Atomi egységek

hossz:	$a_0 = \frac{\hbar^2}{me_0^2} = 0.5\,\text{Å}$
tömeg:	$m = m_e = 9 \cdot 10^{-31}\,\text{kg}$
töltés:	$e_0 = \frac{1}{\sqrt{4\pi\varepsilon_0}} \cdot e = 4.8 \cdot 10^{-16}\,\sqrt{\text{Jm}}$ pl. Coulomb-erő: $F = e_0^2/r^2$
Planck-állandó:	$\hbar = \frac{6.62}{2\pi} \cdot 10^{-34}\,\text{Js} \approx 10^{-34}\,\text{Js}$
energia:	$\frac{e_0^2}{a_0} = 27\text{eV} = 43 \cdot 10^{-19}\,\text{J}$

Hidrogénszerű bázisfüggvények

$$H\Phi_{nlm}(\underline{r}) = E_{nlm}\Phi_{nlm}(\underline{r})$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze_0^2}{r}$$

$$E_{nlm} = E_n = -\frac{1}{2}\frac{Z^2e_0^2}{a_0}\frac{1}{n^2}, \qquad \Phi_{nlm}(\underline{r}) = \Phi(nlm|\underline{r}) = R_{nl}(r)Y_l^m(\vartheta,\varphi)$$

$$R_{10}(r) = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} 2\mathrm{e}^{-\frac{Zr}{a_0}} \qquad Y_0^0(\vartheta,\varphi) = \frac{1}{\sqrt{4\pi}}$$

$$R_{20}(r) = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\sqrt{8}}\left(2 - \frac{Zr}{a_0}\right)\mathrm{e}^{-\frac{Zr}{2a_0}} \qquad Y_1^0(\vartheta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\vartheta$$

$$R_{21}(r) = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\sqrt{24}}\frac{Zr}{a_0}\mathrm{e}^{-\frac{Zr}{2a_0}} \qquad Y_1^{\pm1}(\vartheta,\varphi) = \sqrt{\frac{3}{8\pi}}\sin\vartheta\mathrm{e}^{\pm i\varphi}$$

$$\int d\Omega Y_{l'}^{m'*}(\vartheta,\varphi)Y_l^m(\vartheta,\varphi) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \sin\vartheta Y_{l'}^{m'*}(\vartheta,\varphi)Y_l^m(\vartheta,\varphi) = \delta_{l,l'}\delta_{m,m'}$$

$$\int_0^{\infty} drr^2 R_{n'l'}(r)R_{nl}(r) = \begin{cases} \delta_{n,n'} & \text{ha } l = l' \\ A(n,l,n',l') \text{ szám} & \text{ha } l \neq l' \end{cases}$$

Hasznos formulák

$$\frac{1}{r_{12}} = \frac{1}{|\underline{r}_1 - \underline{r}_2|} = \sum_{l=0}^{\infty} \frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}} \sum_{m=-l}^{l} Y_l^{m*}(\vartheta_1,\varphi_1)Y_l^m(\vartheta_2,\varphi_2)$$

$$\begin{aligned}
\int_0^{\infty} x^n\mathrm{e}^{-x}dx &= n! \\
\int \mathrm{e}^{-x}dx &= -\mathrm{e}^{-x} \\
\int x\mathrm{e}^{-x}dx &= -(x+1)\,\mathrm{e}^{-x} \\
\int x^2\mathrm{e}^{-x}dx &= -\left(x^2 + 2x + 2\right)\mathrm{e}^{-x} \\
\int x^3\mathrm{e}^{-x}dx &= -\left(x^3 + 3x^2 + 6x + 6\right)\mathrm{e}^{-x}
\end{aligned}$$

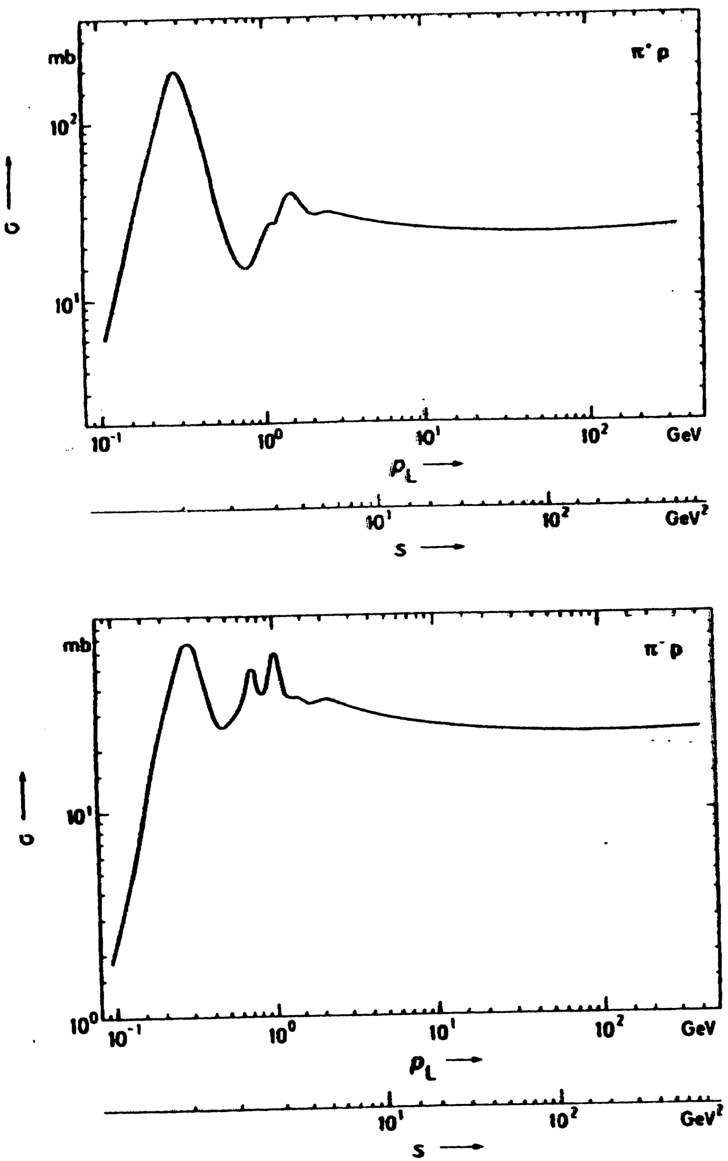

Figure 16.1 The total $\pi^+ p$ and $\pi^- p$ cross-sections (after Particle Data Group 1984).

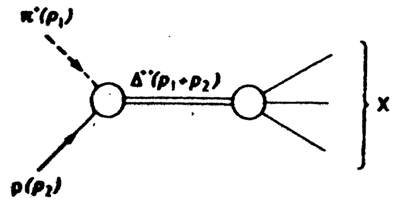

Figure 16.2 Diagram for the production and decay of the Δ^{++} resonance in $\pi^+ p$ scattering. The final state X consists to nearly 100% of $\pi^+ p$ again.

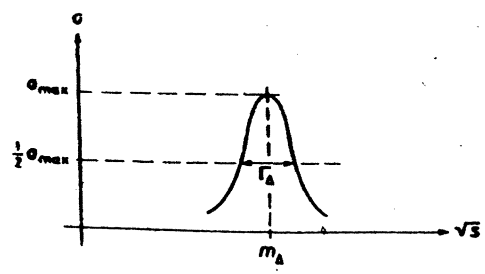

Figure 16.3 Resonance curve and width Γ_Δ corresponding to (16.7) (schematic).

29.2.6. *The group SU(3)*

The fundamental representation of the group SU(3) is given by the matrices

$$U = \exp\left(\tfrac{1}{2}\lambda_i \omega_i\right), \qquad i = 1, 2, \ldots, 8,$$

where λ_i are the Gell-Mann matrices, and ω_i are eight real parameters. Usually the matrices λ_i are chosen in the form:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The matrices λ satisfy the following relations:

$$\mathrm{Tr}\,\lambda_i\lambda_j = 2\delta_{ij},$$

$$\left[\lambda_i\lambda_j\right] = 2if_{ijk}\lambda_k,$$

$$\left[\lambda_i\lambda_j\right]_+ = \tfrac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k, \qquad \text{where } i, j, k = 1, 2, \ldots, 8.$$

Here f_{ijk} are structure constants of the group SU(3), d_{ijk} are symmetrical and f_{ijk} are antisymmetrical with respect to permutations of any pair of indices. Direct calculations easily give 54 non-zero constants f_{ijk} and 58 non-zero constants d_{ijk}:

ijk	f_{ijk}	ijk	d_{ijk}	ijk	d_{ijk}
123	1	118	$1/\sqrt{3}$	355	$\frac{1}{2}$
147	$\frac{1}{2}$	146	$\frac{1}{2}$	366	$-\frac{1}{2}$
156	$-\frac{1}{2}$	157	$\frac{1}{2}$	377	$-\frac{1}{2}$
246	$\frac{1}{2}$	228	$1/\sqrt{3}$	448	$-\frac{1}{2}\sqrt{3}$
257	$\frac{1}{2}$	247	$-\frac{1}{2}$	558	$-\frac{1}{2}\sqrt{3}$
345	$\frac{1}{2}$	256	$\frac{1}{2}$	668	$-\frac{1}{2}\sqrt{3}$
367	$-\frac{1}{2}$	338	$1/\sqrt{3}$	778	$-\frac{1}{2}\sqrt{3}$
458	$\frac{1}{2}\sqrt{3}$	344	$\frac{1}{2}$	888	$-1/\sqrt{3}$
678	$\frac{1}{2}\sqrt{3}$				

($54 = 9 \times 6$ where 6 is the number of permutations of indices $i \neq j \neq k$, and $58 = 4 \times 6 + 11 \times 3 + 1$). Note that $d_{ijk} = 0$ if the number of indices 2, 5, 7 is odd. On the other hand, $f_{ijk} = 0$ if the number of these indices is even. These indices, 2, 5, 7, are special because the corresponding matrices λ are antisymmetric.

29.2.7. *Fierz identities for λ matrices*

Using the completeness of the nine matrices δ^α_β, λ^α_β, we can write:

$$\delta^\alpha_\beta\delta^\gamma_\delta = A\delta^\alpha_\delta\delta^\gamma_\beta + B\lambda^\alpha_\delta\lambda^\gamma_\beta,$$

$$\lambda^\alpha_\beta\lambda^\gamma_\delta = C\delta^\alpha_\delta\delta^\gamma_\beta + D\lambda^\alpha_\delta\lambda^\gamma_\beta,$$

where A, B, C and D are coefficients to be determined and where

$$\lambda \cdot \lambda = \lambda_i\lambda_i, \qquad i = 1, 2, \ldots, 8.$$

Multiplication of these two equalities by $\delta^\delta_\alpha\delta^\beta_\gamma$ yields

$$3 = 9A, \qquad 16 = 9C,$$

and multiplication by $\delta^\beta_\alpha\delta^\delta_\gamma$ yields

$$9 = 3A + 16B, \qquad 0 = 3C + 16,$$

whence

$$\delta^\alpha_\beta\delta^\gamma_\delta = \tfrac{1}{3}\delta^\alpha_\delta\delta^\gamma_\beta + \tfrac{1}{2}\lambda^\alpha_\delta \cdot \lambda^\gamma_\beta,$$

$$\lambda^\alpha_\beta \cdot \lambda^\gamma_\delta = \tfrac{16}{9}\delta^\alpha_\delta\delta^\gamma_\beta - \tfrac{1}{3}\lambda^\alpha_\delta \cdot \lambda^\gamma_\beta.$$

Now it is not difficult to show that

$$8\delta^\alpha_\beta\delta^\gamma_\delta + 3\lambda^\alpha_\beta \cdot \lambda^\gamma_\beta = +\left(8\delta^\alpha_\delta\delta^\gamma_\beta + 3\lambda^\alpha_\delta \cdot \lambda^\gamma_\beta\right),$$

$$4\delta^\alpha_\beta\delta^\gamma_\delta - 3\lambda^\alpha_\beta \cdot \lambda^\gamma_\delta = -\left(4\delta^\alpha_\delta\delta^\gamma_\beta - 3\lambda^\alpha_\delta \cdot \lambda^\gamma_\beta\right).$$

Applied to the product of two triplet spinors, the first of these expressions selects the state 6, and the second one selects the state $\bar{3}$ (recall that $3 \times 3 = 6 + \bar{3}$).

29.2.8. *SU(3) multiplets*

A contravariant three-component spinor t^α is transformed by the matrices $U = \exp(\frac{1}{2}i\omega_i\lambda_i)$; it is denoted by 3. A covariant spinor t_α is transformed by complex conjugate matrices $U^* = \exp(-\frac{1}{2}i\omega_i\lambda_i^*)$; it will be denoted by $\bar{3}$. Representations of higher dimensions can be constructed out of 3 and $\bar{3}$ by making use of the invariant tensors δ^α_β, $\varepsilon_{\alpha\beta\gamma}$, and $\varepsilon^{\alpha\beta\gamma}$:

$3\times\bar{3} = 8+1$: singlet, $1 \sim t^\alpha t_\beta \delta^\beta_\alpha$;
octet, $8 \sim T^\alpha_\beta = t^\alpha t_\beta - \frac{1}{3}\delta^\alpha_\beta(t^\gamma t_\gamma)$.

$3\times 3 = 6+\bar{3}$: antitriplet, $\bar{3} \sim T_\gamma = t^\alpha t^\beta \varepsilon_{\alpha\beta\gamma}$;
sextet, $6 \sim T^{\alpha\beta} = t^\alpha t^\beta + t^\beta t^\alpha$.

$3\times 6 = 8+10$: $8 \sim T^\gamma_\delta = t^\alpha t^{\beta\gamma}\varepsilon_{\alpha\beta\delta}$;
decuplet, $10 \sim T^{\alpha\beta\gamma}$.

$\bar{3}\times 6 = 3+15$: $3 \sim T^\gamma = t_\alpha T^{\alpha\gamma}$;
$15 \sim T^{\beta\gamma}_\alpha$.

$8\times 8 = 1+8+8+10+\overline{10}+27$: $\overline{10} \sim T_{\alpha\beta\gamma}$;
$27 \sim T^{\gamma\delta}_{\alpha\beta}$.

An arbitrary tensor can be written in the form

$$T^q_p = T^{\beta_1\beta_2\ldots\beta_q}_{\alpha_1\alpha_2\ldots\alpha_p},$$

where symmetrization is carried out separately over all upper and lower indices, and the trace for any pair $\alpha_i\beta_k$ is zero. The total number of components of the multiplet T^q_p is found easily:

$$N = \tfrac{1}{2}(p+1)(q+1)(p+q+2).$$

Examples of physical SU(3) multiplets:

$$q^\alpha = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \text{quark triplet,}$$

$$\bar{q}_\alpha = (\bar{u}, \bar{d}, \bar{s}) \quad \text{antiquark (anti)triplet,}$$

$$P^\alpha_\beta = \begin{pmatrix} \sqrt{\frac{1}{6}}\,\eta^0 + \sqrt{\frac{1}{2}}\,\pi^0 & \pi^+ & K^+ \\ \pi^- & \sqrt{\frac{1}{6}}\,\eta^0 - \sqrt{\frac{1}{2}}\,\pi^0 & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\eta^0}{\sqrt{6}} \end{pmatrix} \quad \text{octet of pseudo-scalar mesons,}$$

$$B^\alpha_\beta = \begin{pmatrix} \sqrt{\frac{1}{6}}\,\Lambda^0 + \sqrt{\frac{1}{2}}\,\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \sqrt{\frac{1}{6}}\,\Lambda^0 - \sqrt{\frac{1}{2}}\,\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{1}{6}}\,2\Lambda^0 \end{pmatrix} \quad \text{octet of baryons.}$$

When the isotopic subgroup SU(2) of group SU(3) is singled out, it is convenient to plot the particles of the multiplet on the so-called T_3Y diagrams. Examples are given in figs. 29.1, 2, 3.

By combining d and s (or s and u) quarks, instead of u and d, into an SU(2) doublet we single out the U (or V) spin subgroup* of SU(3) (see fig. 29.4). Figs. 29.1–4 demonstrate that particles within one U-multiplet have identical charges. The composition of U-multiplets is obvious in these figures, with the exception of the central particles on the T_3Y diagram for the octet. The point is that the Σ^0 and Λ^0 states possess a definite T-spin but no definite U-spin. It is their linear superpositions

$$\Sigma^0_U = -\tfrac{1}{2}\Sigma^0 + \tfrac{1}{2}\sqrt{3}\,\Lambda^0, \quad \Lambda^0_U = -\tfrac{1}{2}\sqrt{3}\,\Sigma^0 - \tfrac{1}{2}\Lambda^0,$$

that possess definite U-spin: unity for the first and zero for the second.

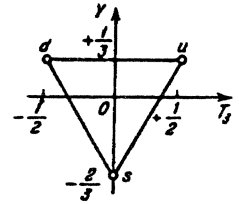

Fig. 29.1

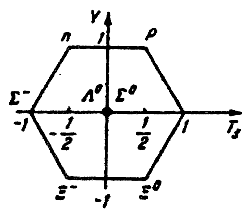

Fig. 29.2

*Sometimes the minus sign is assigned to some of the particles of the SU(3) multiplet in order to make positive the matrix elements of the ladder operators of a given SU(2) subgroup (see J. J. de Swart, *Rev. Mod. Phys.* 35 (1963) 916).