STATFIZ.B HF I.

5.

$$\Omega_0 = \frac{a^{dN}}{h^{dN}} \frac{(2mE)^{\frac{dN}{2}}}{N!} \cdot \frac{\pi^{\frac{dN}{2}}}{\Gamma\left(\frac{dN}{2}+1\right)}$$

a.) $d=1$, $N=3$

$$\Omega_0 = \frac{a^3}{h^3} \frac{(2mE)^{3/2}}{3!} \frac{\pi^{\frac{3}{2}}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{a^3}{h^3} \frac{(2mE)^{3/2}}{6} \frac{\pi^{3/2}}{3/4\sqrt{\pi}} =$$

$$= \frac{a^3}{h^3} (2mE)^{3/2} \cdot \frac{2}{9} \pi$$

$\Gamma(5/2)$

$\Gamma(1/2) = \sqrt{\pi}$

$\Gamma\left(\frac{1}{2}+n\right) = \frac{2n!}{4^n n!}\sqrt{\pi}$

$\Gamma\left(\frac{1}{2}+2\right) = \frac{4!}{4^2 2!}\sqrt{\pi}$

b.) $d=2$, $N=2$

$$\Omega_0 = \frac{a^4}{h^4} \frac{(2mE)^2}{2} \cdot \frac{\pi^2}{\underbrace{\Gamma(2+1)}} = \frac{a^4}{h^4} \frac{(2mE)^2}{2} \frac{\pi^2}{2!} =$$

$\Gamma(n+1) = n!$

$$= \frac{a^4}{h^4} \frac{(2mE)^2}{4} \pi^2 = \frac{a^4}{h^4} (mE\pi)^2$$

8.

$$E = \frac{\hbar\omega N}{2} + \hbar\omega M \Rightarrow \Omega(N,M) = \frac{(N-1+M)!}{(N-1)!\, M!}$$

$$S = k_B \ln \Omega(N,M) = k_B \Big[(N-1+M)\ln(N-1+M) - (N-1+M) -$$
$$- (N-1)\ln(N-1) + (N-1) - M\ln M + M \Big]$$

Ez mennyire különbözik az előző (klasszikus) eredményektől?

$(\log(n!) = n \cdot \ln - n) \rightarrow \ln(\Omega)$?

$$\log\left[(N-1+M)! - (N-1)!\,M!\right] = \Big[(N-1+M)\cdot\ln(N-1+M) - (N-1+M)$$

$$-\Big[\big((N-1)\cdot\ln(N-1) - (N-1)\big) + \big(M\cdot\ln(M) - M\big)\Big]\Big] =$$

$$= \Big[(N-1+M)\cdot\ln(N-1+M) - N + 1 - M - (N-1)\ln(N-1) + N - 1$$

$$- M\ln(M) + M\Big] = \underbrace{\Big[(N-1+M)\ln(N-1+M) - (N-1)\ln(N-1) - M\ln(M)\Big]}_{N \text{ és } M \gg 1 \text{, ezért:}}$$

$$\log(\Omega) \cong \Big[(N+M)\ln(N+M) - N\ln N - M\ln M\Big] = \quad \text{bővítünk}$$

$$= \underbrace{\Big[(N+M)\ln(N+M) - N\ln N \underbrace{- M\ln \overbrace{M + M\ln N}^{-M\ln\left(\frac{M}{N}\right)} - M\ln N}_{-(N+M)\cdot\ln N}}_{(N+M)\ln\left(\frac{N+M}{N}\right)}\Big] =$$

$$= \Big[(N+M)\ln\left(\frac{N+M}{N}\right) - M\ln\left(\frac{M}{N}\right)\Big]$$

tudjuk, hogy $E = \frac{\hbar\omega N}{2} + \hbar\omega M \Rightarrow \frac{E}{\hbar\omega} = \frac{1}{2}N + M$

$$\begin{cases} M = \frac{E}{\hbar\omega} - \frac{1}{2}N \\ N = 2\left(\frac{E}{\hbar\omega} - M\right) \end{cases}$$

$$M+N = \frac{E}{\hbar\omega} - \frac{1}{2}N + \frac{2E}{\hbar\omega} - 2M =$$

$$= \frac{3E}{\hbar\omega} - \frac{1}{2}N - 2M = \frac{3E}{\hbar\omega} - \frac{1}{2}N - \frac{2E}{\hbar\omega} + N = \frac{E}{\hbar\omega} + \frac{1}{2}N$$

$$M/N = \frac{E/\hbar\omega - \frac{1}{2}N}{\frac{2E}{\hbar\omega} - 2M} = \frac{E/\hbar\omega - \frac{1}{2}N}{\cancel{\frac{2E}{\hbar\omega}} - \cancel{\frac{2E}{\hbar\omega}} + N} = \frac{E/\hbar\omega - \frac{1}{2}N}{+N} = \frac{+E}{\hbar\omega N} - \frac{1}{2}$$

$$\log\Omega = \left[\left(\frac{E}{\hbar\omega} + \frac{1}{2}N\right)\cdot\ln\left(\frac{E}{\hbar\omega N} + \frac{1}{2}\right) - \left(\frac{E}{\hbar\omega} - \frac{1}{2}N\right)\cdot\ln\left(\frac{E}{\hbar\omega N} - \frac{1}{2}\right)\right]$$

$$S = k_B \cdot \ln \Omega = k_B \cdot N \cdot \left[\left(\frac{E}{\hbar\omega N} + \frac{1}{2} \right) \ln\left(\frac{E}{\hbar\omega N} + \frac{1}{2} \right) - \left(\frac{E}{\hbar\omega N} - \frac{1}{2} \right) \ln\left(\frac{E}{\hbar\omega N} - \frac{1}{2} \right) \right]$$

A klasszikus eredmény (4-es feladat): $S = k_B N \ln\left(\frac{E/N}{\hbar\omega} \right)$

Azért kellenek extra tagok, hogy kvantumos esetben figyelembe vegyük a ~~t~~ zérusponti energiát.

STATFIZ. B. HF 2.

1 / fajhő?

$$\langle E\rangle = (\varepsilon_2-\varepsilon_1)\frac{e^{-\beta(\varepsilon_2-\varepsilon_1)}}{1+e^{-\beta(\varepsilon_2-\varepsilon_1)}} \to C = \frac{-1}{kT^2}\frac{\partial\langle E\rangle}{\partial\beta} = ?$$

$\Delta\varepsilon = \varepsilon_2-\varepsilon_1$ bevezetése

$$C = \frac{-1}{kT^2}\frac{\partial}{\partial\beta}\left[\Delta\varepsilon\cdot\frac{e^{-\beta\Delta\varepsilon}}{1+e^{-\beta\Delta\varepsilon}}\right] =$$

$$= \frac{-\Delta\varepsilon}{kT^2}\left[\frac{-\Delta\varepsilon\, e^{-\beta\Delta\varepsilon}\cdot(1+e^{-\beta\Delta\varepsilon}) - e^{-\beta\Delta\varepsilon}\cdot -\Delta\varepsilon\, e^{-\beta\Delta\varepsilon}}{(1+e^{-\beta\Delta\varepsilon})^2}\right] =$$

$$= \frac{-\Delta\varepsilon}{kT^2}\cdot -\Delta\varepsilon\, e^{-\beta\Delta\varepsilon}\left[\frac{(1+\cancel{e^{-\beta\Delta\varepsilon}})-\cancel{e^{-\beta\Delta\varepsilon}}}{(1+e^{-\beta\Delta\varepsilon})^2}\right] =$$

$$= \frac{\Delta\varepsilon^2}{kT^2}\frac{e^{-\beta\Delta\varepsilon}}{(1+e^{-\beta\Delta\varepsilon})^2} = \frac{\Delta\varepsilon^2}{kT^2}\frac{e^{-\beta\Delta\varepsilon}}{1+2e^{-\beta\Delta\varepsilon}+e^{-2\beta\Delta\varepsilon}}\cdot\frac{e^{\beta\Delta\varepsilon}}{e^{\beta\Delta\varepsilon}} =$$

$$= \frac{\Delta\varepsilon^2}{kT^2}\frac{1}{e^{\beta\Delta\varepsilon}+2+e^{-\beta\Delta\varepsilon}}\left(\operatorname{ch} x = \frac{e^x+e^{-x}}{2}\right) = \boxed{\frac{\Delta\varepsilon^2}{kT^2}\frac{1}{2\operatorname{ch}(\beta\Delta\varepsilon)+2}}$$

2. 3 állapotú rendszer: $E_0=0$, $E_1=\varepsilon_1$, $E_2=\varepsilon_2$.

$Z=?$, $\langle E\rangle = ?$, $C_{(T)}=?$

nincs degeneráció: 1

$$Z=\sum_i \Omega(E_i)\cdot e^{-\beta E_i} = e^{-\beta 0}+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2} = 1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}$$

$$\langle E\rangle = \sum_i \frac{E_i\,\Omega(E_i)e^{-\beta E_i}}{Z} = \frac{0\cdot 1\cdot e^{-\beta 0}+\varepsilon_1\cdot 1\cdot e^{-\beta\varepsilon_1}+\varepsilon_2\cdot 1\cdot e^{-\beta\varepsilon_2}}{1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}} =$$

$$= \frac{\varepsilon_1\cdot e^{-\beta\varepsilon_1}+\varepsilon_2\cdot e^{-\beta\varepsilon_2}}{1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}}$$

$$C(T) = \frac{-1}{kT^2}\frac{d\langle E\rangle}{d\beta} = \qquad \frac{f'\cdot g - f\cdot g'}{g^2}$$

$$= \frac{-1}{kT^2}\cdot \frac{\left(-\varepsilon_1^2\cdot e^{-\beta\varepsilon_1} - \varepsilon_2^2 e^{-\beta\varepsilon_2}\right)\left(1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}\right) - \left[\left(\varepsilon_1\cdot e^{-\beta\varepsilon_1}+\varepsilon_2 e^{-\beta\varepsilon_2}\right)\cdot\left(-\varepsilon_1\cdot e^{-\beta\varepsilon_1} - \varepsilon_2 e^{-\beta\varepsilon_2}\right)\right]}{\left(1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}\right)^2} =$$

$$= \frac{-1}{kT^2}\,\frac{\left(-\varepsilon_1^2 e^{-\beta\varepsilon_1} - \cancel{\varepsilon_1^2 e^{-2\beta\varepsilon_1}} - \varepsilon_1^2 e^{-\beta(\varepsilon_1+\varepsilon_2)} - \varepsilon_2^2 e^{-\beta\varepsilon_2} - \varepsilon_2^2 e^{-\beta(\varepsilon_2+\varepsilon_1)} - \cancel{\varepsilon_2^2 e^{-2\beta\varepsilon_2}}\right) \; (\dots) \; \ominus\left(\cancel{+\varepsilon_1^2 e^{-2\beta\varepsilon_1}} + \varepsilon_1\varepsilon_2 e^{-\beta(\varepsilon_1+\varepsilon_2)} + \varepsilon_2\varepsilon_1 e^{-\beta(\varepsilon_2+\varepsilon_1)} + \cancel{\varepsilon_2^2 e^{-2\beta\varepsilon_2}}\right)}{e^{-2\beta\varepsilon_2} + 2e^{-\beta\varepsilon_2} + 2e^{\beta(-\varepsilon_2)-\varepsilon_1\beta} + e^{-2\varepsilon_1\beta} + 2e^{\varepsilon_1(-b)} + 1} =$$

$$= \frac{-1}{kT^2}\left[\frac{-\varepsilon_1^2 e^{-\beta\varepsilon_1} - \varepsilon_1^2 e^{-\beta(\varepsilon_1+\varepsilon_2)} - \varepsilon_2^2 e^{-\beta\varepsilon_2} - \varepsilon_2^2 e^{-\beta(\varepsilon_2+\varepsilon_1)} + 2\cdot\varepsilon_1\varepsilon_2 e^{-\beta(\varepsilon_1+\varepsilon_2)}}{e^{-2\beta\varepsilon_2} + 2e^{-\beta\varepsilon_2} + 2e^{-\beta(\varepsilon_1+\varepsilon_2)} + e^{-2\varepsilon_1\beta} + 2e^{-\varepsilon_1\beta} + 1}\right] =$$

neu sezt

$$\underbrace{-\varepsilon_1^2 e^{-\beta(\varepsilon_1+\varepsilon_2)} - \varepsilon_2^2 e^{-\beta(\varepsilon_2+\varepsilon_1)} + 2\cdot\varepsilon_1\varepsilon_2 e^{-\beta(\varepsilon_1+\varepsilon_2)}}$$

$$\left(2\varepsilon_1\varepsilon_2 - \varepsilon_1^2 - \varepsilon_2^2\right) e^{-\beta(\varepsilon_2+\varepsilon_1)} = -(\varepsilon_1 - \varepsilon_2)^2 e^{-\beta(\varepsilon_2+\varepsilon_1)}$$

$$= \frac{-1}{kT^2}\left[\frac{-\varepsilon_1^2 e^{-\beta\varepsilon_1} - \varepsilon_2^2 e^{-\beta\varepsilon_2} - (\varepsilon_1-\varepsilon_2)^2 e^{-\beta(\varepsilon_2+\varepsilon_1)}}{\left(1+e^{-\beta\varepsilon_1}+e^{-\beta\varepsilon_2}\right)^2}\right]$$

STATFIZ.B. HF 3

⑥ 2 független, 3 állapotú rendszerből álló rendszer. Mikroállapotok? Ezek megtalálási valószínűségei T-n? Állapotösszeg? Összefügg az 1 db 3 állapotú rendszerével? Hotel? Trivago!

- 3 állapot : $-\varepsilon, 0, \varepsilon$ energiákkal

2 rendszer:

⊖	⊖	-2ε	$\to \Omega(-2\varepsilon)=1$
⊕	⊕	2ε	$\to \Omega(2\varepsilon)=1$
0	0	0	$\to \Omega(0)=1$
⊖	⊕	$-\varepsilon+\varepsilon=0$	$\Omega(-\varepsilon+\varepsilon)=2$
⊕	⊖	$\varepsilon+-\varepsilon=0$	
⊖	0	$-\varepsilon$	$\Omega(-2\varepsilon)=2$
0	⊖	$-\varepsilon$	
⊕	0	ε	$\Omega(2\varepsilon)=2$
0	⊕	ε	

- megtalálási valószínűségek: $P_n = \frac{1}{Z} e^{-\beta \varepsilon_n}$

$$P_1 \sim e^{\beta 2\varepsilon}$$

$$P_2 \sim e^{-\beta 2\varepsilon}$$

$$P_3 \sim 1$$

$$P_5 = P_4 \sim 1$$

$$P_7 = P_6 \sim e^{+\beta\varepsilon}$$

$$P_8 = P_9 \sim e^{-\beta\varepsilon}$$

- állapotösszeg: $Z_N = \sum_n \Omega(E_n) \cdot e^{-\beta E_n}$

1 rendszer esetén: $1 \cdot e^{-\beta \cdot 0} + 1 \cdot e^{-\beta \varepsilon} + 1 \cdot e^{\beta \varepsilon}$

$$Z_1 = 1 + e^{-\beta \varepsilon} + e^{\beta \varepsilon}$$

2 rendszer esetén: $Z_2 = 1 \cdot e^{\beta 2 \varepsilon} + 1 \cdot e^{-\beta 2 \varepsilon} + 1 \cdot e^{-\beta \cdot 0} +$

$$+ 2 \cdot e^{-\beta(-\varepsilon + \varepsilon)} + 2 \cdot e^{\beta 2 \varepsilon} + 2 \cdot e^{-\beta 2 \varepsilon} =$$

$$= e^{\beta 2 \varepsilon} + e^{-\beta 2 \varepsilon} + 3 + 2 e^{\beta 2 \varepsilon} + 2 e^{-\beta 2 \varepsilon}$$

$$Z_1^2 = 1 + e^{-2\beta\varepsilon} + e^{2\beta\varepsilon} + 2e^{-\beta\varepsilon} + 2e^{\beta\varepsilon} + 2e^{\beta\varepsilon + -\beta\varepsilon} =$$

$$= 1 + e^{-2\beta\varepsilon} + e^{2\beta\varepsilon} + 2e^{-\beta\varepsilon} + 2e^{\beta\varepsilon} + 2$$

$\downarrow$

ez alapján $Z_2 = Z_1^2$, vagyis teljesül az órán levezetett azonosság: $Z_N = Z_1^N$ független rendszerek esetén.

9. N db független részecske van. 3 energiaszinten lehetnek: $0, \varepsilon, 2\varepsilon$, ezek multiplicitása $1, 3, 5$.

$E(T) = ?$

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln(Z)$$ órán vettük

$$Z_N = (Z_1)^N = \left(\Omega(0)\cdot e^{-\beta\cdot 0} + \Omega(\varepsilon)\cdot e^{-\beta\cdot\varepsilon} + \Omega(2\varepsilon)\cdot e^{-\beta 2\varepsilon}\right)^N =$$

$$= \left(1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}\right)^N$$

$$\langle E \rangle = -\frac{\partial}{\partial\beta}\left[\ln\left(\left(1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}\right)^N\right)\right] =$$

$$= -N\cdot\frac{\partial}{\partial\beta}\left[\ln\left(1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}\right)\right] =$$

$$= -N\cdot\frac{1}{1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}}\cdot\frac{\partial}{\partial\beta}\left[1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}\right] =$$

$$= \frac{-N}{1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}}\cdot\left[0 - 3\varepsilon\cdot e^{-\beta\varepsilon} - 10\varepsilon\, e^{-\beta 2\varepsilon}\right] =$$

$$= \frac{N\left(3\varepsilon e^{-\beta\varepsilon} + 10\varepsilon\cdot e^{-\beta 2\varepsilon}\right)}{1 + 3e^{-\beta\varepsilon} + 5\cdot e^{-\beta 2\varepsilon}} = \boxed{\frac{N\varepsilon\cdot\left(3 + 10e^{-\beta\varepsilon}\right)}{e^{\beta\varepsilon} + 3 + 5e^{-\beta\varepsilon}}}$$

$\beta = \frac{1}{k_B T}$, ezért, ha csak β-tól függ, akkor T-től függ.

STATFIZ. B. HF IV.

(1.) $T_f = ?$ konkrét számértékekkel?

$\Theta = 4.59 \cdot 10^{-48}\ kg\,m^2$, $\hbar = 1.055 \cdot 10^{-34}\ Js$, $k = 1.38 \cdot 10^{-23}\ J/K$

$$T_f = \frac{\hbar^2}{\Theta \cdot k} = \frac{(1.055 \cdot 10^{-34}\ J \cdot s)^2}{4.59 \cdot 10^{-48}\ kgm^2 \cdot 1.38 \cdot 10^{-23}\ J/K} = \boxed{175.7167\ K}$$

(3.) $T_r = ?$

$\omega = 4.2 \cdot 10^{14}\ \frac{1}{s}$

$$T_r = \frac{\hbar\omega}{k} = \frac{1.055 \cdot 10^{-34}\ Js \cdot 4.2 \cdot 10^{14}\ \frac{1}{s}}{1.38 \cdot 10^{-23}\ J/K} = \boxed{3210.8696\,K}$$

(4.) Külső elektronok esetén $E_{kötési} = 0.1\ eV \rightarrow T_{ion}$?

Belső elektronoknál $E_{kötési} = 30\ eV \rightarrow T_{ion.\ teljes}$?

Protonok, neutronok : $E_{kötési} \approx 1\ MeV$ → Van esélyünk megfigyelni?

$k_B = 8.6 \cdot 10^{-5}\ eV/K$

$$E \approx \hbar\omega \approx kT$$

$$T_{e_1} = \frac{E_{e_1}}{k} = \frac{0.1\ eV}{8.6 \cdot 10^{-5}\ eV/K} = 1162.7907\ K$$

$$T_{e_2} = \frac{30\ eV}{8.6 \cdot 10^{-5}\ eV/K} = 348837.2093\ K$$

$$T_{n,p} = \frac{1 \cdot 10^{6}\ eV}{8.6 \cdot 10^{-5}\ eV/K} = 1.1628 \cdot 10^{10}\ K \rightarrow \text{kollapszár}$$

szupernóva neutroncsillag magja

(vagy CERN)

STATFIZ B. V. HF

(1.) Állapotszám: $\Omega_{0(p)} = \dfrac{V 4\pi p^3}{h^3}$ (3D-s gömb)

Diszperziós reláció: $\varepsilon(p) \to \dfrac{p^2}{2m}$, ha $\varepsilon \ll mc^2$

$\varepsilon(p) \to c\sqrt{p^2+m^2c^2} - mc^2$, ha $\varepsilon \gg mc^2$

Mi a $\rho(\varepsilon) = \dfrac{d\Omega_0}{d\varepsilon}$ állapotsűrűség?

ultrarelativisztikus

$\varepsilon \gg mc^2$

$\varepsilon_{kin} = c \cdot p$

klasszikus

$\varepsilon \ll mc^2$

$\varepsilon_{kin} = \dfrac{p^2}{2m}$

$$\varepsilon = c\sqrt{p^2+m^2c^2} - mc^2 \to p = \sqrt{\left(\frac{\varepsilon+mc^2}{c}\right)^2 - m^2c^2} = \sqrt{\frac{\varepsilon^2+2\varepsilon mc^2}{c^2}}$$

$$\Omega_{0(p)} = V \cdot \frac{4\pi}{3}\frac{p^3}{h^3}$$

$$\Omega_0 = V \cdot \frac{4\pi}{3}\frac{1}{h^3}\left(\frac{\varepsilon^2+2\varepsilon mc^2}{c^2}\right)^{3/2}$$

$$\Omega_0 = V\frac{4\pi}{3}\frac{\varepsilon^3}{(hc)^3}$$

$$\rho(\varepsilon) = \frac{d\Omega_0}{d\varepsilon}$$

m.o.

$$\Omega_0 = V\frac{4}{3}\pi\frac{(2\varepsilon m)^{3/2}}{h^3}$$

$$\rho(\varepsilon) = \frac{d\Omega_0}{d\varepsilon}$$

m.o.

2 dimenzió: $p^2\pi \rightarrow \Omega_{0(p)} = \frac{A\cdot p^2\pi}{h^2} = \frac{A\pi}{h^2}\left(\frac{\varepsilon^2+2\varepsilon mc^2}{c^2}\right)$

$$\Omega_0 = \frac{a^{dN}}{h^{dN}}\frac{(2me)^{\frac{dN}{2}}}{N!}\cdot\frac{\pi^{\frac{dN}{2}}}{\Gamma\left(\frac{dN}{2}+1\right)}$$

$(A=a^2,\ L=a)$

- $\varepsilon \ll mc^2$: klasszikus

$$\Omega_0 = \frac{A\pi}{h^2}\cdot\frac{2\varepsilon mc^2}{c^2} = \frac{A\pi}{h^2}2\varepsilon m$$

$$\rho(\varepsilon) = \frac{d\Omega_0}{d\varepsilon} = \frac{d}{d\varepsilon}\left(\frac{A\pi}{h^2}2\varepsilon m\right) = \boxed{\frac{A\pi}{h^2}2m}$$

- $\varepsilon \gg mc^2$: ultrarelativisztikus

$$\Omega_0 = \frac{A\pi}{h^2}\frac{\varepsilon^2}{c^2}$$

$$\rho(\varepsilon) = \frac{d}{d\varepsilon}\left(\frac{A\pi}{h^2}\frac{\varepsilon^2}{c^2}\right) = \boxed{\frac{2A\pi}{h^2c^2}\varepsilon} \sim \varepsilon$$

1 dimenzió: $p \rightarrow \Omega_{0(p)} = \frac{L\cdot p}{h} = \frac{L}{h}\left(\frac{\varepsilon^2+2\varepsilon mc^2}{c^2}\right)^{1/2}$

- $\varepsilon \ll mc^2$: klasszikus

$$\Omega_0 = \frac{L}{h}(2\varepsilon m)^{1/2}$$

$$\rho(\varepsilon) = \frac{d}{d\varepsilon}\left(\frac{L}{h}\sqrt{2\varepsilon m}\right) = \frac{L}{h}\frac{1}{2}\frac{1}{\sqrt{2\varepsilon m}}\cdot 2m =$$

$$= \boxed{\frac{L}{h}\frac{m}{\sqrt{2\varepsilon m}}} \sim \varepsilon^{-1/2}$$

- $\varepsilon \gg mc^2$: ultrarel.

$$\Omega_0 = \frac{L}{h}\left(\frac{\varepsilon^2}{c^2}\right)^{1/2} = \frac{L}{hc}\sqrt{\varepsilon^2} = \frac{L}{hc}\varepsilon \quad (c \text{ és } \varepsilon > 0)$$

$$\rho(\varepsilon) = \frac{d}{d\varepsilon}\left(\frac{L}{hc}\varepsilon\right) = \boxed{\frac{L}{hc}}$$

(2.) Energiaintegrál?

$$E = \int_0^\infty \varepsilon \cdot \rho(\varepsilon) \cdot e^{-\beta(\varepsilon-\mu)} \, d\varepsilon = (\dots)$$

$$\rho(\varepsilon) = V\, 2\pi \frac{\sqrt{2m}^3}{h^3} \sqrt{\varepsilon}$$

$$(\dots) = \int_0^\infty \varepsilon \cdot V 2\pi \frac{\sqrt{2m}^3}{h^3} \sqrt{\varepsilon} \cdot e^{-\beta(\varepsilon-\mu)} \, d\varepsilon =$$

$$= V 2\pi \frac{\sqrt{2m}^3}{h^3} e^{\beta\mu} \cdot \underbrace{\int_0^\infty \underbrace{\varepsilon \cdot \sqrt{\varepsilon}}_{\varepsilon^{3/2}} \cdot e^{-\beta\varepsilon} \, d\varepsilon}_{} =$$

$$\int_0^\infty x^n \cdot e^{-ax} \, dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

$$= V 2\pi \frac{\sqrt{2m}^3}{h^3} e^{\beta\mu} \cdot \left[\frac{\Gamma(5/2)}{\beta^{5/2}} \right] = \quad \Gamma(5/2) = 3/4 \sqrt{\pi}$$

$$= V 2\pi \frac{\sqrt{2m}^3}{h^3 \beta^{3/2}} e^{\beta\mu} \cdot \frac{3}{4} \sqrt{\pi} = \boxed{V \frac{3}{2} \frac{(2\pi m)^{3/2}}{h^3 \beta^{3/2}} e^{\beta\mu}}$$

③ Fermi hőmérséklet?

$$T_{fermi} = \frac{E_{Fermi}}{k_B} \quad , \text{ ha } \frac{N}{V} = 10^{22} \frac{1}{cm^3} = 10^{28} \frac{1}{m^3}$$

$$N_1 = V 4\pi \frac{\sqrt{2m}^3}{h^3} \frac{2}{3} \mu_0^{3/2} \quad (\text{ klassz.})$$

$$\mu_0 = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

$$N_2 = 8V\pi \frac{\mu_0^3}{h^3 c^3} \quad (\text{ relat.})$$

$$\mu = \hbar c \left(3\pi^2 \frac{N}{V}\right)^{1/3}$$

óráról tudjuk

$$\mu_{01} = \frac{(1.0546 \cdot 10^{-34} \, Js)^2}{2 \, m_e} \left(3\pi^2 \, 10^{28} \frac{1}{m^3}\right)^{2/3} = \frac{1}{m_e} 2.44 \cdot 10^{-49} =$$

$$= \frac{1}{9.109 \cdot 10^{-31}} \cdot 2.47 \cdot 10^{-49} = 2.712 \cdot 10^{-19}$$

$$\mu_{02} = 1.05457 \cdot 10^{-34} \, Js \cdot 3 \cdot 10^8 \left(3\pi^2 \cdot 10^{28} \frac{1}{m^3}\right)^{1/3} = 2.1086 \cdot 10^{-16}$$

$$T_1 = \frac{2.712 \cdot 10^{-19}}{k_B} = 19649.3 \text{ K}$$

$$T_2 = \frac{2.1086 \cdot 10^{-16}}{k_B} = 15279710.14 \text{ K}$$

Nem hiszem, hogy van rá esély. Ha már olyan körülmények uralkodnak, hogy elektrongáz van, akkor elég valószínűtlen, hogy klasszikus, és ideális is legyen. (pl.: fehér törpék)