Polyakov hatás:

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{-g}\, g^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

téregyenletek: X^μ: $\quad \partial_\alpha\left(\sqrt{-g}\, g^{\alpha\beta}\partial_\beta X^\nu g_{\mu\nu}\right) = 0$

$$g^{\alpha\beta} \stackrel{?}{=} \gamma^{\alpha\beta}$$

$$\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu$$

$g^{\alpha\beta}$: $\quad \sqrt{-g}\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} - \frac{1}{2}\sqrt{-g}\, g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma X^\mu \partial_\delta X^\nu \eta_{\mu\nu} = 0$

$$g_{\alpha\beta} = \underbrace{2f(\sigma,\tau)}_{\frac{1}{g^{\gamma\delta}\partial_\gamma X^\mu \partial_\delta X^\nu \eta_{\mu\nu}}} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

egy ilyen skalár nem számít

$g_{\alpha\beta} \to f g_{\alpha\beta}$ invariancia $\quad \sqrt{-g} \to f\sqrt{-g}$, $\quad g^{\alpha\beta} \to \frac{1}{f} g^{\alpha\beta}$ (inverz) $\quad \sqrt{-g}\, g^{\alpha\beta}$ invariáns

Nambu-Goto $\equiv$ Polyakov hatás, mert $fg = g'$ $\quad \boxed{\sqrt{-g}\, g^{\alpha\beta} \to \sqrt{-g'}\, g'^{\alpha\beta}}$

Polyakov hatás szimm:

- Lorentz $\quad X^\mu \to \Lambda^\mu{}_\nu X^\nu + c^\mu$
- diffeomorf $\quad (\sigma,\tau) \to (\tilde\sigma,\tilde\tau)$
- Weyl: $\quad X^\mu \to X^\mu \qquad g_{\alpha\beta} \to \Omega^2(\sigma)\, g_{\alpha\beta}$

Szögek nem változnak

Mérték rögzítés

konform mérték:

$$g_{\alpha\beta} = \begin{pmatrix} g_{\tau\tau} & g_{\tau\sigma} \\ g_{\sigma\tau} & g_{\sigma\sigma} \end{pmatrix} \longrightarrow e^{\phi(\sigma)} \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{\eta_{\alpha\beta}}$$

ilyen alakra hozzuk

$$g_{\alpha\beta}\, d\sigma^\alpha d\sigma^\beta = \tilde g_{\alpha'\beta'}\, d\tilde\sigma^{\alpha'} d\tilde\sigma^{\beta'}$$

konform a laposhoz

Weyl $\to \eta_{\alpha\beta}$

$T = \frac{1}{2\pi\alpha'} \qquad \alpha' = l_s^2$

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X_\mu \Rightarrow \partial_\alpha \partial^\alpha X^\mu = 0$$

mozg. egy.

Szabad tömegtelen skalár

energia-impulzus

$$\frac{\delta S}{\delta g^{\alpha\beta}} \to T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\alpha\beta}} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} - \frac{1}{2}\eta_{\alpha\beta}\eta^{\gamma\delta}\partial_\gamma X^\mu \partial_\delta X^\nu \eta_{\mu\nu} = 0$$

$$\boxed{T_{01} = \dot X \cdot X' = 0 \qquad T_{00} = \frac{1}{2}\left(\dot X^2 + X'^2\right) = T_{11} = 0}$$

kényszer egyenletek, Virasoro kényszerek