Sugárzás és anyag kölcsönhatása

Ütközési folyamatok → részecskekeltés (kvantum terek)

- $i\hbar \partial_t |\Psi(t)\rangle_S = \hat{H} |\Psi(t)\rangle_S$ → Schrödinger-kép

$$|\Psi(t)\rangle_S = \underbrace{e^{-\frac{i}{\hbar}\hat{H}(t-t_0)}}_{\hat{U}(t,t_0)} |\Psi(t_0)\rangle_S$$

$\hat{U}(t,t_0)$ időfejlesztő operátor → unitér

$\langle\Psi(0)|\Psi(0)\rangle = 1$

adjungált tér: ${}_S\langle\Psi(t)| = {}_S\langle\Psi(t_0)| \hat{U}^+(t,t_0)$

$$\hat{U}^+ = \hat{U}^{-1}(t,t_0)$$

$${}_S\langle\Psi(t)|\Psi(t)\rangle_S = {}_S\langle\Psi(0)| \hat{U}^{-1}(t,t_0)\hat{U}(t,t_0)|\Psi(0)\rangle_S$$

⇒ megtartja a normáltságot

- Kölcsönhatás: $\hat{H} = \hat{H}_0 + \hat{H}_I(t)$

↳ pl: $-e\underline{x}\underline{E}(t)$

$\hat{H}_I(t=-\infty) = \hat{H}_I(t=+\infty) = 0$

Dirac-kép: $|\Psi(t)\rangle_I = e^{\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_S = \hat{U}_0^+(t,0)|\Psi(t)\rangle_S$

$$i\hbar\partial_t\left(e^{-\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_I\right) = \left(\hat{H}_0 + \hat{H}_I(t)\right)e^{-\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_I$$

$$\hat{H}_0 e^{-\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_I + e^{-\frac{i}{\hbar}\hat{H}_0 t} i\hbar\partial_t|\Psi(t)\rangle_I =$$

$$= \hat{H}_0 e^{-\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_I + \hat{H}_I(t) e^{-\frac{i}{\hbar}\hat{H}_0 t}|\Psi(t)\rangle_I$$

$$i\hbar\partial_t|\Psi(t)\rangle_I = \underbrace{\hat{U}_0^+(t,0)\hat{H}_I(t)\hat{U}_0(t,0)}_{\overline{H}_I}|\Psi(t)\rangle_I$$

(van Heisenberg-kép is)

$${}_S\langle\Psi(t)|\hat{A}_S|\Psi(t)\rangle_S = \langle A\rangle = {}_I\langle\Psi(t)|\hat{A}_I|\Psi(t)\rangle_I =$$

$$= {}_S\langle\Psi(t)|U_0(t,0)\hat{A}_I U_0^+(t,0)|\Psi(t)\rangle_S$$

$$\hat{A}_I(t) = \hat{U}_0^+(t,0)\hat{A}_S\hat{U}_0(t,0)$$

$\Psi(\underline{x},t) = \Psi(\underline{x})e^{-i\hbar\frac{\omega}{\hbar}t}$ → leválasztjuk a harmonikus időfejlődést

Szórásmátrix = S-mátrix:

- $|\Psi(t_i)\rangle_I = |i\rangle$ → kezdeti állapot

$\hat{S}(t,t_i)|i\rangle = |\Psi(t)\rangle_I$

$|f\rangle$ → végállapot $= |\Psi(t=t_f)\rangle_I$

$\langle f | S(t_f, t_i) | i \rangle \rightarrow$ átmeneti valószínűségi amplitúdó

baj: $\bar{H}_I(t_1)\bar{H}_I(t_2) \neq \bar{H}_I(t_2)\bar{H}_I(t_1)$

$$|f\rangle = \left(1 - \frac{i}{\hbar}\bar{H}_I(t_f - \Delta t)\Delta t\right) \dots \left(1 - \frac{i}{\hbar}\bar{H}_I(t_i + \Delta t)\Delta t\right) \cdot \left(1 - \frac{i}{\hbar}\bar{H}_I(t_i)\Delta t\right)|i\rangle$$

$\hat{T} \rightarrow$ időrendező operátor

$$\hat{T}\left[\hat{A}(t_2)\hat{A}(t_1)\right] = \hat{T}\left[\hat{A}(t_1)\hat{A}(t_2)\right] = \begin{cases} \hat{A}(t_1)\hat{A}(t_2), & \text{ha } t_1 > t_2 \\ \hat{A}(t_2)\hat{A}(t_1), & \text{ha } t_2 > t_1 \end{cases}$$

$$U(t_f, t_i) = \hat{T}\exp\left\{-\frac{i}{\hbar}\int_{t_i}^{t_f} \bar{H}_I(t')\,dt'\right\}$$

$$S_{fi} = \langle f | \hat{S} | i \rangle = \langle f | \underbrace{\hat{T}\exp\left\{-\frac{i}{\hbar}\int_{t_i}^{t_f}\bar{H}_I(t')\,dt'\right\}}_{\hat{S}} | i \rangle$$

általában: $t_i = -\infty$, $t_f = +\infty$ választás

Perturbációszámítás:

$$\hat{H} = \hat{H}_0 + \hat{H}_I$$

$$S_{fi} = \delta_{fi} - \frac{i}{\hbar}\int_{-\infty}^{+\infty} dt \langle f | \bar{H}_I(t') | i \rangle - \frac{1}{2\hbar^2}\int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dt'' \langle f | \hat{T}\left[\bar{H}_I(t')\bar{H}_I(t'')\right] | i \rangle + \dots$$

$$S_{fi} = \delta_{fi} - i(2\pi)^4 \delta^{(4)}(P_f - P_i)\langle f | \hat{T} | i \rangle$$

T-mátrix

össz. negyesimp.

$w_{f \leftarrow i} = ?$

Fermi-aranyszabály

$$(2\pi)^4 \delta^{(4)}(P_f - P_i) = \int_V d^3\underline{x} \int_{-T/2}^{T/2} dt\, e^{-i(P_{0f} - P_{0i})t + i(\underline{P}_f - \underline{P}_i)\underline{x}} = VT$$

nagy tartomány

$$\frac{dw_{f \leftarrow i}}{T} = (2\pi)^4 \delta^{(4)}(P_f - P_i) V |\langle f | \hat{T} | i \rangle|^2 \cdot \prod_{f\ell}^{n}\left(\frac{d^3 p_{f\ell}}{(2\pi\hbar)^3} V\right) \cdot \left(\prod_{\ell}^{n} \frac{1}{2p^0_{f\ell} V} \frac{1}{2p^0_{i1} V\, 2p^0_{i2} V}\right)$$

V kvantálási térfogatban egyenletesen oszlik el a rendszer

azt is figyelembe kell venni, hogy hány részecske van a V térfogatban → részecskéket síkhullámmal reprezentáljuk → egy részecskére akarjuk normálni → térfogatfüggést eltüntetjük

$$\rho = 2p_0 \qquad N = 2p_0 V$$

$$\underset{j_{be}}{\overset{2}{\Rrightarrow}}\ \underset{1}{\square} \qquad d\sigma = \frac{d\omega_{fi}}{T}\cdot\frac{1}{j_{be}}$$

$$j_{be} = \frac{v_{i2}}{V}$$

$$d\sigma = (2\pi)^4\,\delta^{(4)}(P_f - P_i)\,|\langle f|\hat{T}|i\rangle|^2 \cdot$$

$$\cdot \prod \underbrace{\left(\frac{d^3 p_{fe}}{(2\pi\hbar)^3\, 2p^0_{fe}}\right)}_{\text{Lorentz-invariáns}} \frac{1}{2m_1c^2}\,\frac{1}{2p^0_{i2}v_{i2}}$$

$$\left[(p_1p_2)^2 - p_1^2p_2^2\right]^{1/2}$$

$$\left[(m_1c^2)^2 p_2^{0\,2} - (m_1c^2)^2(m_2c^2)^2\right]$$

$$m_1c^2\,|p|\,c$$

$$\frac{v}{c} = \frac{|p|}{p^0} \qquad p^0_{fe}(|p|, m) = \sqrt{(pc)^2 + (mc^2)^2}$$

$$d\sigma = (2\pi)^4\,\delta^{(4)}(P_f - P_i)\,|\langle f|\hat{T}|i\rangle|^2 \cdot$$

$$\cdot \prod_{l}^{n} \frac{d^3p_{fe}}{(2\pi\hbar)^3\, 2p^0_{fe}}\;\frac{1}{4\left[(p_{i1}p_{i2})^2 - p_{i1}^2p_{i2}^2\right]^{1/2}}$$

Anyag → anyagtér

téridő minden pontjához egy számot rendelünk

$\Phi(\underline{x},t)$ skalártér

$E^2 = (pc)^2 + m^2c^4$

$(\hbar\omega)^2 = (\hbar \underline{k} c)^2 + m^2c^4$

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \Rightarrow$$

Klein-Gordon-egyenlet: $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2c^2}{\hbar^2}\right)\phi = 0$

Hatáselv: $\mathcal{L} = \frac{1}{2}\left[(\partial_0\phi)^2 - (\underline{\nabla}\phi)^2 - \frac{m^2c^2}{\hbar^2}\phi^2\right]$

Lagrange-sűrűség

$$S = \int_T dt \int_V d^3x\, \mathcal{L}$$

$$\delta_\phi S = S[\Phi + \delta\Phi] - S[\Phi] = 0$$

$$\mathcal{L}[\phi + \delta\phi] \cong \mathcal{L}[\phi] + \partial_0\phi\,\partial_0\delta\phi - \underline{\nabla}\phi\,\underline{\nabla}\delta\phi - \frac{m^2c^2}{\hbar^2}\phi\,\delta\phi$$

$$0 = \delta_\phi S = \int dt \int d^3x \left[\partial_0\phi\,\partial_0\delta\phi - \underline{\nabla}\phi\,\underline{\nabla}\delta\phi - \frac{m^2c^2}{\hbar^2}\phi\,\delta\phi\right]$$

$$= \int dt \int d^3x\, \delta\phi\left[-\partial_0^2\phi + \underline{\nabla}^2\phi - \frac{m^2c^2}{\hbar^2}\phi\right]$$

kanonikus impulzus: $\pi(\underline{x},t) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)} = \partial_0\phi$

Hamilton: $\mathcal{H} = \pi\partial_0\phi - \mathcal{L} = \pi^2 - \frac{1}{2}\pi^2 + \frac{1}{2}(\underline{\nabla}\phi)^2 + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2 =$

$$\mathcal{H} = \frac{1}{2}\left[\pi^2 + (\underline{\nabla}\phi)^2 + \frac{m^2c^2}{\hbar^2}\phi^2\right]$$

$H = \int d^3x\, \mathcal{H}$ a teljes Hamilton-fv.

$$\phi(\underline{x},t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[a_1(\underline{k})e^{i\underline{k}\underline{x} - i\omega_k t} + a_2(\underline{k})e^{-i\underline{k}\underline{x} + i\omega_k t}\right] =$$

$\underline{k} \to -\underline{k}$ $\quad \omega_k = \omega_{-k}$

$$= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[a_1(\underline{k})e^{i\underline{k}\underline{x} - i\omega_k t} + a_2(-\underline{k})e^{-i\underline{k}\underline{x} + i\omega_k t}\right]$$

$$\phi^*(\underline{x},t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[a_1^*(\underline{k})e^{-i\underline{k}\underline{x} + i\omega_k t} + a_2^*(-\underline{k})e^{i\underline{k}\underline{x} - i\omega_k t}\right]$$

$$a_1(\underline{k}) = a_2^*(-\underline{k}) = a(\underline{k})$$

$$\phi(\underline{x},t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k}\left[a(\underline{k})\, e^{i(\underline{k}\underline{x}-\omega_k t)} + a^*(\underline{k})\, e^{-i(\underline{k}\underline{x}-\omega_k t)}\right]$$

$$\pi(\underline{x},t) = \partial_0 \phi(\underline{x},t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k}\left(-\frac{i\omega_k}{c}\right)\left[a(\underline{k})\, e^{i(\underline{k}\underline{x}-\omega_k t)} - a^*(\underline{k})\, e^{-i(\underline{k}\underline{x}-\omega_k t)}\right]$$

kanonikus kvantálás:

$$\left[\hat\phi(\underline{x},0), \hat\pi(\underline{y},0)\right] = i\hbar\, \delta^{(3)}(\underline{x}-\underline{y})$$

$$[\hat a_k, \hat a_{k'}] = [\hat a_k^+, \hat a_{k'}^+] = 0$$

$$[\hat a_k, \hat a_{k'}^+] = (2\pi)^3\, \hbar\, 2\omega_k\, \delta^{(3)}(\underline{k}-\underline{k}')$$

$$\hat H = \int d^3x\, \frac{1}{2}\Big[\underbrace{\hat\pi^2}_{\hat H_1} + \underbrace{(\nabla\hat\phi)^2}_{\hat H_2} + \underbrace{\frac{m^2c^2}{\hbar^2}\hat\phi^2}_{\hat H_3}\Big]$$

$$\hat H_3 = \frac{1}{2}\frac{m^2c^2}{\hbar^2}\underbrace{\int d^3x}_{\Rightarrow\,\delta(\underline{k}\pm\underline{k}')} \int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\frac{1}{4\omega_k\omega_{k'}}\Big[\hat a(\underline{k})\hat a(\underline{k})\, e^{i\underline{x}(\underline{k}+\underline{k}')} +$$
$$+ \hat a(\underline{k})\hat a^+(\underline{k}')\, e^{i(\underline{k}-\underline{k}')\underline{x}} + \hat a^+(\underline{k})\hat a(\underline{k}')\, e^{i(\underline{k}'-\underline{k})\underline{x}} +$$
$$+ \hat a^+(\underline{k})\hat a^+(\underline{k}')\, e^{-i(\underline{k}+\underline{k}')\underline{x}}\Big] =$$
$$= \frac{1}{2}\frac{m^2c^2}{\hbar^2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_k^2}\left[\hat a(\underline{k})\hat a(-\underline{k}) + \hat a(\underline{k})\hat a^+(\underline{k}) + \hat a^+(\underline{k})\hat a(\underline{k}) + \hat a^+(\underline{k})\hat a^+(-\underline{k})\right]$$

$$\hat H_2 = \frac{1}{2}\int d^3x\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\frac{1}{4\omega_k\omega_{k'}}\Big[-\underline{k}\underline{k}'\,\hat a(\underline{k})\hat a(\underline{k}')\, e^{i\underline{x}(\underline{k}+\underline{k}')} +$$
$$+ \underline{k}\underline{k}'\,\hat a(\underline{k})\hat a^+(\underline{k}')\, e^{i\underline{x}(\underline{k}-\underline{k}')} + \underline{k}\underline{k}'\,\hat a^+(\underline{k})\hat a(\underline{k}')\, e^{i\underline{x}(\underline{k}-\underline{k}')} -$$
$$- \underline{k}\underline{k}'\,\hat a^+(\underline{k})\hat a^+(\underline{k}')\, e^{-i\underline{x}(\underline{k}+\underline{k}')}\Big] =$$
$$= \frac{1}{8}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k^2}\,\underline{k}^2\left[\hat a(\underline{k})\hat a(-\underline{k}) + \hat a(\underline{k})\hat a^+(\underline{k}) + \hat a^+(\underline{k})\hat a(\underline{k}) + \hat a^+(\underline{k})\hat a^+(-\underline{k})\right]$$

$$\hat H_1 = \frac{1}{2}\int d^3x\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\left(-\frac{i}{2c}\right)^2\Big[\hat a(\underline{k})\hat a(\underline{k})\, e^{i(\underline{k}+\underline{k})\underline{x}} -$$
$$- \hat a(\underline{k})\hat a^+(\underline{k}')\, e^{i(\underline{k}-\underline{k}')\underline{x}} - \hat a^+(\underline{k})\hat a(\underline{k}')\, e^{i(-\underline{k}+\underline{k}')\underline{x}} +$$
$$+ \hat a^+(\underline{k})\hat a^+(\underline{k}')\, e^{-i(\underline{k}+\underline{k}')\underline{x}} =$$

$$= \frac{1}{8c^2}\int\frac{d^3k}{(2\pi)^3}\left[\hat{a}(\underline{k})\hat{a}^+(\underline{k})+\hat{a}^+(\underline{k})\hat{a}(\underline{k})-\hat{a}^+(\underline{k})\hat{a}^+(-\underline{k})-\hat{a}(\underline{k})\hat{a}(-\underline{k})\right]$$

$\hat{H}_2+\hat{H}_3 \Rightarrow \frac{1}{8\omega_k^2}\left[\underline{k}^2+\frac{m^2c^2}{\hbar^2}\right]$ diszp. rel. látszik

$$\hat{H}=\hat{H}_1+\hat{H}_2+\hat{H}_3=\frac{1}{8}\int\frac{d^3k}{(2\pi)^3}\left\{\left[\frac{1}{c^2}+\frac{1}{\omega_k^2}\left(\underline{k}^2+\frac{m^2c^2}{\hbar^2}\right)\right]\left(\hat{a}^+(\underline{k})\hat{a}(\underline{k})+\right.\right.$$
$$\left.\left.+\hat{a}(\underline{k})\hat{a}^+(\underline{k})\right)+\underbrace{\left[\frac{1}{c^2}-\frac{1}{\omega_k^2}\left(\underline{k}^2-\frac{m^2c^2}{\hbar^2}\right)\right]}_{0}\left(\hat{a}(\underline{k})\hat{a}(-\underline{k})+\hat{a}^+(\underline{k})\hat{a}^+(-\underline{k})\right)\right\}$$

$$\hat{H}=\frac{1}{4c^2}\int\frac{d^3k}{(2\pi)^3}\left[\hat{a}^+(\underline{k})\hat{a}(\underline{k})+\hat{a}(\underline{k})\hat{a}^+(\underline{k})\right]$$

↳ olyan, mint az oszcillátor

betöltési szám operátor: $\hat{n}(\underline{k})=\hat{a}^+(\underline{k})\hat{a}(\underline{k})$

$$\hat{H}=\frac{1}{2c^2}\int\frac{d^3k}{(2\pi)^3}\left[\hat{a}^+(\underline{k})\hat{a}(\underline{k})+(2\pi)^3\,2\,\omega_k\hbar\,\delta^{(3)}(\underline{k}-\underline{k})\right]$$

zérusponti energia végtelen akar lenni

⇒ újradefiniáljuk a Hamiltont

Normálrendezett szorzat:

$$:a_1^+ a_2^n a_3^{+m}\dots: \;\equiv\; a_1^+ a_3^{+m}\dots a_2^n\dots$$

(keltőoperátorok előre)

$$\hat{H}=\int d^3x\,\frac{1}{2}\left[:\hat{\pi}^2(\underline{x},t):+:(\underline{\nabla}\hat{\phi})^2:+\frac{m^2c^2}{\hbar^2}:\hat{\phi}^2:\right]$$

Összefoglalva: $\boxed{\hbar=1}$ $\boxed{c=1}$ egységrendszer

$$\hat{\phi}(\underline{x},t)=\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\hat{a}(\underline{k})e^{i(\underline{k}\underline{x}-\omega_k t)}+\hat{a}^+(\underline{k})e^{-i(\underline{k}\underline{x}-\omega_k t)}\right]$$

$\hat{\phi}=\hat{\phi}^+$ valós tér

$\omega_k^2=\underline{k}^2+m^2$

$$\hat{\pi}(\underline{x},t)=\dot{\hat{\phi}}(\underline{x},t)=-\frac{i}{2}\int\frac{d^3k}{(2\pi)^3}\left[\hat{a}(\underline{k})e^{i(\underline{k}\underline{x}-\omega_k t)}-\hat{a}^+(\underline{k})e^{-i(\underline{k}\underline{x}-\omega_k t)}\right]$$

$\hat{\pi}=\hat{\pi}^+$

$$\left[\hat{\phi}(\underline{x},0),\hat{\pi}(\underline{y},0)\right]=i\,\delta^{(3)}(\underline{x}-\underline{y})$$

$$\hat{\mathcal{H}}=\frac{1}{2}\int d^3x\left[:\hat{\pi}^2:+:(\underline{\nabla}\hat{\phi})^2:+m^2:\hat{\phi}^2:\right]=$$

$$=\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\hat{a}^+(\underline{k})\hat{a}(\underline{k})$$

Alapállapot : Ψ_0

$$\hat{\mathcal{H}}\Psi_0 = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\hat{a}^+(\underline{k})\,\hat{a}(\underline{k})\,\Psi_0 = 0$$

$$\Longleftrightarrow \quad \hat{a}(\underline{k})\Psi_0 = 0 \qquad \text{(eltüntető operátor)}$$

$$\hat{a}^+(k_0)\Psi_0 = \Psi_{k_0} \qquad \text{(keltő operátor)}$$

$$\hat{\mathcal{H}}\Psi_{k_0} = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\hat{a}^+(\underline{k})\,\underbrace{\hat{a}(\underline{k})\,\hat{a}^+(\underline{k}_0)}_{[\hat{a}(\underline{k}),\,\hat{a}^+(\underline{k}_0)]}\,\Psi_0 =$$

$$= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\hat{a}^+(\underline{k})\,(2\pi)^3\omega_k\,\delta(\underline{k}-\underline{k}_0)\,\Psi_0 =$$

$$= \omega_{k_0}\,\hat{a}^+(\underline{k}_0)\Psi_0 = \omega_{k_0}\Psi_{k_0}$$

$\hat{\mathcal{H}}$-nak sajátállapota Ψ_{k_0} ω_{k_0} sajátértékkel, ezt hívjuk egyrészecske állapotnak

Norma:

$$\left(\hat{a}^+(\underline{k})\Psi_0,\,\hat{a}^+(\underline{k}')\Psi_0\right) = \left(\Psi_0,\,\hat{a}(\underline{k})\,\hat{a}^+(\underline{k}')\Psi_0\right) =$$

$$= \left(\Psi_0,\,[\hat{a}(\underline{k}),\,\hat{a}^+(\underline{k}')]\Psi_0\right) = (2\pi)^3\,2\omega_k\,\delta(\underline{k}-\underline{k}')\,\underbrace{(\Psi_0,\Psi_0)}_{1}$$

ez nem igazi fizikai állapot $\Rightarrow$

Hullámcsomag:

$$\bar{\Phi} = \int\frac{d^3k}{(2\pi)^3}\,\frac{1}{\sqrt{2\omega_k}}\,\phi(\underline{k})\,\hat{a}^+(\underline{k})\Psi_0$$

$$(\bar{\Phi},\bar{\Phi}) = 1 = \int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\,\frac{1}{2\sqrt{\omega_k\omega_{k'}}}\,\phi^*(k')\,\phi(k)\,\cdot$$

$$\cdot\left(\hat{a}^+(k')\Psi_0,\,\hat{a}^+(\underline{k})\Psi_0\right) =$$

$$= \int\frac{d^3k}{(2\pi)^3}\,|\phi(\underline{k})|^2$$

(Energiamegmaradás:

$$\frac{d\hat{\mathcal{H}}}{dt} = \int_V d^3x\,\frac{\partial\hat{H}}{\partial t} = \frac{1}{2}\int d^3x\,\underline{\nabla}\left(\hat{\Phi}\,\underline{\nabla}\hat{\pi} - \hat{\pi}\,\underline{\nabla}\hat{\Phi}\right)$$

energiasűrűség $\frac{1}{2}(\hat{\Phi}\,\underline{\nabla}\hat{\pi} - \hat{\pi}\,\underline{\nabla}\hat{\Phi}) = \underline{j}_E = \underline{\rho}_P$

(impulzust is viszi)

$$\hat{\underline{P}} = \frac{1}{2}\int d^3x\,\left[\hat{\Phi}\,\underline{\nabla}\hat{\pi} - \hat{\pi}\,\underline{\nabla}\hat{\Phi}\right])$$

Hamilton-egyenletek:

$$\dot{\pi} = -\frac{\delta \mathcal{H}}{\delta \phi}$$

$$\dot{\phi} = \frac{\delta \mathcal{H}}{\delta \pi}$$

funkcionálderivált:

$$\phi \to \phi + \delta\phi$$

$$\delta_\phi \int d^3x \, \frac{1}{2} (\nabla\phi)^2 = \int d^3x \, \frac{1}{2}\left(\nabla(\phi+\delta\phi)\nabla(\phi+\delta\phi) - \nabla\phi^2\right) =$$

$$= \int d^3x \, \underline{\nabla}\phi \underline{\nabla}\delta\phi + \mathcal{O}(\delta\phi^2) = -\int d^3x \, \Delta\phi \, \delta\phi$$

$$\frac{\delta \frac{1}{2}\int d^3x (\nabla\phi)^2}{\delta\phi(y)} = -\Delta\phi(y)$$

$$\frac{d\mathcal{H}}{dt} = \int_V d^3x \, \frac{\partial H}{\partial t} = \int_V d^3x \, \frac{1}{2}\frac{\partial}{\partial t}\left(\pi^2 + (\nabla\phi)^2 + m^2\phi^2\right) =$$

$$= \int_V \left(\pi\dot{\pi} + (\nabla\phi)(\nabla\dot{\phi}) + m^2\phi\dot{\phi}\right) d^3x =$$

$$\dot{\pi} = \Delta\phi - m^2\phi \qquad \dot{\phi} = \pi$$

$$= \int_V d^3x \left(\pi(\Delta\phi - \cancel{m^2\phi}) + (\underline{\nabla}\phi)(\underline{\nabla}\pi) + \cancel{m^2\phi\pi}\right) =$$

$$= \int_V d^3x \left(\pi\Delta\phi + (\nabla\phi)(\nabla\pi)\right) = \qquad \text{parc. int.}$$

$$= \int_V d^3x \, \underline{\nabla}(\pi\underline{\nabla}\phi)$$

energiaáramsűrűség ennek az antiszimmetrikus része: $\underline{j}_E = \underline{g}_P = \frac{1}{2}(\phi\underline{\nabla}\pi - \pi\underline{\nabla}\phi)$

impulzust is visz

$$\underline{\hat{P}} = \frac{1}{2}\int d^3x \left[\hat{\phi}\underline{\nabla}\hat{\pi} - \hat{\pi}\underline{\nabla}\hat{\phi}\right] = \dots =$$

$$= \frac{1}{4\omega_k}\int \frac{d^3k}{(2\pi)^3} \, \underline{k}\left(\hat{a}(\underline{k})\hat{a}^+(\underline{k}) + \hat{a}^+(\underline{k})\hat{a}(\underline{k})\right) =$$

kell normálrendezés

$$= \frac{1}{2\omega_k}\int \frac{d^3k}{(2\pi)^3} \, \underline{k}\left(\hat{a}^+(\underline{k})\hat{a}(\underline{k})\right)$$

sajátértékei: $\underline{\hat{P}}\,\psi_{k_0} = \underline{k}_0\,\psi_{k_0}$

Elektromágneses tér kvantálása

$$\mathcal{H} = \int d^3x \left[\frac{1}{2}\varepsilon_0 \underline{E}^2 + \frac{1}{2\mu_0}\underline{B}^2 \right]$$

$$\underline{E} = -\underline{\nabla} A_0 - \dot{\underline{A}}$$

$$\underline{B} = \text{rot}\, \underline{A}$$

Coulomb-mérték: $\underline{\nabla}\,\underline{A} = 0$

$$\frac{1}{2}\varepsilon_0 \int d^3x \left((\nabla A_0)^2 + \underbrace{2\dot{\underline{A}}\underline{\nabla} A_0}_{-2A_0 \underbrace{\underline{\nabla}\dot{\underline{A}}}_{0}} + \dot{\underline{A}}^2 \right)$$

parc. int.

$$\Rightarrow \frac{\varepsilon_0}{2} \int d^3x \left((\underline{\nabla} A_0)^2 + \dot{\underline{A}}^2 \right)$$

$$\frac{1}{2\mu_0} \int d^3x\ \varepsilon_{ijk}\partial_j A_k \varepsilon_{imn}\partial_m A_n =$$

$$= \frac{1}{2\mu_0} \int d^3x \left((\partial_j A_k)^2 - \underbrace{\partial_j A_k \partial_k A_j}_{-(\underbrace{\partial_j A_j}_{0})^2} \right) =$$

2 parc. int.

$$= \frac{1}{2\mu_0} \int d^3x\, (\partial_j A_k)^2$$

$$\mathcal{H}_C = \frac{\varepsilon_0}{2} \int d^3x \left[\dot{\underline{A}}^2 + (\underline{\nabla} A_0)^2 + c^2 (\partial_j \underline{A}\, \partial_j \underline{A}) \right]$$

nincs dinamikája → nem kvantáljuk

véges térfogatot veszünk periodikus határfeltétellel

$$\underline{A}(\underline{x},t) = \frac{1}{\sqrt{V}} \sum_{\underline{k},\alpha} \left[\hat{a}_k^\alpha \underline{e}_k^\alpha e^{i(\underline{k}\underline{x} - \omega_k t)} + \hat{a}_k^{+\alpha} \underline{e}_k^\alpha e^{-i(\underline{k}\underline{x} - \omega_k t)} \right]$$

div $\underline{A} = 0 \rightarrow \underline{k}\,\underline{e}_k^\alpha = 0 \rightarrow$ így elégíthető ki a Coulomb-mérték mellékfeltétele

$$k_n = \frac{2\pi}{L} n$$

$\underline{e}_k^\alpha \quad \alpha = 1,2 \quad \underline{e}_k^\alpha \underline{e}_k^\beta = \delta_{\alpha\beta}$ polarizációs vektor

$\omega_k = c|\underline{k}|$ diszperziós reláció

$$\dot{\hat{\underline{A}}}(\underline{x},t) = \frac{1}{\sqrt{V}} \sum_{\underline{k},\alpha} (-i\omega_k) \left[\hat{a}_k^\alpha \underline{e}_k^\alpha e^{i\underline{k}\underline{x} - i\omega_k t} - \underline{e}_k^\alpha \hat{a}_k^{+\alpha} e^{-i\underline{k}\underline{x} + i\omega_k t} \right]$$

$$\frac{\varepsilon_0}{2}\int d^3x\, \dot{\underline{A}}^2 = \frac{1}{V}\int d^3x \sum_{\underline{k},\alpha}\sum_{\underline{k}',\beta} \omega_{\underline{k}}\omega_{\underline{k}'}\Big[\hat{a}^{\alpha}_{\underline{k}}\,\underline{e}^{\alpha}_{\underline{k}}\, e^{i\underline{k}\underline{x}-i\omega_k t} -$$

$$-\hat{a}^{\alpha+}_{k}\,\underline{e}^{\alpha}_{k}\, e^{-i\underline{k}\underline{x}+i\omega_k t}\Big]\Big[\hat{a}^{\beta+}_{k'}\,\underline{e}^{\beta}_{k'}\, e^{-i\underline{k}'\underline{x}+i\omega_{k'} t} - \hat{a}^{\beta}_{\underline{k}'}\,\underline{e}^{\beta}_{k'}\, e^{i\underline{k}'\underline{x}-i\omega_{k'}t}\Big] = *$$

$$\frac{1}{V}\int d^3x\, e^{i(k-k')\underline{x}} = \delta_{\underline{k}\underline{k}'} \qquad (\omega_{\underline{k}} = \omega_{-\underline{k}})$$

$$* = \sum_{\underline{k},\alpha,\beta}\omega^2_{\underline{k}}\Big[\hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta+}_{\underline{k}}\underline{e}^{\alpha}_{\underline{k}}\underline{e}^{\beta}_{\underline{k}} - \hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta}_{-\underline{k}}\underline{e}^{\alpha}_{\underline{k}}\underline{e}^{\beta}_{-\underline{k}} - \hat{a}^{\alpha+}_{\underline{k}}a^{\beta+}_{-\underline{k}}\underline{e}^{\alpha}_{k}\underline{e}^{\beta}_{-\underline{k}} + \hat{a}^{\alpha+}_{\underline{k}}\hat{a}^{\beta}_{\underline{k}}\underline{e}^{\alpha}_{\underline{k}}\underline{e}^{\beta}_{\underline{k}}\Big] =$$

$$= \sum_{\underline{k},\alpha}\omega^2_{\underline{k}}\Big[\hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\alpha+}_{\underline{k}} + \hat{a}^{\alpha+}_{\underline{k}}\hat{a}^{\alpha}_{k} - \sum_{\beta}\big(\hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta}_{-\underline{k}} + \hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta}_{-\underline{k}}\big)\underline{e}^{\alpha}_{\underline{k}}\underline{e}^{\beta}_{-\underline{k}}\Big]$$

$$\frac{\varepsilon_0 c^2}{2}\int d^3x\,(\mathrm{rot}\,\underline{A})^2$$

$$\mathrm{rot}(\underline{A}) = \frac{i}{\sqrt{V}}\sum_{\underline{k},\alpha}\Big[\hat{a}^{\alpha}_{k}(\underline{k}\times\underline{e}^{\alpha}_{\underline{k}})e^{i\underline{k}\underline{x}-i\omega_k t} - \hat{a}^{\alpha+}_{k}(\underline{k}\times\underline{e}^{\alpha}_{\underline{k}})e^{-ikx+i\omega_k t}\Big]$$

$$(\underline{k}\times\underline{e}^{\alpha}_{\underline{k}})(\underline{k}\times e^{\beta}_{k}) = k^2\,\delta_{\alpha\beta}$$

$$(\underline{k}\times\underline{e}^{\alpha}_{k})(-\underline{k}\times\underline{e}^{\beta}_{-k}) = (\underline{k}\times\underline{e}^{\alpha}_{k})(\underline{e}^{\beta}_{-k}\times\underline{k}) = \underline{e}^{\beta}_{-k}\,\underbrace{\big(\underline{k}\times(\underline{k}\times\underline{e}^{\alpha}_{k})\big)}_{\underline{k}(\underline{k}\,\underline{e}^{\alpha}_{k}) - \underline{e}^{\alpha}_{k}k^2} =$$

$$= -k^2\,\underline{e}^{\alpha}_{k}\,\underline{e}^{\beta}_{-k}$$

$$\frac{\varepsilon_0 c^2}{2V}\int d^3x \sum_{\substack{\underline{k},\alpha\\ \underline{k}',\beta}}(\underline{k}\times\underline{e}^{\alpha}_{\underline{k}})(\underline{k}'\times\underline{e}^{\beta}_{\underline{k}'})\Big[\hat{a}^{\alpha}_{\underline{k}}e^{i\underline{k}\underline{x}} - \hat{a}^{\alpha+}_{k}e^{-i\underline{k}\underline{x}}\Big]\cdot$$

$$\cdot\Big[\hat{a}^{\beta+}_{\underline{k}'}e^{-i\underline{k}'\underline{x}} - \hat{a}^{\beta}_{\underline{k}'}e^{i\underline{k}'\underline{x}}\Big] =$$

$$= \frac{\varepsilon_0 c^2}{2}\sum_{\underline{k},\alpha,\beta}\Big[\hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta+}_{\underline{k}}k^2\delta_{\alpha\beta} + \hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\beta}_{-\underline{k}}k^2\underline{e}^{\alpha}_{\underline{k}}\underline{e}^{\beta}_{-\underline{k}} + \hat{a}^{\alpha+}_{\underline{k}}\hat{a}^{\beta+}_{-\underline{k}}\underline{e}^{\alpha}_{k}\underline{e}^{\beta}_{-k} +$$

$$+ \hat{a}^{\alpha+}_{\underline{k}}\hat{a}^{\beta}_{\underline{k}}k^2\delta_{\alpha\beta}$$

$$\mathcal{H}_C = \varepsilon_0\sum_{\underline{k},\alpha}\omega^2_{\underline{k}}\big(\hat{a}^{\alpha}_{\underline{k}}\hat{a}^{\alpha+}_{\underline{k}} + \hat{a}^{\alpha+}_{\underline{k}}\hat{a}^{\alpha}_{\underline{k}}\big)$$

$$\hat{a}^{\alpha}_{\underline{k}} = \sqrt{\frac{\hbar}{2\varepsilon_0\omega_k}}\,\hat{b}^{\alpha}_{\underline{k}}$$

$$\mathcal{H}_C = \frac{1}{2}\sum_{\underline{k},\alpha}\hbar\omega\big(\hat{b}^{\alpha}_{\underline{k}}\hat{b}^{\alpha+}_{\underline{k}} + \hat{b}^{\alpha+}_{\underline{k}}\hat{b}^{\alpha}_{\underline{k}}\big)$$

kvantálás: $\big[b^{\alpha}_{k},\, b^{\beta+}_{k'}\big] = \delta_{kk'}\delta_{\alpha\beta}$

$$:\hat{\mathcal{H}}_C: = \sum_{\underline{k},\alpha}\hbar\omega_k\,\hat{b}^{\alpha+}_{\underline{k}}\hat{b}^{\alpha}_{\underline{k}}$$

$$\underline{\hat{A}}(\underline{x},t) = \sum_{\underline{k},\alpha} \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_k V}}\, \underline{e}^{\alpha}_{\underline{k}} \left(\hat{b}^{\alpha}_{\underline{k}} e^{i\underline{k}\underline{x} - i\omega_k t} + \hat{b}^{\alpha +}_{\underline{k}} e^{-i\underline{k}\underline{x} + i\omega_k t} \right)$$

$$\varepsilon_0 \dot{\hat{\underline{A}}} = \hat{\underline{\Pi}}_A = -i \sum_{\underline{k},\alpha} \sqrt{\frac{\hbar \omega_k}{2\varepsilon_0 V}}\, \varepsilon_0 \underline{e}^{\alpha}_{\underline{k}} \left(\hat{b}^{\alpha}_{\underline{k}} e^{i\underline{k}\underline{x} - i\omega_k t} - \hat{b}^{\alpha +}_{\underline{k}} e^{-i\underline{k}\underline{x} + i\omega_k t} \right)$$

$$\left[\hat{A}_i(\underline{x},0), \hat{\Pi}_{Aj}(\underline{y},0) \right] = i\hbar\, \delta_{ij}\, \delta(\underline{x}-\underline{y})$$

Sugárzási tér kölcsönhatása az anyaggal

$$L_{\text{atom külső em térben}} = \frac{1}{2} m \dot{\underline{x}}^2 + e A_0(\underline{x}) - e \dot{\underline{x}} \underline{A}(\underline{x}) \qquad q = -e$$

$$\underline{\Pi} = \frac{\partial L}{\partial \dot{\underline{x}}} = m\dot{\underline{x}} - e\underline{A}(\underline{x})$$

$$\dot{\underline{x}} = \frac{1}{m}\left(\underline{\Pi} + e\underline{A}(\underline{x}) \right)$$

$$H = \underline{\Pi}\dot{\underline{x}} - L = \frac{1}{m}\underline{\Pi}\left(\underline{\Pi} - e\underline{A}(x)\right) - \frac{1}{2} m \frac{1}{m^2}\left(\underline{\Pi} + e\underline{A}(\underline{x})\right)^2 - eA_0(x) +$$
$$+ \frac{1}{m}\left(\underline{\Pi} + e\underline{A}(\underline{x})\right) e\underline{A}(\underline{x})$$

$$H = \frac{1}{2m}\left(\underline{\Pi} + e\underline{A}\right)^2 - eA_0$$

$$\dot{\underline{\Pi}} = -\frac{\partial H}{\partial \underline{x}} \qquad \dot{\underline{x}} = \frac{\partial H}{\partial \underline{\Pi}}$$

$$m\ddot{\underline{x}} = \dot{\underline{\Pi}} + e\dot{\underline{A}}(\underline{x}) + e\frac{\partial \underline{A}}{\partial x_j}\dot{x}_j =$$

$$= \underbrace{-\frac{1}{m}\left(\Pi_i + eA_i\right)}_{\dot{x}_i} e\frac{\partial A_i}{\partial \underline{x}} + \underbrace{e\frac{\partial A_0}{\partial \underline{x}} + e\dot{\underline{A}}}_{e\underline{E}} + e\frac{\partial \underline{A}}{\partial x_j}\dot{x}_j$$

$$m\ddot{\underline{x}} = -e\underline{E} - e\left(\dot{\underline{x}} \times \underline{B}\right)$$

$$H_I = \frac{e}{2m}\left(\underline{\Pi}_a \underline{A} + \underline{A}\underline{\Pi}_a \right)$$

atom kanonikus impulzusa; atom helyén kell venni

Hosszúhullámú közelítés:

$$|\underline{k}\underline{x}_a| \ll 1 \iff \frac{d}{\lambda} \ll 1$$

$$\hat{H}_I = \frac{e}{m}\hat{\underline{\Pi}}_a \hat{\underline{A}}(0) \Rightarrow e\dot{\hat{\underline{x}}}_a \hat{\underline{A}}(0) \quad \text{dipólközelítés}$$

$$\langle f|S|i\rangle = -\frac{ie}{\hbar}\int_{-\infty}^{\infty} dt\, \langle b_{atom}|\dot{\hat{\underline{x}}}|a_{atom}\rangle \langle B_{foton}|\hat{\underline{A}}(0,t)|A_{foton}\rangle = *$$

atom végállapot; atom kezdeti; foton kezdeti

① a_{atom}: atomi alapállapot és
b_{atom}: atomi gerjesztett állapot
A_{foton}: n db $\underline{\varepsilon}$ foton
B_{foton}: $n-1$ db $\underline{\varepsilon}$ foton

abszorbció

② a_{atom}: gerjesztett állapot
b_{atom}: alapállapot
A_{foton}: n foton
B_{foton}: $n+1$ foton

emisszió

$$* = \frac{ie}{\hbar}\int_{-\infty}^{\infty} dt \, \langle b_{atom} | \hat{\underline{x}}_a | a_{atom} \rangle \langle B_{foton} | \hat{\underline{A}}(0,t) | A_{foton} \rangle =$$

$$= \frac{i}{\hbar}\int_{-\infty}^{\infty} dt \, \langle b_{atom} | \hat{\underline{d}}_a | a_{atom} \rangle \langle B_{foton} | \hat{\underline{E}}(0,t) | A_{foton} \rangle$$

$$\langle \text{alap} | \hat{d} | \text{gerjesztett} \rangle = \int d^3x \, \Psi_a^*(\underline{x}) e^{i\omega_a t} (-e\underline{x}) \Psi_g(\underline{x}) e^{i\omega_g t} = \underline{d}_{ag} \, e^{i(\omega_a - \omega_g)t}$$

$$\langle n+1 | \hat{E} | n \rangle = \langle (n+1)^\beta_{\varepsilon_0} | \, i \sum_{\varepsilon, \alpha} \sqrt{\frac{\hbar\omega_\varepsilon}{2\varepsilon_0 V}} \, \underline{e}^\alpha_\varepsilon \left(\hat{a}^\alpha_\varepsilon e^{-i\omega_\varepsilon t} - \hat{a}^{\alpha +}_\varepsilon e^{i\omega_\varepsilon t} \right) | n^\beta_{\varepsilon_0} \rangle = -i \sqrt{\frac{\hbar\omega_{\varepsilon_0}}{2\varepsilon_0 V}} \sqrt{n^\beta_{\varepsilon_0}+1} \, e^{i\omega_{\varepsilon_0} t} \, \underline{e}^\beta_{\varepsilon_0}$$

$$S_{fi} = \frac{1}{\hbar}\int_{-\infty}^{\infty} dt \, \underline{d}_{ag} \, e^{i(\omega_a - \omega_g)t} \, \underline{e}^\beta_{\varepsilon_0} \sqrt{\frac{\hbar\omega_{\varepsilon_0}}{2\varepsilon_0 V}} \sqrt{n^\beta_{\varepsilon_0}+1} \, e^{i\omega_{\varepsilon_0} t} =$$

$$= \frac{2\pi}{\hbar} \underbrace{\delta(\omega_a - \omega_g + \omega_{\varepsilon_0})}_{\text{energiamegmaradás}} \sqrt{\frac{\hbar\omega_{\varepsilon_0}}{2\varepsilon_0 V}} \sqrt{n^\beta_{\varepsilon_0}+1} \, \underline{d}_{ag} \, \underline{e}^\beta_{\varepsilon_0} =$$

$$= 2\pi \, \delta(E_{alap} + \hbar\omega_{\varepsilon_0} - E_{gerj}) \, \underline{e}^\beta_{\varepsilon_0} \underline{d}_{ag} \sqrt{n^\beta_{\varepsilon_0}+1} \sqrt{\frac{\hbar\omega_{\varepsilon_0}}{2\varepsilon_0 V}} = *$$

$$g^\beta_{\varepsilon_0} := \sqrt{\frac{\omega_{\varepsilon_0}}{2\varepsilon_0 V \hbar}} \, \underline{e}^\beta_{\varepsilon_0} \underline{d}_{ag}$$

$$* = 2\pi\hbar \, g^\beta_{\varepsilon_0} \sqrt{n^\beta_{\varepsilon_0}+1} \, \delta(E_{alap} + \hbar\omega_{\varepsilon_0} - E_{gerj})$$

Fermi-féle aranyszabály:

emisszió: $\frac{w_{fi}}{T} = 2\pi\,\delta(\omega_a + \omega_{k_0} - \omega_g)\,|g^\beta_{k_0}|^2 (n^\beta_{k_0} + 1)$

spontán

abszorpció: $\frac{w_{fi}}{T} = 2\pi\,\delta(\omega_a + \omega_{k_0} - \omega_g)\,|g^\beta_{k_0}|^2\, n^\beta_{k_0}$

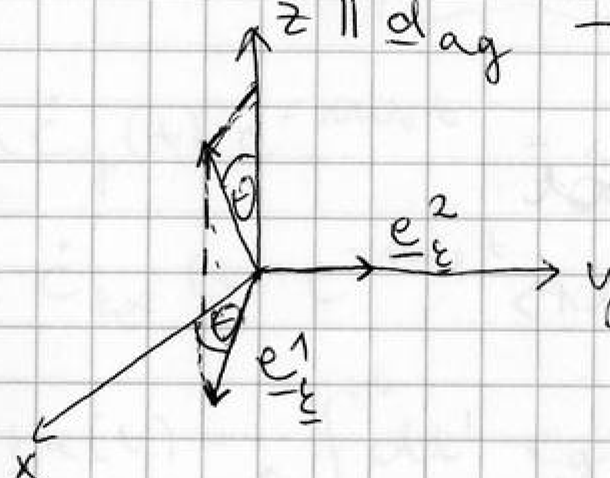
sugárzási teljesítmény =

$$= \sum_{k,\alpha} 2\pi\,\delta(\omega_a - \omega_g + \omega_k)\,\frac{\omega_k}{2\varepsilon_0 V\hbar}\,|\underline{e}^\alpha_k \underline{d}_{ag}|^2\,\hbar\omega_k =$$

$$= \cancel{V}\int \frac{d^3k}{(2\pi)^3} \sum_\alpha 2\pi\delta(\omega_a - \omega_g + \omega_k)\,\frac{\omega_k^2}{2\varepsilon_0 \cancel{V}}\,|\underline{e}^\alpha_k d_{ag}|^2 =$$

$$= \frac{1}{4\pi}\int d\Omega_k \int_0^\infty \frac{d\omega\,\omega^2}{c^3}\,\delta(\omega_a - \omega_g - \omega)\,\frac{\omega^2}{2\varepsilon_0}\sum_\alpha |\underline{e}^\alpha_k \underline{d}_{ag}|^2 = *$$

$z \parallel \underline{d}_{ag} \rightarrow$ így választjuk



$$* = \frac{1}{8\pi^2\varepsilon_0 c^3}(\omega_a - \omega_g)^4 \int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)\left[\sin^2\theta\,|d_{ag}|^2 + 0\right] =$$

$$= \frac{|\underline{d}_{ag}|^2(\omega_a - \omega_g)^4}{3\pi\varepsilon_0 c^3}$$

elnyelési hatáskeresztmetszet:

$$\frac{w_{abs}}{t} = \underbrace{\frac{1}{2}\sum_{\alpha_0=1}^{2}}_{\text{polarizálatlan beeső fény}} \int dE_g\,\underbrace{\rho(E_g)}_{\text{állapotsűrűség}}\,2\pi\,\delta(\omega_a - \omega_g + \omega_{k_0})\,\frac{\omega_{k_0}}{2\varepsilon_0\hbar V}\,|\underline{e}^{\alpha_0}_{k_0}\underline{d}_{ga}|^2 \cdot n^{\alpha_0}_{k_0}$$

$$\frac{1}{2}\sum_{\alpha_0} |\underline{e}^{\alpha_0}_{k_0}\underline{d}_{ga}|^2 = \frac{1}{3}|\underline{d}_{ga}|^2$$

$$\frac{w_{abs}}{t} = \frac{1}{3}|\underline{d}_{ga}|^2\,\rho(E_a + \hbar\omega_{k_0})\,\frac{\pi\,\omega_{k_0} n_{k_0}}{\varepsilon_0 V}$$

bejövő foton energiaáramsűrűsége

$$\frac{\langle n_{k_0}|:\hat{E}_{k_0}\times\hat{H}_{k_0}:|n_{k_0}\rangle}{\hbar\omega_{k_0}} = j_{foton} = \ldots = \frac{c}{V}\,\underline{k}_0\, n_{k_0}$$

$$\sigma_{abs} = \frac{W_{abs}}{t}\frac{1}{|j_{foton}|} = \frac{1}{3}|\underline{d}_{ga}|^2 \rho(E_a + \hbar\omega_{k_0})\frac{\pi\omega_{k_0}}{\varepsilon_0 c}$$

<u>Sugárzási visszahatás az atomi szintekre</u>

elektromos dipól kölcsönhatás: $\hat{\mathcal{H}}_{ED} = -\underline{d}_{atom}\,\underline{\hat{E}}(0,t)$

$$\underline{\hat{E}}(0,t) = i\sum_{k,\alpha}\sqrt{\frac{\omega_k}{2\varepsilon_0 V}}\left(\hat{a}_k^\alpha e^{-i\omega_k t} - \hat{a}_k^{\alpha+} e^{i\omega_k t}\right)$$

Weisskopf-Wigner-féle közelítés → két szint

φ_{alap} φ_{gerj}

φ_1 φ_2

$|a1\rangle$ $|a2\rangle$

$$\langle ai|\hat{\mathcal{H}}_{ED}|aj\rangle = i\sum_{k,\alpha}\hbar(g_k^\alpha)_{ij}\left(\hat{a}_k^\alpha e^{-i\omega_k t} + \hat{a}_k^{\alpha+} e^{i\omega_k t}\right)$$

$$(g_k^\alpha)_{ij} = \sqrt{\frac{\omega_k}{2\varepsilon_0 V}}\,\underline{e}_k^\alpha\,\underline{d}_{ij}$$

atomi "vákuum" = alapállapot

$\hat{\pi}^+|a1\rangle = |a2\rangle$ $\quad\quad \hat{\pi}|a2\rangle = |a1\rangle$

$\hat{\pi}|a1\rangle = 0$

$$\hat{\mathcal{H}}_{ED} = i\hbar\sum_{k,\alpha}\left(\hat{a}_k^\alpha e^{-i\omega_k t} - \hat{a}_k^{\alpha+} e^{i\omega_k t}\right)\left(\hat{\pi}^+ g_{21} + \hat{\pi} g_{21}^*\right)$$

Schrödinger-kép $|\Psi\rangle_t \Rightarrow \left(|\{n_k^\alpha\}, ai\rangle\right) \rightarrow$ ezekből rakjuk össze az időben változó állapotvektort

$$|\Psi\rangle_t = c_g(t)\,\hat{\pi}^+|0,a1\rangle e^{-i\omega_0 t} + \sum_{q,\beta} c_{q,\beta}(t)\,\hat{a}_q^{\beta+} e^{-i\omega_q t}$$

$$\omega_0 = \omega_g - \omega_a$$

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle_t = \left(\hat{\mathcal{H}}_{sug} + \hat{\mathcal{H}}_{atom} + \hat{\mathcal{H}}_{ED}\right)|\Psi\rangle_t$$

kezdeti feltételek:

$$\left.\begin{array}{l} c_g(0) = 1 \\ c_{q\beta}(0) = 0 \end{array}\right\}$$

$$\hat{\mathcal{H}}_{sug} = \sum_{k,\alpha}\hbar\omega_k\,\hat{a}_k^{\alpha+}\hat{a}_k^\alpha$$

$$\hat{\mathcal{H}}_{atom} = \hbar\omega_0\,\hat{\pi}^+\hat{\pi}$$

$$\hat{\mathcal{H}}_{atom}|\Psi\rangle_t = \hbar\omega_0 c_g(t) e^{-i\omega_0 t} \hat{\pi}^+ |0, a1\rangle$$

$$\hat{\mathcal{H}}_{sug}|\Psi\rangle_t = \sum_{q,\beta} \hbar\omega_q c_{q,\beta}(t) \hat{a}_q^{\beta+} |0, a1\rangle e^{-i\omega_q t}$$

$$\hat{\mathcal{H}}_{ED}|\Psi\rangle_t = -c_g(t) i\hbar \sum_{\xi,\alpha} \hat{a}_\xi^{\alpha+} |0, a1\rangle g_{21}^* e^{-i\omega_0 t} +$$

$$+ i\hbar \sum_{\xi,\alpha} c_{\xi\alpha}(t) \hat{\pi}^+ |0, a1\rangle e^{-i\omega_\xi t} g_{21}$$

$$i\hbar \frac{\partial}{\partial t}|\Psi\rangle_t = \dot{c}_g(t) i\hbar \hat{\pi}^+ |0, a1\rangle e^{-i\omega_0 t} +$$

$$+ \hbar\omega_0 c_g(t) \hat{\pi}^+ |0, a1\rangle e^{-i\omega_0 t} +$$

$$+ i\hbar \sum_{q,\beta} \dot{c}_{q,\beta}(t) \hat{a}_q^{\beta+} |0, a1\rangle e^{-i\omega_q t} +$$

$$+ \sum_{q,\beta} \hbar\omega_q c_{q,\beta}(t) \hat{a}_q^{\beta} |0, a1\rangle e^{-i\omega_q t}$$

$$i\hbar \dot{c}_g(t) e^{-i\omega_0 t} = i\hbar \sum_{\xi,\alpha} c_{\xi,\alpha}(t) e^{-i\omega_\xi t} g_{21}$$

$$i\hbar \dot{c}_{\xi\alpha}(t) e^{-i\omega_\xi t} = -c_g(t) i\hbar g_{21}^* e^{-i\omega_0 t}$$

$$c_{\xi,\alpha}(t) = -\int_{-\infty}^{t} dt' \, c_g(t') g_{21}^* e^{i(\omega_\xi - \omega_0)t'}$$

$$\dot{c}_g(t) = -\sum_{\xi,\alpha} \int_{-\infty}^{t} dt' \, c_g(t') g_{21}^* e^{i(\omega_\xi - \omega_0)(t'-t)} g_{21} =$$

$$= -\int_{-\infty}^{t} \Gamma(t'-t) c_g(t') \, dt' \rightarrow \text{ahol}$$

$$\Gamma(t'-t) = \sum_{\xi,\alpha} |g_{21}^\alpha|^2 e^{i(\omega_\xi - \omega_0)(t'-t)} \text{ magfv.}$$

$c_g(0) = 1$ kezdőfeltétellel meg kell oldani

vezessünk be egy új magfv-t:

$$\Gamma(t'-t) \equiv \frac{d\gamma(t'-t)}{dt}$$

formálisan: $\gamma(t-t') = \sum_{\xi\alpha} |g_{21}^\alpha|^2 \dfrac{e^{i(\omega_\xi - \omega_0)(t'-t)}}{i(\omega_\xi - \omega_0)}$

$$\dot{c}_g(t) = -\int_{-\infty}^{t} \frac{d\gamma(t-t')}{dt'} c_g(t') \, dt' = \text{parc. int.}$$

$$= \int_{-\infty}^{t} \gamma(t-t') \dot{c}_g(t') dt' - \gamma(0) c_g(t)$$

kvantummechanika szerint $\dot{c}_g(t) = 0 \to$ a sugárzási térrel való kölcsönhatás változtatja $\to$ lassú változás $\to$ közelítünk

$$c_g = c_g^{(0)} + c_g^{(1)} + \dots$$

$$c_g^{(0)}(t) = c_g(0)\, e^{-\gamma(0) t}$$

$$c_g^{(1)}(t) = \Theta(\gamma^2)$$

$\mathrm{Re}\,\gamma(0) = \dfrac{1}{2\tau} \to$ élettartam

$p_\gamma = |c_g(t)|^2 = e^{-2(\mathrm{Re}\,\gamma(0))\, t} \to$ gerjesztett állapot valószínűsége

azt szeretnénk, hogy $\Gamma(t'-t)$, $\gamma(t'-t) = 0$, ha $t' > t \Rightarrow$ értelmezzük így:

$$\gamma(t-t') = \sum_{k,\alpha} |g_{21}^\alpha|^2 \frac{e^{i(\omega_k - \omega_0)(t'-t)}}{i(\omega_k - \omega_0 - i\delta)}$$

ahol $\delta > 0$ infiterimális, így $\gamma(t'-t)$ eltűnik $t' > t$-re vagyis $c_g(t)$ nem függ a jövőtől

$$\gamma(0) = -i \sum_{k,\alpha} |g_{21}^\alpha|^2 \frac{1}{\omega_k - \omega_0 - i\delta}$$

az imaginárius rész eltolja ω_0-t

a valós rész érdekel minket:

$$\mathrm{Re}\,\gamma(0) = \mathrm{Im}\left(\sum_{k,\alpha} |g_{21}^\alpha|^2 \frac{1}{\omega_k - \omega_0 - i\delta} \right)$$

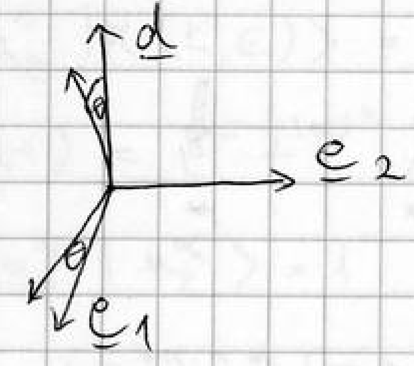
$$\frac{1}{\omega_k - \omega_0 - i\delta} = \mathcal{P}\frac{1}{\omega_k - \omega_0} + i\pi\,\delta(\omega_k - \omega_0)$$

$\hookrightarrow$ főértéktételt alkalmazzuk és V-vel a végtelenbe tartunk $\Rightarrow \sum_k \to V \int \frac{d^3k}{(2\pi)^3}$

$$\mathrm{Re}\,\gamma(0) = V \int \frac{d^3k}{(2\pi)^3} \sum_\alpha |g_{21}^\alpha|^2\, \pi\, \delta(\omega_k - \omega_0) =$$

$$= \cancel{V} \int_0^\infty \int \frac{dk\, k^2}{8\pi^2}\, d\Omega_k \sum_\alpha \frac{\omega_k}{2\varepsilon_0 \cancel{V}} |e_k^\alpha d_{21}|^2\, \delta(\omega_k - \omega_0) =$$

$$\int_0^{2\pi} d\varphi \int_{-1}^{1} d\cos\theta \,(1-\cos^2\theta) = 2\pi\left(2-\frac{2}{3}\right) = \frac{8\pi}{3}$$



$$|\underline{e}_\varepsilon^1 \underline{d}_{21}|^2 = |\underline{d}_{21}|^2 \sin^2\theta$$

$$|\underline{e}_\varepsilon^2 \underline{d}_{21}|^2 = 0$$

$$* = \frac{\omega_0^3 |\underline{d}_{21}|^2}{6\pi\varepsilon_0 c^3}$$

$$\tau = \frac{1}{2\mathrm{Re}\,\gamma(0)} = \frac{3\pi\varepsilon_0 c^3}{\omega_0^3 |d_{21}|^2}$$ az élettartam

$$c_{\varepsilon\alpha}(t) = -\int_0^t c_g(t')\,dt'\, g_{21}^* e^{i(\omega_\varepsilon-\omega_0)t} \cong$$

$$\cong -g_{21}^* \frac{1}{i(\omega_\varepsilon-\omega_0)-\gamma(0)}\left(e^{i(\omega_\varepsilon-\omega_0)t-\gamma(0)t}-1\right)$$

valós rész

$$c_{\varepsilon\alpha}(t\to\infty) = \frac{g_{21}^*}{i(\omega_\varepsilon-\omega_0)-\gamma(0)}$$

$$|c_{\varepsilon\alpha}(\infty)|^2 = \frac{|g_{21}|^2}{(\omega_\varepsilon-\omega_0)^2+\gamma^2(0)}$$ $\Rightarrow$ $\gamma(0)$ szélességgel meg vannak engedve ω_ε frekvenciát is

$\mathrm{Re}\,\gamma(0) \approx \frac{1}{2\tau}$, $\mathrm{Re}\,\gamma(0) = \Delta\omega_\varepsilon \Rightarrow$ bizonytalanság

két jelentése $\mathrm{Re}\,\gamma(0)$-nak $\Rightarrow$ természetes vonalszélesség

$$\Delta\omega_\varepsilon \tau \approx \frac{1}{2} \Rightarrow \hbar\Delta\omega_\varepsilon = \Delta(E_g - E_a)$$

$$\Rightarrow \Delta E_g \tau \approx \frac{\hbar}{2}$$ (energia-idő határozatlansági reláció)

be lehet vezetni a félértékszélességet:

$$\frac{1}{\tau} = \gamma = 2\mathrm{Re}\,\gamma(0)$$

$$|c_{\varepsilon\alpha}^{(\infty)}|^2 = |g_{21}^\alpha|^2 \frac{1}{(\omega_\varepsilon-\omega_0)^2+\frac{\gamma^2}{4}}$$

időegységre jutó átmeneti valószínűséget lehet definiálni $\frac{W_{fi}}{\tau} \sim |c_{\varepsilon\alpha}(\infty)|^2$

$$\frac{w_{fi}}{t} = w_{fi}\gamma = 2|g_{21}^{\alpha}|^2 \frac{\frac{\gamma}{2}}{(\omega_\varepsilon-\omega_0)^2+\frac{\gamma^2}{4}} \xrightarrow[\gamma\to 0]{} 2\pi\,\delta(\omega_\varepsilon-\omega_0)|g_{21}|^2$$

$$\left.\begin{aligned} &\int d\omega_\varepsilon \frac{\frac{\gamma}{2}}{(\omega_\varepsilon-\omega_0)^2+\frac{\gamma^2}{4}} = \pi \\ &\lim_{\gamma\to 0} \frac{1}{\pi}\frac{\frac{\gamma}{2}}{(\omega_\varepsilon-\omega_0)^2+\frac{\gamma^2}{4}} = \delta(\omega_\varepsilon-\omega_0)\end{aligned}\right\}$$

$$\Downarrow$$

$*$

$*$ pont ezt számoltuk ki a spontán emisszióra is

pontos rezonáns abszorbció: $\frac{w_{abs}}{t} \to 2|g_{21}|^2\frac{2}{\gamma}n_\varepsilon - *$

mostmár véges értéke, nem robban fel

$$\frac{1}{\gamma} = \frac{3\pi\varepsilon_0 c^3}{\omega_0^3|d_{21}|^2}, \quad |g_{21}|^2 = \frac{|e_\varepsilon^\alpha d_{21}|^2\omega_\varepsilon}{2\varepsilon_0\hbar V} \xrightarrow[\text{átlagolás}]{} \frac{1}{3}|d_{21}|^2\frac{\omega_\varepsilon}{2\varepsilon_0\hbar V}$$

a lézerre ez lesz a jó képlet: $* = \frac{4\pi c^3}{V\omega_0^2}n_\varepsilon = G n_\varepsilon$

V = aktív térfogat

ω_0 = működési frekvencia

$P_{abszorbció} = G n_\varepsilon$

$P_{emisszió} = G(n_\varepsilon+1)$

Kaotikus és koherens fény statisztikája

Kaotikus: $p_n(n\text{ foton}) = (1-e^{-\beta\hbar\omega})e^{-\beta n\hbar\omega}$

$$T=\frac{1}{\beta} \quad k_B = 1$$

$$\langle n\rangle = \sum_n p_n n = \sum_n (1-e^{-\beta\hbar\omega})\left(-\frac{d}{d(\beta\hbar\omega)}\right)e^{-\beta n\hbar\omega} =$$

$$= (1-e^{-\beta\hbar\omega})\left(-\frac{d}{d(\beta\hbar\omega)}\right)\underbrace{\sum_n e^{-\beta n\hbar\omega}}_{\frac{1}{1-e^{-\beta\hbar\omega}}} = \frac{1}{1-e^{-\beta\hbar\omega}}e^{-\beta\hbar\omega} =$$

$$= \frac{1}{e^{\beta\hbar\omega}-1}$$

$$\langle n^2\rangle - \langle n\rangle^2 = \langle n\rangle^2 + \langle n\rangle = \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$$

Koherens:

$$\hat{a}^{\epsilon}_{\underline{k}} |\alpha(\underline{k},\epsilon)\rangle = \alpha(\underline{k},\epsilon)|\alpha(\underline{k},\epsilon)\rangle$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n^{\epsilon}_{\underline{k}}\rangle \to$$ határozott foton-számú állapotokból

$$\langle n^{\alpha}_{\underline{k}} | n^{\alpha}_{\underline{k}}\rangle = 1$$

$$c_n (\hat{a}^{+\alpha}_{\underline{k}})^n |0\rangle$$

$$\langle \{\alpha^{\epsilon}_{\underline{k}}\} | \hat{\underline{A}}(\underline{x},t) | \{\alpha^{\epsilon}_{\underline{k}}\}\rangle =$$
$$= \sum_{\underline{k},\epsilon}\sqrt{\frac{\hbar}{2\varepsilon_0 V\omega_{\underline{k}}}}\left(\alpha^{\epsilon}_{\underline{k}} e^{i\underline{k}\underline{x}-i\omega_{\underline{k}}t} + \alpha^{\epsilon *}_{\underline{k}} e^{-i\underline{k}\underline{x}+i\omega_{\underline{k}}t}\right)\underline{\epsilon}_{\underline{k}}$$

$$p_n = |\langle n^{\epsilon}_{\underline{k}} | \alpha^{\epsilon}_{\underline{k}}\rangle|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$$ Poisson - eloszlás $|\alpha|^2$ paraméterrel

$$\langle n\rangle = |\alpha|^2, \quad \langle n^2\rangle - \langle n\rangle^2 = \langle n\rangle$$

c_n-ek: $\langle 0|(a^+)^n (a^+)^n|0\rangle = n! \Rightarrow c_n = \sqrt{n!}$

$$|\alpha\rangle = \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle e^{-\frac{1}{2}|\alpha|^2}$$

$$\langle n|m\rangle = \delta_{nm}$$

$$\langle\alpha|\alpha\rangle = \sum_{n,m}^{\infty}\frac{\alpha^n\alpha^m}{\sqrt{n!\,m!}}\langle m|n\rangle e^{-|\alpha|^2} = \sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2} = 1$$

$$a(a^+)^n|0\rangle = ([a,a^+]+a^+a)(a^+)^{n-1}|0\rangle =$$
$$= 1(a^+)^{n-1}|0\rangle + a^+([a,a^+]+a^+a)(a^+)^{n-2}|0\rangle =$$
$$= 2(a^+)^{n-1}|0\rangle + (a^+)^2 a(a^+)^{n-2}|0\rangle = \ldots =$$
$$= n(a^+)^{n-1}|0\rangle$$

$$a|\alpha\rangle = \alpha|\alpha\rangle = a\left|\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}(a^+)^n|0\rangle e^{-\frac{1}{2}|\alpha|^2}\right\rangle =$$
$$= \sum_{n=1}^{\infty}\alpha^n\frac{1}{(n-1)!}(a^+)^{n-1}|0\rangle e^{-\frac{1}{2}|\alpha|^2} =$$
$$= \alpha\sum_{m=0}^{\infty}\frac{\alpha^m}{m!}(a^+)^m|0\rangle e^{-\frac{1}{2}|\alpha|^2} = \alpha|\alpha\rangle$$:)

$$\langle n\rangle = \langle\alpha|a^+a|\alpha\rangle = |\alpha|^2$$

$$|\{\alpha_{\underline{k}}\}\rangle = \prod_{\underline{k}}\sum_{n_{\underline{k}}=0}^{\infty}\frac{\alpha^{n_{\underline{k}}}}{\sqrt{n_{\underline{k}}!}}|n_{\underline{k}}\rangle e^{-|\alpha|^2}$$ az állapottér $(\epsilon,\underline{k})$

$$\hat{\underline{E}} = -\dot{\hat{\underline{A}}} \qquad \Delta E^{\epsilon}_{\underline{k}}\cdot\Delta A^{\epsilon}_{\underline{k}} = \frac{1}{2}$$ (ideális)

Kétszintes atomi rendszerhez csatolt foton-állapot statisztikája, Részletes egyensúly

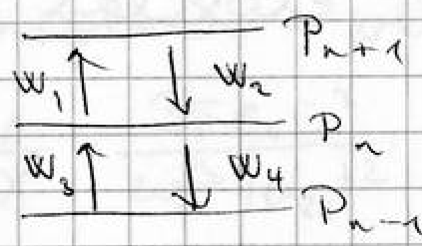
Pontos rezonáns abszorbció időegységre jutó valószínűsége: $G \cdot n$

emisszióra: $G(n+1)$

$$G = \frac{4\pi c^3}{V\omega^2}$$

Kinetikus egyenletet akarunk felírni a fotonszámra:

N_1 alapállapotú, N_2 gerjesztett atom



$$\begin{aligned}\frac{dP_n}{dt} = & -N_2 G (n+1) P_n(t) + && w_1 \\ & + N_1 G (n+1) P_{n+1}(t) + && w_2 \\ & + N_2 G\, n P_{n-1}(t) - && w_3 \\ & - N_1 G\, n P_n(t) && w_4\end{aligned}$$

stacionárius állapotban $\frac{dP_n}{dt} = 0$ feltétel

0 fotonos állapot csak az egy fotonossal van kapcsolatban: $w_1 = w_2 \Rightarrow$ 1 fotonosra feltétel $w_3 = w_4$

$\Rightarrow$ részletes egyensúly elve: n és $n+1$ kommunikációjának valószínűségei legyenek egyensúlyban

$$N_2 P_n = N_1 P_{n+1}$$

$$E_2 - E_1 = \hbar\omega \qquad e^{-\frac{E_2-E_1}{k_B T}} = \frac{N_2}{N_1} = \frac{P_{n+1}}{P_n} = e^{-\frac{\hbar\omega}{k_B T}} = q$$

P_n kifejezhető P_0-lal

$\sum_{n=0}^{\infty} P_n = 1$ teljességi feltételből $P_0 = 1 - \frac{N_2}{N_1}$

$$P_n = \left(1 - \frac{N_2}{N_1}\right) e^{-\frac{n\hbar\omega}{k_B T}}$$

$$\bar{n} = \sum_{n=0}^{\infty} n P_n = P_0 \sum_{n=0}^{\infty} n q^n = P_0\, q \frac{1}{(1-q)^2} = \frac{q}{1-q}$$

$$\frac{d\bar{n}}{dt} = \sum_n n \frac{dP_n}{dt} = -N_2 G(\overline{n^2}+\bar{n}) + N_1 G(\overline{n^2}-\bar{n}) +$$
$$+ N_2 G(\overline{n^2}+2\bar{n}+1) - N_1 G \overline{n^2} = *$$

$$\sum_{n=0}^{\infty} n(n+1)P_n = \overline{n^2} + \bar{n}$$

$$\sum_{n=0}^{\infty} n(n+1)P_{n+1} = \sum_{n=1}^{\infty} n(n+1)P_{n+1} = \sum_{m=0}^{\infty} (m-1)m P_m = \overline{n^2} - \bar{n}$$

$$\sum_{n=0}^{\infty} n^2 P_{n-1} = \sum_{n=1}^{\infty} n^2 P_{n-1} = \overline{n^2} + 2\bar{n} + 1$$

$$\sum_{n=0}^{\infty} n^2 P_n = \overline{n^2}$$

$$* = (N_2 - N_1) G \bar{n} + N_2 G$$

$$\bar{n}(t) = \bar{n}(0) e^{(N_2-N_1)Gt} + \frac{N_2}{N_2-N_1}\left(e^{-(N_2-N_1)Gt} - 1\right)$$

indukált spontán emisszió

ha $N_2 < N_1$, akkor csökkenő jelleg

ha $N_2 > N_1$, populációinverzió

$$\frac{d}{dt}\underbrace{(\overline{n^2} - 2\bar{n}^2 - \bar{n})}_{\sigma} = 2G(N_2-N_1)\underbrace{(\overline{n^2} - 2\bar{n}^2 - \bar{n})}_{\sigma}$$

$$\sigma(t) = \sigma(0) e^{2(N_2-N_1)Gt}$$

ha $\sigma(0) = 0 \Rightarrow \sigma(t) = 0 \ \forall t$

$\sigma = 0$, ha a fény kaotikus

koherens fényre: $\overline{n_0^2} - \bar{n}_0^2 - \bar{n}_0 = 0$

$\sigma(0) = -n_0^2$

$\sigma(t) = -n_0^2 e^{2(N_2-N_1)Gt}$

egyre kevésbé termikus a statisztika időben

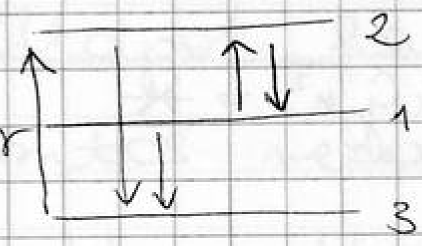
ha koherens a fény és populációinverzió van

$$\bar{n}(t) = \bar{n}(0) e^{(N_2-N_1)Gt} + \frac{N_2}{N_2-N_1}\left(e^{(N_2-N_1)Gt} - 1\right)$$

koherens inkoherens

$$\frac{n \text{ inkoherens}}{n \text{ koherens}} \xrightarrow{t \to \infty} \frac{N_2}{N_2-N_1}\frac{1}{\bar{n}_0}$$

Klasszikus 3 szintes lézer modell



$r \to$ pumpálás

$R_n^\alpha \to n$ foton jelenlétében α állapotban lévő atom valószínűsége

$P(\alpha | n) = \frac{R_n^\alpha}{P_n} \to$ feltételes valószínűség

$$\frac{dR_n^2}{dt} = rP_n - 2\gamma_2 R_n^2 - R_n^2 G(n+1) + R_{n+1}^1 G(n+1) \approx 0$$

$\uparrow$ félszélesség

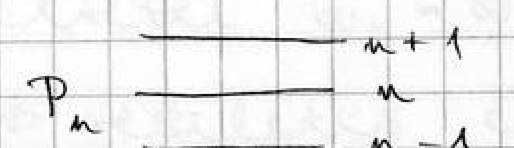
$$\frac{dR_{n+1}^1}{dt} = -2\gamma_1 R_{n+1}^1 + R_n^2 G(n+1) - R_n^1 G(n+1) \approx 0$$

fotondinamika lassú az atomihoz képest $\Uparrow$

$$\begin{pmatrix} 2\gamma_1 + G(n+1) & -G(n+1) \\ -G(n+1) & 2\gamma_2 + G(n+1) \end{pmatrix} \begin{pmatrix} R_{n+1}^1 \\ R_n^2 \end{pmatrix} = \begin{pmatrix} 0 \\ rP_n(t) \end{pmatrix}$$

eredmény: $R_{n+1}^1 = \frac{G(n+1)}{4\gamma_1\gamma_2 + 2(\gamma_1+\gamma_2)G(n+1)} rP_n$

$$R_n^2 = \frac{2\gamma_2 + G(n+1)}{4\gamma_1\gamma_2 + 2(\gamma_1+\gamma_2)G(n+1)} rP_n$$



$N_2 = NR_n^2$

$N_1 = NR_n^1$

átmeneteket ki lehet számolni

$$\frac{dP_n}{dt} = -\frac{\alpha\beta}{\beta+n+1} P_n(n+1) + \frac{\alpha\beta}{\beta+n} P_{n-1}\, n +$$

$$+ CP_{n+1}(n+1) - CP_n\, n$$

$\hookrightarrow$ C a fénykicsatolás miatti technikai állandó

$$\alpha = \frac{r}{2\gamma_2} NG$$

$$\beta = \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)G} \gg 1 \text{ általában}$$

$cP_n = \frac{\alpha\beta}{\beta+n} P_{n-1}$ a stacionaritás (folyamatos üzem) feltétele → rekurzió.

$\frac{\alpha}{c} \gg 1$ általában

közelítőleg $\bar{n} \ll \beta \Rightarrow P_n \approx \frac{\alpha}{c} P_{n-1}$

$\Rightarrow \bar{n} \approx \frac{\frac{\alpha}{c}}{1-\frac{\alpha}{c}}$ → termikus ha $\frac{\alpha}{c} < 1$

de lézernél $\frac{\alpha}{c} \gg 1$

rátától függ

α nő → ~~...~~ megváltozik az eloszlás

$\Rightarrow P_n \approx \left(\frac{\alpha\beta}{c}\right)^{n+\beta} \frac{1}{(n+\beta)!} e^{-\frac{\alpha\beta}{c}}$

$$(n+\beta)! = \beta(\beta+1)(\beta+2) \dots (\beta+n)$$

ha $\bar{n} \gg \beta$, akkor Poisson-eloszlásba megy

$\bar{n} = \left(\frac{\alpha}{c} - 1\right)\beta$ így lehet a határeseteket nézni → α és c fv-eben lehet nézni azt, ahol a termikus eloszlás átmegy Poisson-eloszlásba → lézerküszöb $\frac{\alpha}{c} \sim 1$