Nehéz kvarkónium kötött állapotok:

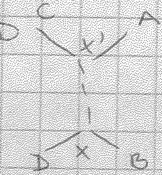
$$V(\underline{r}\,[redacted], s_1, s_2) = V_{\text{spinfüggetlen}} + V_{\text{spinfüggő}}$$

$$= \left(m_1 + m_2 + V_0 - \frac{4}{3}\alpha_S \frac{1}{r} + \sigma r\right) + V_{\text{spinfüggő}}$$

$$\mathcal{H} = V(\underline{r}, s_1, s_2) + \frac{1}{2m_{red}} p^2 \quad - \text{ nem-relativisztikus közelítés}$$

$$-\left(\frac{1}{m_1^3} + \frac{1}{m_2^3}\right)(p^2)^2 \quad \text{Darwin tag}$$

$$V_{\text{spinfügg}} = -\frac{4}{3}\alpha_S V_{\text{Breit-Fermi}}$$

QED 

$$= -\int d^4x \int d^4x' \, j^{\mu}_{CA}(x') D_c(x-x') j_{\mu DB}(x) =$$

$$\cong \int d^3x \int d^3x' \; \varphi^*_C(\underline{x}') \varphi^*_D(\underline{x}') V(\underline{x}, \underline{x}') \varphi_A(\underline{x}), \varphi_B(\underline{x}) =$$

$$\alpha \int \Psi^*_C(\underline{x}') \Psi^*_D(\underline{x}) \left\{ \frac{1 - \underline{\alpha}_B \underline{\alpha}_A}{|\underline{x} - \underline{x}'|} e^{i|\omega_A - \omega_C||\underline{x} - \underline{x}'|/c} \right\} \Psi_A(\underline{x}) \Psi_B(\underline{x})$$

$$\underline{\alpha} = \gamma^0 \underline{\gamma}$$

nem-rel. $\mathcal{O}\left(\frac{1}{c^2}\right)$

$$\hat{\mathcal{H}}_{Dirac} = c\underline{\alpha}\,\underline{p} + \beta m c^2 + e A_0^{\text{Coulomb}}(\underline{x})$$

$$\approx \frac{1}{|\underline{x} - \underline{x}'|} + i \frac{|\omega_A - \omega_C|}{c} - \frac{(\omega_A - \omega_C)^2}{2c^2} |\underline{x} - \underline{x}'| + \dots$$

Fourier transzformált hf.:

$$u(\underline{p}) = \sqrt{2m}\left(1 + \frac{p^2}{8m^2}\right)\begin{pmatrix} \chi \\ \frac{\underline{\sigma}\,\underline{p}}{m}\chi \end{pmatrix}$$

(szórási amplitúdó Fourier-transzformáltja a V)

$$S^{(2)}_{fi} = \bar{u}_C \gamma_\mu u_A \frac{e^2}{q^2} \bar{u}_D \gamma^\mu u_B \cong$$

$$\frac{1}{q^2} \cong \frac{1}{\underline{q}^2} + \frac{1}{4m^2} + \frac{(\underline{q}\,\underline{p}_1)(\underline{q}\,\underline{p}_2)}{m^2(\underline{q}^2)^2}$$

$$\cong (2m)^2 \chi_C^{T*} \chi_D^{T*} V(\underline{q}) \chi_A \chi_B.$$

$$V(\underline{q}) = \alpha \left\{ \frac{1}{\underline{q}^2} - \frac{1}{4m^2} + \frac{i}{4\underline{q}^2 m^2} \left[(\underline{q} \times \underline{p}_A)\underline{\sigma}_A - (\underline{q} \times \underline{p}_B)\underline{\sigma}_B + \right.\right.$$

$$\left. + 2(\underline{q} \times \underline{p}_A)\underline{\sigma}_B - 2(\underline{q} \times \underline{p}_B)\underline{\sigma}_A \right] +$$

$$+\frac{(q p_A)(q p_B)}{m^2 (q^2)^2} - \frac{p_A p_B}{m^2 q^2} + \frac{(q\sigma_A)(q\sigma_B)}{4m^2 q^2} - \frac{\sigma_A\sigma_B}{4m^2}\Big\}$$

spinpálya k.h.

$$V^{(ee)}(\underline{r}) = \frac{\alpha}{r} - \pi\frac{\alpha}{m^2c^2}\delta^{(3)}(\underline{r}) - \frac{\alpha}{4m^2c^2}\frac{1}{r^3}\Big[(\underline{r}\times p_A)\underline{\sigma}_A +$$

$$+(\underline{r}\times p_B)\underline{\sigma}_B + 2(\underline{r}\times p_A)\underline{\sigma}_B - 2(\underline{r}\times p_B)\underline{\sigma}_A\Big] -$$

$$-\frac{\alpha}{2m^2c^2}\Big[\frac{1}{r}p_A p_B + \frac{1}{r^3}\underline{r}(\underline{r}p_A)p_B\Big] + \frac{\alpha}{4m^2c^2}\Big[\frac{\sigma_A\sigma_B}{r^3} -$$

$$-\frac{3(\underline{r}\sigma_A)(\underline{r}\sigma_B)}{r^5} - \frac{8\pi}{3}\sigma_A\sigma_B\delta^{(3)}(\underline{r})\Big]$$

← spin-spin k.h.

$$V^{(e\bar{e})}(\underline{r}) = -V^{(ee)} + \frac{\pi}{2}\frac{\alpha}{m^2c^2}\left(3+\underline{\sigma}_A\underline{\sigma}_B\delta^{(3)}(\underline{r})\right)$$

kicserélődési kölcsönhatás

$$V_{Breit-Fermi} = V_B + V_{SS} + V_{SO} + V_T$$

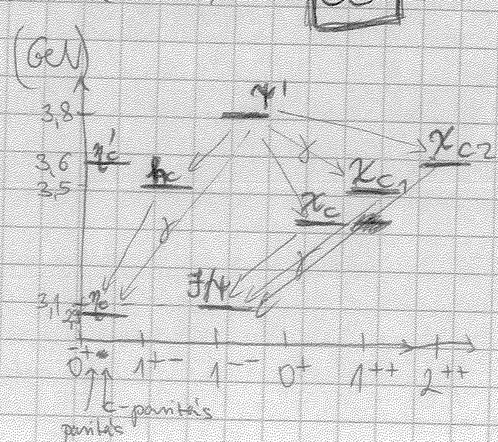
$$V_B = \frac{1}{2m^2 r}\left(p^2 + \frac{(\underline{r}\underline{p})^2}{r^2}\right)$$

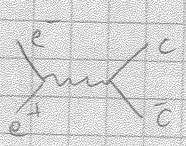
$$V_{SS} = -\frac{\pi}{2}\delta^{(3)}(\underline{r})\left(\frac{2}{m^2} + \frac{\hat{\underline{S}}_Q\hat{\underline{S}}_{\bar{Q}}}{2m^2}\right)$$

$$V_{SO} = \frac{3}{2m^2r^3}\underline{S}\,\underline{L} \qquad \underline{S} = \underline{S}_Q + \underline{S}_{\bar{Q}}$$

$$V_T = \frac{3}{2m^2r^3}\left((\underline{S}\hat{\underline{r}})(\underline{S}\hat{\underline{r}}) - \frac{1}{3}\underline{S}^2\right)$$

Barbieri (1976) $\boxed{c\bar{c}}$





$1^{-} \Rightarrow$ csak ψ' és J/ψ

hasonló a pozitrónium spektrumhoz

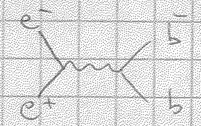
spinfüggetlen részt Schrödinger-egyenlettel megoldják → az ott kapott hfs-eket használjuk a többihez

3 paraméter : m_c , α_s , σ húrfesz. $m_c = 1,6$ GeV

jobb, mint 5% pontosságú jóslatok

$\boxed{b\bar{b}}$ Υ állapotok

9,5 GeV	1s	
10,0 GeV	2s	1^{--}
10,75 GeV	3s	



σ ugyanaz, mint $c\bar{c}$-nél

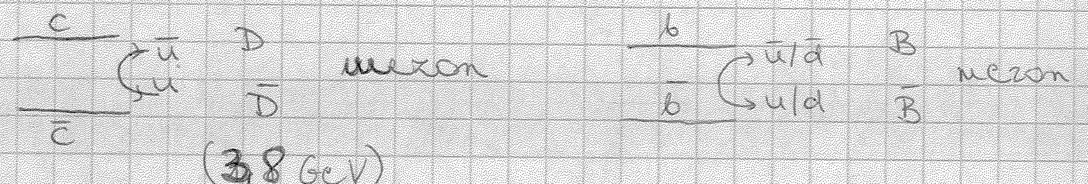
α_s aszimptotikus szabadság szerint csökken

m_b szabad param $= 4,5$ GeV

Darwin tag járuléka < 10% ⟹ jogos a nem-relativisztikus közelítés

u, d kvarkra nem használható

Zweig-szabály:



nagy $c\bar{c}$ tömegnél polarizálódik a vákuum

$b\bar{b}$ (10,6 GeV)

erősen bomlik a $c\bar{c}$ / $b\bar{b}$ rendszer

Zweig megengedett bomlások