

Relativitáselmélet tételsor

1. Tér-idő alapfogalmai. Ívhossz, sajátidő. Lorentz-transzformáció. Sebességek összeadása
2. Négyes formalizmus. Kontra- és kovariáns komponensek, metrikus tenzor. Lorentz transzformáció mátrixa. Négyes vektorok és tenzorok. Differenciálope-rátorok.
3. Kinematika. Négyessebesség, négyesgyorsulás, egyenletesen gyorsuló mozgás.
4. Tömegpont relativisztikus leírása. Hatás, Lagrange- és Hamilton függvény. Négyesimpulzus, mozgásegyenlet kovariáns alakja.
5. Részecskék bomlása, ütközése. Rugalmas szórás. Rugalmatlan ütközés.
6. Impulzusmomentum. Tömegközéppont relativisztikus definíciója.
7. Nulla tömegű részecskék. Doppler-effektus.
8. Töltött részecske mozgása elektromágneses térben. Hatás, Lagrange- és Hamilton függvény. Kovariáns mozgásegyenletek.
9. Térerősség tenzor. Térerősségek transzformációja, invariánsai.
10. Maxwell egyenletek kovariáns alakja. Mértékszabadság. Mozgó töltés elektromágneses tere.
11. Klasszikus térelmélet. Lagrange-sűrűség fogalma, téregyenletek. Energia-impulzus tenzor.
12. Elektromágneses mező hatásintegrálja, Lagrange-sűrűsége, energia-impulzus tenzora. Megmaradási törvények.
13. Klein-Gordon egyenlet és síkhullám megoldásai. Dirac egyenlet szabad részecskére.
14. Dirac egyenlet síkhullám megoldásai. Helicitás sajátállapotok.
15. Dirac egyenlet megoldása centrális potenciál esetén. Hidrogénszerű atomok spektruma.

Javasolt irodalom:

- Landau-Lifszitz: Eléleti Fizika II. Klasszikus erőterek
- Jackson: Klasszikus elektrodinamika
- Greiner: Relativistic Quantum Mechanics

Kate Gerdor

Jordan II. Klassikus ent'et'el

Jackson Klassikus elektrodinamika

Greiner: Relativistic Quantum Mechanics

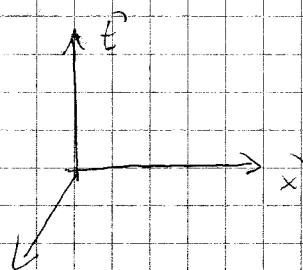
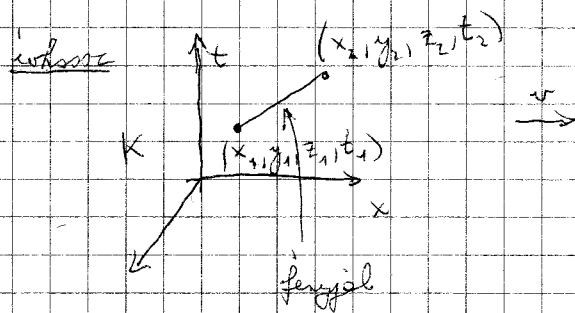
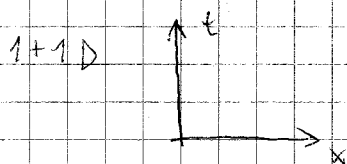
Relativitás' elve: a fizika törvényei minden inerciarendszerben ugyanazok

hírlétszó K koordinátarendszerben az ottani (x, y, z, t) koordinátákban felírt egyenletek alapján vannak

Törvényszerű: a fény sebessége $\forall$ koordinátarendszerben azonos

Járó: események helyszíne

ahol K rendszerben az (x, y, z, t) pontok ismeretében



fény által megkötött két pont $\ell = c(t_2 - t_1)$

másképp $\ell^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$$c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 = 0$$

$$c^2(t_2' - t_1')^2 - (x_2' - x_1')^2 - (y_2' - y_1')^2 - (z_2' - z_1')^2 = 0$$

whose

$$s_{12} = \sqrt{c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2}$$

c illendősége $\Rightarrow s_{12} = 0$ egy K rendszerben,

akkor $s'_{12} = 0$ minden K' rendszerben

ívelen nézzük

$$\begin{array}{ccccccc} t_2 - t_1 & , & x_2 - x_1 & , & y_2 - y_1 & , & z_2 - z_1 \text{ kicsi} \\ \parallel & & \parallel & & \parallel & & \parallel \\ dt & & dx & & dy & & dz \end{array}$$

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

whom, ívelen: továbbfejlesztés

4D Minkowski geometria és a továbbfejlesztés

Hogyan viszonyul egymáshoz ds^2 és ds'^2 ?

$$ds'^2 = a \cdot ds^2 \quad \text{mivel függhet a?}$$

térrel homogénitás $\rightarrow$ a nem függhet x, y, z, t -től

csak a v sebességtől függhet

v invariáns minden függ (invariancia)

$a(v)$ 3 rendszer: $K \xrightarrow{v_1} K_1 \xrightarrow{v_{12}} K_2$

$$ds_1'^2 = a(v_1) \cdot ds^2$$

$$ds_2'^2 = a(v_2) \cdot ds^2$$

$$a(v_{12}) = \frac{a(v_2)}{a(v_1)}$$

$$ds_2'^2 = a(v_{12}) \cdot ds_1'^2$$

mivel $|v_{12}|$ nem csak $|v_1|$ és $|v_2|$ függvénye $\Rightarrow a = \text{konst} \Rightarrow a = 1$

$$\boxed{ds'^2 = ds^2} \Rightarrow \gamma = 1$$

speciális esetben

$$t_1 \neq t_2 \quad (x_1, y_1, z_1) \neq (x_2, y_2, z_2)$$

Legyen (x_1, y_1, z_1, t_1) és (x_2, y_2, z_2, t_2) két esemény K -ben

a) letezik-e olyan K' koordin. rendsz., melyben a két esemény egy helyen történik?

b) letezik-e olyan K' ----- egyidejű?

legyen $t_{12} = t_2 - t_1$ $\quad l_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$$t_{12} \neq 0$$

$$l_{12} \neq 0$$

$$\Delta_{12}^2 = c^2 t_{12}^2 - l_{12}^2$$

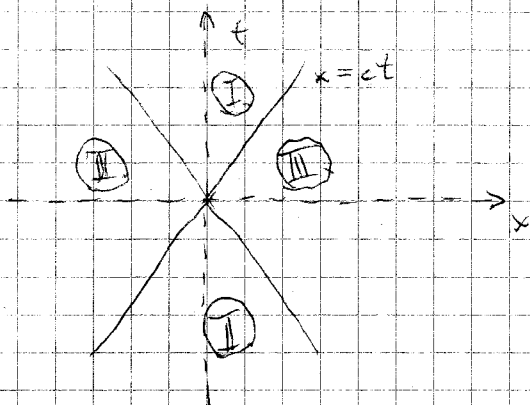
$$\Delta_{12}'^2 = c^2 t_{12}'^2 - l_{12}'^2$$

a) $l_{12}^2 = 0 \Rightarrow \Delta_{12}^2 = \Delta_{12}'^2 = c^2 t_{12}^2 > 0 \quad \Delta^2 > 0$ "idősebb"

b) $t_{12}^2 = 0 \Rightarrow \Delta_{12}^2 = \Delta_{12}'^2 = -l_{12}^2 < 0 \quad \Delta^2 < 0$ "távolabb"

a) állított idő K' -ben $t_{12}' = \frac{1}{c} \Delta_{12} = \frac{1}{c} \sqrt{c^2 t_{12}^2 - l_{12}^2}$

b) távolság: $l_{12}' = \sqrt{-\Delta_{12}^2} = |i \cdot \Delta_{12}| = \sqrt{l_{12}^2 - c^2 t_{12}^2}$



I. $c^2 t^2 - x^2 > 0$

nincsen esemény "idősebb" O -hoz képest

$\Leftrightarrow$ nincs olyan K' , ahol

egyidejű lenne O -val ($t > 0$)

abszolút jó"

II. abszolút múlt

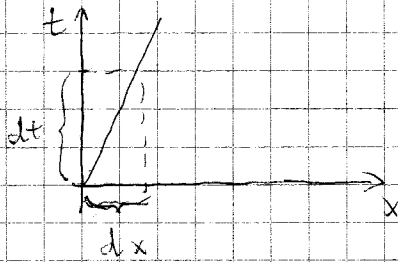
III. $\Delta s^2 < 0$ az egyidejűség K' függő, lehet

múlt és jövő is, nem lehet egy helyen O -val

$\Rightarrow$ semmilyen hatás nem juthat el ide az O -tól

$\Rightarrow$ "távolabb" megengedett pontok

sajátidő



$d\hat{t} = ?$ az együttmozgó rendszerben

t K rendszerben dt idő alatt

$$\sqrt{dx^2 + dy^2 + dz^2} \text{ távolságot}$$

tesz meg a test

t K' rendszerben (együtt mozgó) $dx' = dy' = dz' = 0$ $d\hat{t} = ?$

$$ds^2 = d\hat{s}^2 \Rightarrow c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 d\hat{t}^2$$

$$d\hat{t} = \frac{ds}{c} \quad dt = d\hat{t} \sqrt{1 - \frac{dx^2 + dy^2 + dz^2}{c^2 dt^2}}$$

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} = v^2 \text{ a K rendszerben}$$

$$d\hat{t} = dt \sqrt{1 - \frac{v^2}{c^2}} \text{ az teljes mozgásra}$$

$$t_2' - t_1' = \int_{t_1}^{t_2} d\hat{t} = \int_{t_1}^{t_2} dt \sqrt{1 - \frac{v^2}{c^2}} \leq t_2 - t_1$$

a mozgás óta mindig kevesebbbet jár

sajátidő $\Delta\tau = \int d\hat{t} = \frac{1}{c} \int ds$ sajátidő ~ inkonz

(anti) kronológiai egyenlőtlenség

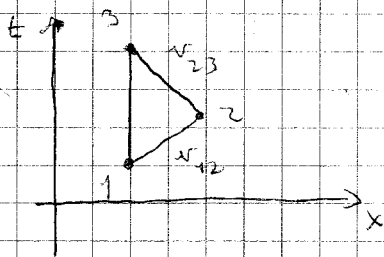
3.

O_1, O_2, O_3 3 esemény egy helyen

t_{12}, t_{13}, t_{23} időköz

$$t_1 < t_2 < t_3$$

$\Rightarrow$ $\exists$ olyan $\bar{K}$, ahol $O_1' = O_3$ egy helyen van



$$s_{13} = \frac{1}{c} \int ds_{13} = t_3 - t_1$$

$$s_{12} + s_{23} = \int_{t_1}^{t_2} dt \sqrt{1 - \frac{v_{12}^2}{c^2}} + \int_{t_2}^{t_3} dt \sqrt{1 - \frac{v_{13}^2}{c^2}} < \int_{t_1}^{t_3} dt = t_3 - t_1$$

$s_{13} > s_{12} + s_{23}$ két pont között a "legkisebb" út az egyenes

Lorentz transzformáció

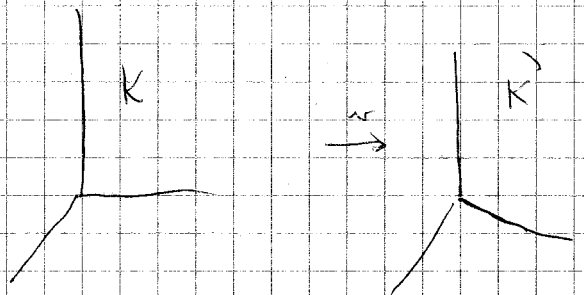
- milyen kapcsolat van a K és $\bar{K}$ rendszerbeli koordináták között?

Galilei tr. $x = \bar{x} + v\bar{t}$

$$y = \bar{y}$$

$$z = \bar{z}$$

$$t = \bar{t}$$



sem mindig neg. az időköz

$\rightarrow$ olyan transzformációk keresünk, ami megőrzik az időköz

időköz $\sim$ távolság $\Rightarrow$ relatív, invariancia, invariancia

6 elemi fogatás $\underbrace{xy, xz, yz, xt, yt, zt}$ invariancia
szokásos fogatás

relatív az xt fogatás

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \begin{pmatrix} c\bar{t} \\ \bar{x} \end{pmatrix}$$

$$d\vec{s}^2 = a_{tt} d\vec{t}^2 \text{ in allen } t:$$

$$d\vec{s}^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 =$$

$$= c^2 \left(\frac{\partial \vec{t}}{\partial x} dx + \frac{\partial \vec{t}}{\partial y} dy + \frac{\partial \vec{t}}{\partial z} dz + \frac{\partial \vec{t}}{\partial t} dt \right)^2$$

$$= a_{tt}^2 dt^2 + a_{xx}^2 dx^2 + a_{yy}^2 dy^2 + a_{zz}^2 dz^2 +$$

$$+ a_{tx}^2 c dt \cdot dx + \dots + a_{yz}^2 dy \cdot dz$$

$$\text{also wegen: } c^2 dt^2 = d\vec{s}^2 + dx^2 + dy^2 + dz^2$$

$$d\vec{s}^2 = a_{tt}^2 d\vec{s}^2 + a_{xx}^2 dx^2 + a_{yy}^2 dy^2 + a_{zz}^2 dz^2 +$$

$$+ a_{tx}^2 c dt \cdot dx + \dots + a_{yz}^2 dy \cdot dz$$

$$(d\vec{s}^2 = 0 \Leftrightarrow d\vec{s}^2 = 0) \Rightarrow a_{xx}^2 = 0, a_{yy}^2 = 0, \dots, a_{yz}^2 = 0$$

$$d\vec{s}^2 = a_{tt}^2 d\vec{s}^2$$

x, t forgotten

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

$$\text{alternativ: } (ct)^2 - x^2 = (ct')^2 - x'^2 \quad \forall t', x'$$

$$(\alpha \cdot ct' + \beta \cdot x')^2 - (\gamma \cdot ct' + \delta \cdot x')^2 = (ct')^2 - x'^2 \quad \forall t', x'$$

(~~alternativ~~)

$$\underbrace{(\alpha^2 - \gamma^2)}_1 (ct')^2 + \underbrace{2(\alpha\beta - \gamma\delta)}_0 ct' x' - \underbrace{(\delta^2 - \beta^2)}_{-1} x'^2 = (ct')^2 - x'^2$$

$$\gamma^2 - \beta^2 = 1$$

$$\gamma = \cosh X$$

$$\beta = \pm \sinh X$$

$$\gamma \beta - \beta \gamma = 0$$

$$\beta = \sinh X$$

$$\beta = \pm \sinh X$$

$$\gamma^2 - \beta^2 = 1$$

γ -lan + : nincs időtérítés

β -lan + : nincs térítés

$$\cosh X \cdot \sinh X - \sinh X \cdot \cosh X = 0$$

$$\cosh X = \cosh X \Rightarrow X = X$$

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh X & \sinh X \\ \sinh X & \cosh X \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

X : rapiditás

hátrébb K -ban a K' megfigyelő megáll

$$x' = 0$$

$$\left. \begin{aligned} ct &= \cosh X \cdot ct' \\ x &= \sinh X \cdot ct' \end{aligned} \right\} \Rightarrow \frac{x}{ct} = \tanh X$$

$$\boxed{\frac{v}{c} = \tanh X}$$

$$\sinh X = \frac{\tanh X}{\sqrt{1 - \tanh^2 X}}$$

$$\cosh X = \frac{1}{\sqrt{1 - \tanh^2 X}}$$

$$= \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

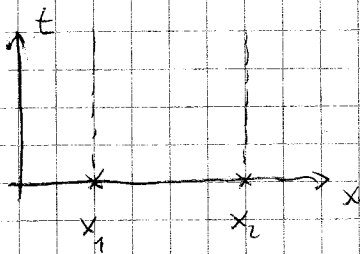
$$\boxed{\begin{aligned} t &= \frac{t' + \frac{v}{c^2} x'}{\sqrt{1 - \frac{v^2}{c^2}}} & x &= \frac{vt' + x'}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}}$$

invers transzformáció

$$v \rightarrow -v$$

2. feladat

1. Lorentz-kontrakció



A K rendszerben $\Delta x = x_2 - x_1$ hosszúságú
rúd helyezkedik el a K' rendszerben
adott t' -nél hol van
a rúd vége? (x_1' , x_2')

$$x_1' = \frac{vt' + x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{vt' + x_2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

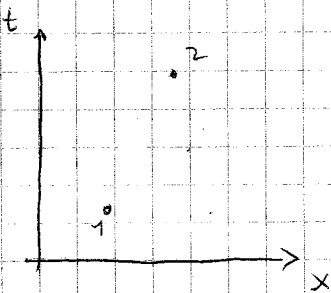
$$t_1' = t_2' = t'$$

$$t_1 \neq t_2$$

$$\Delta x = x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

2. relativitás igazsága

2 esemény (t_1, x_1) (t_2, x_2)



$$t_1' = \frac{t_1 + \frac{v}{c^2} x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2' = \frac{t_2 + \frac{v}{c^2} x_2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

egyszerűsítés: K' : $x_1' = x_2' = x'$

$$\Delta t = t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativitätstheorie 1D-Len

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh X_1 & \sinh X_1 \\ \sinh X_1 & \cosh X_1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$



$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh X_2 & \sinh X_2 \\ \sinh X_2 & \cosh X_2 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh X_1 & \sinh X_1 \\ \sinh X_1 & \cosh X_1 \end{pmatrix} \begin{pmatrix} \cosh X_2 & \sinh X_2 \\ \sinh X_2 & \cosh X_2 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\begin{pmatrix} \cosh(X_1 + X_2) & \sinh(X_1 + X_2) \\ \sinh(X_1 + X_2) & \cosh(X_1 + X_2) \end{pmatrix}$$

HF

X additiv (auch 1D-Len)

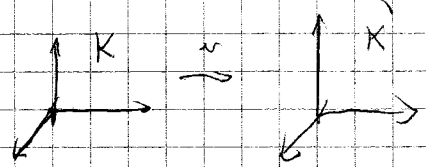
Relativitätstheorie: $\frac{v_1}{c} = \tanh X_1$ $\frac{v_2}{c} = \tanh X_2$

$$\frac{v}{c} = \tanh(X_1 + X_2) = \frac{\sinh(X_1 + X_2)}{\cosh(X_1 + X_2)} = \frac{\cosh X_1 \sinh X_2 + \sinh X_1 \cosh X_2}{\cosh X_1 \cosh X_2 + \sinh X_1 \sinh X_2} =$$

$$= \frac{\tanh X_1 + \tanh X_2}{1 + \tanh X_1 \tanh X_2} = \frac{\frac{v_1}{c} + \frac{v_2}{c}}{1 + \frac{v_1 v_2}{c^2}}$$

$$\Rightarrow \boxed{v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}}$$

sebeségek transzformációi általánosan



3D Lorentz transzformáció

$$\boxed{t = \frac{t' + \frac{v}{c^2} x'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad x = \frac{vt' + x'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad y = y' \quad z = z'}$$

K rendszerben mozogjon egy részecske v sebeséggel

$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt}$$

K' K-hez képest V sebeséggel mozog x irányban

v' és v között mi a kapcsolat?

$$dt = \frac{dt' + \frac{V}{c^2} dx'}{\sqrt{1 - \frac{V^2}{c^2}}} \quad dx = \frac{Vdt' + dx'}{\sqrt{1 - \frac{V^2}{c^2}}} \quad dy = dy' \quad dz = dz'$$

$$v_x = \frac{dx}{dt} = \frac{Vdt' + dx'}{dt' + \frac{V}{c^2} dx'} = \frac{V + v'_x}{1 + \frac{V \cdot v'_x}{c^2}}$$

$$v_y = \frac{dy}{dt} = \frac{dy' \sqrt{1 - \frac{V^2}{c^2}}}{dt' + \frac{V}{c^2} dx'} = \frac{v'_y \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v'_x}{c^2}}$$

$$v_z = \frac{dz}{dt} = \frac{dz' \sqrt{1 - \frac{V^2}{c^2}}}{dt' + \frac{V}{c^2} dx'} = \frac{v'_z \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v'_x}{c^2}}$$

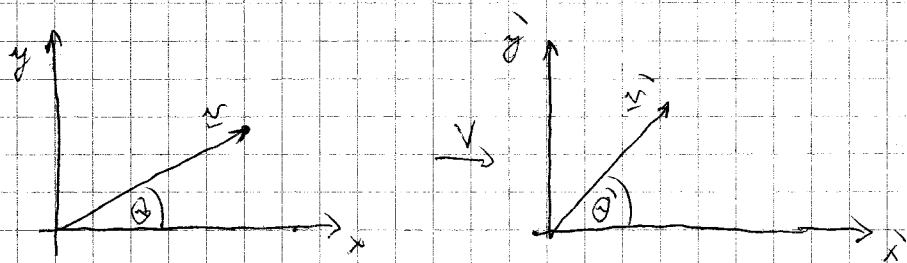
→ további sebeségek is átalakíthatók

→ nem szimmetrikus V és v'_x cserejére

Lorentz transzformáció nem felcserélhető

relativistisk transformering

låt oss se på xy och x'y'



$$r_x = r \cdot \cos \theta$$

$$r_y = r \cdot \sin \theta$$

$$r'_x = r' \cdot \cos \theta'$$

$$r'_y = r' \cdot \sin \theta'$$

$$\tan \theta = \frac{r_y}{r_x} = \frac{r'_y \sqrt{1 - \frac{V^2}{c^2}}}{r'_x + V} = \frac{r' \sqrt{1 - \frac{V^2}{c^2}} \cdot \sin \theta'}{r' \cos \theta' + V}$$

låt oss se på $r = c$, $r' = c$ färgjel

$$\tan \theta = \frac{\sin \theta' \sqrt{1 - \frac{V^2}{c^2}}}{\cos \theta' + \frac{V}{c}}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V}{c} \cos \theta'} \cdot \sin \theta'$$

$$\Rightarrow \cos \theta = \frac{\cos \theta' + \frac{V}{c}}{1 + \frac{V}{c} \cos \theta'}$$

$$\frac{V}{c} \ll 1 \quad \sin \theta = \sin \theta' \left(1 - \frac{V}{c} \cos \theta' \right)$$

$$\sin \theta' - \sin \theta = \frac{V}{c} \cos \theta' \cdot \sin \theta'$$

$$\Delta \Theta = \Theta' - \Theta \quad \text{linear}$$

$$\sin \Theta' - \sin \Theta \approx \cos \Theta \cdot \Delta \Theta$$

$$\cos \Theta \cdot \Delta \Theta = \frac{v}{c} \cos \Theta \cdot \sin \Theta$$

$$\Delta \Theta = \frac{v}{c} \sin \Theta$$

lang ablesien

Minkowski formalismus

$$\text{Lage} \quad x^0 := ct, \quad x^1 := x, \quad x^2 := y, \quad x^3 := z$$

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \rightarrow \text{reges 4erfeld}$$

Lorentz - trafo x invariante messen

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} \cosh X & \sinh X & 0 & 0 \\ \sinh X & \cosh X & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix}$$

Trabeli forgtin x y - vellen

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi & 0 \\ 0 & -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix}$$

lineare

$$x^\mu = \sum_{\nu=0}^3 \Lambda^\mu{}_\nu \cdot x^\nu \quad \mu = 0, \dots, 3$$

$\Lambda^\mu{}_\nu$ Lorentz transformációs mátrix

(index) összegzési konvenció: azonos alsó - felső indexre

automatikus összegzés $\nu = 0 \dots 3$ -ig

$$x^\mu = \Lambda^\mu{}_\nu \cdot x^\nu$$

invariancia: $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 =$

$$= (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

$$= \eta_{\mu\nu} dx^\mu \cdot dx^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad \text{metrikus tensor}$$

Kovariáns és kontravariáns komponensek

$$x^\mu = \eta^{\mu\nu} x_\nu$$

kontravariáns komponensek

kovariáns komponensek

$$x_\mu = \eta_{\mu\nu} x^\nu$$

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x^\mu = (ct, \mathbf{x})$$

$$x_\mu = (ct, -\mathbf{x})$$

wele invariantia

$$ds^2 = d\tilde{s}^2$$

$$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\beta\lambda} dx^{\beta'} dx^{\lambda'}$$

$$dx^\mu = \Lambda^\mu_{\beta} dx^{\beta'}$$

$$dx^\nu = \Lambda^\nu_{\lambda} dx^{\lambda'}$$

$$\eta_{\mu\nu} \Lambda^\mu_{\beta} dx^{\beta'} \Lambda^\nu_{\lambda} dx^{\lambda'} = \eta_{\beta\lambda} dx^{\beta'} dx^{\lambda'}$$

$$(\eta_{\mu\nu} \Lambda^\mu_{\beta} \Lambda^\nu_{\lambda}) dx^{\beta'} dx^{\lambda'} = \eta_{\beta\lambda} dx^{\beta'} dx^{\lambda'}$$

$\forall dx$ -re

$$\eta_{\mu\nu} \Lambda^\mu_{\beta} \Lambda^\nu_{\lambda} = \eta_{\beta\lambda}$$

$$\eta_{\beta\lambda} \eta^{\lambda\sigma} = \delta_{\beta}^{\sigma}$$

$$\boxed{\eta_{\mu\nu} \Lambda^\mu_{\beta} \Lambda^\nu_{\lambda} \eta^{\lambda\sigma} = \delta_{\beta}^{\sigma}}$$

$$\det \Lambda^2 = 1 \Rightarrow \det \Lambda = \pm 1$$

$\left. \begin{array}{l} +1 \text{ times} \\ -1 \text{ times} \end{array} \right\} \text{terids' terids'}$

$$x^\mu = \Lambda^\mu_\nu \cdot x^{\nu'}$$

kovariante komponentel transformatsioji

$$x_\mu = \eta_{\mu\nu} x^\nu = \eta_{\mu\nu} \Lambda^\nu_\sigma \cdot x^{\sigma'} =$$

$$= \underbrace{\eta_{\mu\nu} \Lambda^\nu_\sigma \cdot \eta^{\sigma\kappa}}_{\Lambda_\mu{}^\kappa} \cdot x^\kappa$$

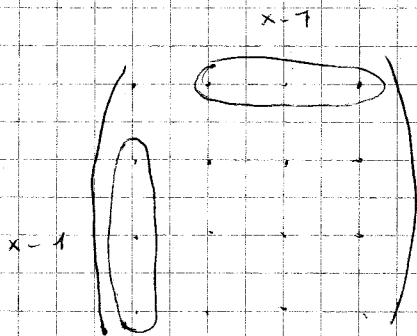
$$\Lambda_\mu{}^\kappa$$

$$\Lambda_\mu{}^\kappa := \eta_{\mu\nu} \cdot \Lambda^\nu_\sigma \cdot \eta^{\sigma\kappa}$$

$$\Lambda^\mu_\sigma \cdot \underbrace{\eta_{\mu\nu} \Lambda^\nu_\lambda \cdot \eta^{\lambda\sigma}}_{\Lambda_\mu{}^\sigma} = \delta_\sigma^\sigma$$

$$\Lambda^\mu_\sigma \cdot \Lambda_\mu{}^\sigma = \delta_\sigma^\sigma$$

$$(\tilde{\Lambda}^T)^\lambda_\mu := \Lambda_\mu{}^\lambda$$



$$(\tilde{\Lambda}^T)^\lambda_\mu \Lambda^\mu_\kappa = \delta_\kappa^\lambda$$

$$\boxed{\tilde{\Lambda}^T \cdot \Lambda = \mathbb{1}}$$

10 faktel $\rightarrow$ 6 parameter

3 forgetas (x_y, x_z, y_z sikholan)

3 lost (x_t, y_t, z_t sikholan)

negativvektoren altlassen

4. da A^μ ein negativvektor ist,
es ist transformierbar, sind ein kovektor,
also

$$A^\mu = \Lambda^\mu_\nu \cdot A^\nu$$

x^μ -lin können kovariant kompensiert

$$A_\mu := \eta_{\mu\nu} A^\nu$$

vektor „kontra“, negativ

$$A^2 := A_\mu A^\mu = \eta_{\mu\nu} A^\mu A^\nu$$

skalarwert

A^μ, B^μ negativvektoren

$$AB := A_\mu B^\mu = \eta_{\mu\nu} A^\mu B^\nu \quad \text{skalarwert}$$

all: skalarwert invariant (skalar):

oder m. - be ergänze an Stelle

all.

spez. sel: A^2 invariant

jetzt: $A^\mu = (A^0, \underline{A})$

$$A_\mu = (A^0, -\underline{A})$$

tensor

1.6.16. $A^{\mu\nu}$ sind bilineares Tensorpaar, da nicht indexellig transformierbar, mit 2 Basisvektoren

$$A^{\mu\nu} = \Lambda^{\mu}_{\sigma} \cdot \Lambda^{\nu}_{\kappa} \cdot A^{\sigma\kappa}$$

indexell "Kreuztensor" (Kovarianz kontravarianz)

$$A^{\mu}_{\nu} := \eta_{\mu\sigma} A^{\sigma\nu} \quad , \quad A^{\mu}_{\nu} := \eta_{\nu\sigma} A^{\mu\sigma}$$

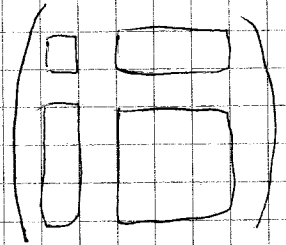
$$A_{\mu\nu} := \eta_{\mu\sigma} \eta_{\nu\sigma} A^{\sigma\sigma}$$

$$A_{00} = A^0_0 = A_0^0 = A^{00}$$

$$(A_{0i} = A^0_i = A_i^0 = A^{0i})$$

$$A_{0i} = -A^0_i = -A^{0i} = A^i_0$$

$$A_{ij} = -A^i_j = -A^{ij} = A^j_i$$



bilindexer tensor

1.6.17. $A^{\mu_1 \dots \mu_n}$ sind indexellig n indexer Tensorpaar, da

$$A^{\mu_1 \dots \mu_n} = \Lambda^{\mu_1}_{\nu_1} \dots \Lambda^{\mu_n}_{\nu_n} \cdot A^{\nu_1 \dots \nu_n}$$

specialis tenoroh

egzistencja

$$\delta^\mu{}_\nu \cdot A^\nu = A^\mu$$

tetralogues A-ra

$$J_{\mathbf{H}} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\sigma_r = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(\delta_{\mu\nu})^{\sigma} = \eta_{\mu\sigma} \delta^{\sigma}_{\nu} = \eta_{\mu\nu}$$

des as η against!

Δ Η Β

crinitus, ha

$$A^T V = A^V T$$

10 Le fuggeltes man

antisimulibus, La

$$A^{\mu\nu} = -A^{\nu\mu}$$

6 de ——— 11 ———

cosk 3D longat'shna

A^i vector A

tenorinal

A 0 2

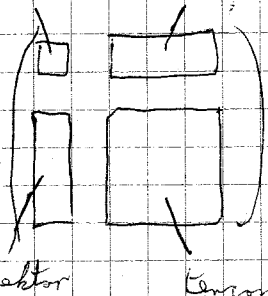
3D vector

A. J.

3D tensor

shaker

vehten



Feljesen antisimmetrisches Tensor

4 indexes

Σ 1790

$$\xi_{0123} = 1$$

beneljik 2 index cserejere antisimetrikus

24 der von 0 aben von

$$\sum m v_{30} = -24 \text{ (kg)}$$

$$\epsilon^{\mu\nu\rho\sigma} \epsilon_{\kappa\lambda\gamma\sigma} = -2 (\delta^\mu_\kappa \delta^\nu_\lambda - \delta^\mu_\lambda \delta^\nu_\kappa) \quad (\text{Def.})$$

$$\epsilon^{\mu\nu\rho\sigma} \epsilon_{\kappa\nu\rho\sigma} = -6 \delta^\mu_\kappa$$

Vertikalisierung

Skalar / pseudoskalar

↑
neu ist
elogelet

↑
elogelet ist

vektor: elogelet ist

axialvektor: neu ist elogelet

pl.: $\underline{C} = \underline{A} \times \underline{B}$
↑
axialvektor

↑
vektor

antisymmetrischer tensor: 3D Formelzeichen

$$\Lambda^{\mu\nu} = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix}$$

$\underline{a}$ vektor

$\underline{b}$ axialvektor

noch: "eindeutigkeit" f"ugge" Skalar, Vektor, Tensor "erlehen"

fr. - el

Skalar: $T(0) = T'(0)$

0 point

koordinatell $T(x) = T'(\Lambda^{-1}x)$

$$x = \Lambda \hat{x}$$

$$\hat{x} = \Lambda^{-1} x$$

vektoren: $V^\mu(0) = \Lambda^\mu_\nu \cdot V^\nu(0)$

$$V^\mu(x) = \Lambda^\mu_\nu \cdot V^\nu(\Lambda^{-1}x)$$

tensor: $A^{\mu\nu}(0) = \Lambda^\mu_\rho \Lambda^\nu_\sigma A^{\rho\sigma}(0)$

$$A^{\mu\nu}(x) = \Lambda^\mu_\rho \Lambda^\nu_\sigma A^{\rho\sigma}(\Lambda^{-1}x)$$

Kovariáns egyenlet: a két skálár közötti transzformáció nemjellel állnak

pl.: $A^\mu = B^\mu$ ~~$A^{\mu\nu} = B^\mu$~~ $v=0$

minden K rendszerben azonos alakú

relativitás elve: $\forall$ fizikai törvény felírható kovariáns egyenlettel

Kovariáns differenciál egyenlet

F legyen egy skálár x^μ függvénye esetén

$$dF = \sum_{\mu} \frac{\partial F}{\partial x^\mu} \cdot dx^\mu$$

dF skálár

dx^μ vektorkomponens



$\frac{\partial F}{\partial x^\mu}$ vektorkomponens kovariáns kifejezés

$$\partial_\mu := \frac{\partial}{\partial x^\mu}$$

$$dF = \partial_\mu F \cdot dx^\mu$$

$$\partial_\mu = \left(\frac{1}{c} \partial_t, \partial_x, \partial_y, \partial_z \right)$$

$$\partial^\mu = \left(\frac{1}{c} \partial_t, -\partial_x, -\partial_y, -\partial_z \right)$$

$$(\partial^\mu = \frac{\partial}{\partial x_\mu})$$

D'Alembert operátor

$$\partial_\mu \partial^\mu = \frac{1}{c^2} \partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 = \frac{1}{c^2} \partial_t^2 - \Delta =: \square$$

skálár operátor

pl: ϕ skalarno $\square \phi = 0$

negativst konv: $\phi(x) = e^{i k_\mu x^\mu}$ k : eigenvektor
ablen

$$\partial_\mu \phi = i k_\mu e^{i k_\mu x^\mu} = i k_\mu e^{i k_\mu x^\mu}$$

$$\partial^\mu \partial_\mu \phi = i k_\mu (i k^\mu) \cdot e^{i k_\mu x^\mu} = - (k_\mu k^\mu) \cdot e^{i k_\mu x^\mu}$$

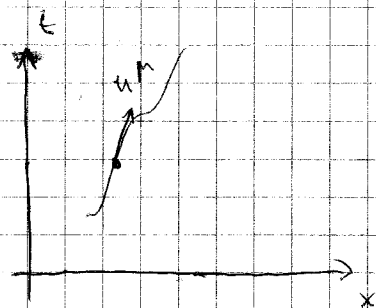
$$\partial^\mu \partial_\mu \phi = 0 \Rightarrow k_\mu k^\mu = 0$$

eigenvektor

$\frac{dx^\mu}{dt}$ 4-er vektor

$dt \rightarrow ds \Rightarrow \frac{dx^\mu}{ds}$ eigenvektor

$u^\mu := \frac{dx^\mu}{ds}$



$$ds = c \cdot dt \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$$u^\mu = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_x}{c \sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_y}{c \sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_z}{c \sqrt{1 - \frac{v^2}{c^2}}} \right)$$

$$= \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{\underline{v}}{c \sqrt{1 - \frac{v^2}{c^2}}} \right)$$

u^μ dimensionslos

$$u_\mu u^\mu = \frac{dx_\mu}{ds} \cdot \frac{dx^\mu}{ds} = \frac{dx_\mu dx^\mu}{ds^2} = 1$$

$$\frac{1}{1 - \frac{v^2}{c^2}} - \frac{v^2}{c^2 \left(1 - \frac{v^2}{c^2}\right)} = \frac{1 - \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = 1$$

'egyszerűsülés'

$$a^\mu = \frac{d^2 x^\mu}{ds^2} = \frac{du^\mu}{ds}$$

értelemszerűen: $u_\mu u^\mu = 1 \quad / \quad \frac{d}{ds}$

$$a_\mu u^\mu + u_\mu a^\mu = 0$$

$$2 a_\mu u^\mu = 0$$

u és a ortogonálisok

egyenletes gyorsulás megfogalmazása

def: a pillanatnyi gyorsulás vektorában legyen konstans a gyorsulás

'egyszerűsülés' (x tengely irányú mozgás)

sziget rendszerben $ds = c dt$

$$a^0 = \frac{1}{c} \frac{d^2 t}{ds^2} = 0$$

$$a^1 = \frac{d^2 x}{ds^2} = \frac{1}{c} a$$

$$a^\mu = \left(0, \frac{a}{c}, 0, 0\right)$$

$$\Rightarrow a_\mu a^\mu = \text{konstans} = -\frac{a^2}{c^2}$$

a^μ later beschreiben

$$a^\mu = \Lambda^\mu_{\nu} \cdot a^{\nu'} = \begin{pmatrix} \cosh X & \sinh X & & \\ \sinh X & \cosh X & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{c}a \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{a}{c^2} \left(\frac{v/c}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, 0, 0 \right)$$

normalisiert

$$u^\mu = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v}{c \sqrt{1 - \frac{v^2}{c^2}}}, 0, 0 \right)$$

$$a^\mu = \frac{du^\mu}{ds} = \frac{\cancel{ds} du^\mu}{c \sqrt{1 - \frac{v^2}{c^2}} dt}$$

in Komponenten

$$\frac{a}{c^2} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{c^2 \sqrt{1 - \frac{v^2}{c^2}}} \frac{d}{dt} \left(\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

$$\frac{d}{dt} \left(\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = a$$

$$\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} = a \cdot t + \cancel{b} \quad t=0 \quad v=0$$

ergibt's:

$$v = \frac{at}{\sqrt{1 + \frac{a^2 t^2}{c^2}}}$$

$$x = x_0 + \int_0^t v \, dt \stackrel{H\ddot{a}}{=} x_0 + \frac{c^2}{a} \left(\sqrt{1 + \frac{a^2 t^2}{c^2}} - 1 \right)$$

↑
setzen $x_0 = 0$

Ähnlichkeit:

$$\left(\frac{ax}{c^2} + 1 \right)^2 = 1 + \frac{a^2 t^2}{c^2}$$

- hyperbolsche Ergänzung

$t \rightarrow x$ ~~weil~~ $\frac{ax}{c^2} = \frac{a \cdot t}{c} \Rightarrow x = ct$

Zeitdilatation

$$\tau = \int_0^t dt \sqrt{1 - \frac{v^2}{c^2}} \stackrel{H\ddot{a}}{=} \frac{c}{a} \cdot \operatorname{arsh}\left(\frac{at}{c}\right)$$

fl.: $a = g = 10 \frac{m}{s^2}$

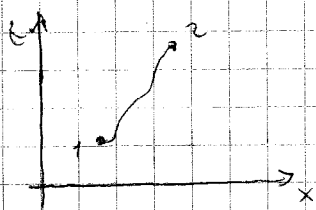
$x = 100 \text{ fänger} \approx 3,46 \cdot 10^{12} \text{ m}$

$t = ? \quad 100,95 \text{ er}$

$\tau = ? \quad 5,04 \text{ er}$

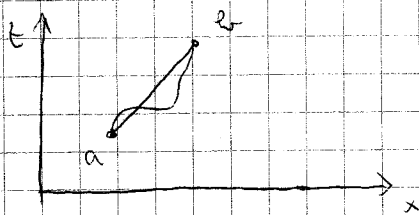
Relativistisches Rechnen

logische Ketten aber



in a Ketten?

Geometrische



Weglängen: invariante

begegnungsinvariante: invariante

$$S = -\alpha \cdot \int_a^b ds \quad \int ds \text{ - und maximiere von } \Rightarrow \alpha > 0$$

Lagrange - fr.

$$S = \int_{t_1}^{t_2} L dt$$

$$ds = c dt \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$$S = -\alpha \int_{t_1}^{t_2} c \cdot dt \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = -\alpha \cdot c \sqrt{1 - \frac{v^2}{c^2}}$$

$$v \ll c \text{ Entwicklung: } L = -\alpha c \left(1 - \frac{v^2}{2c^2} + O\left(\frac{v^4}{c^4}\right) \right)$$

$$= -\alpha c + \frac{\alpha}{2c} v^2 + \dots$$

||

$$= \text{const} + \frac{1}{2} m \cdot v^2$$

$$\Rightarrow \alpha = m \cdot c$$

$$S = -m c \int ds$$

$$L = -m c^2 \sqrt{1 - \frac{v^2}{c^2}}$$

rel. impulzusok:

$$\vec{p}_i = \frac{\partial L}{\partial \vec{v}_i} = \frac{m \vec{v}_i}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v \ll c$$

$$\vec{p}_0 = m \vec{v}_i$$

Mi a helyes $F = m \cdot a$ -val?

hiszen p_i időderiváltja

(a) $|\vec{v}| = \text{állandó}$

$$\frac{d\vec{p}_i}{dt} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{d\vec{v}_i}{dt}$$

(b) $\vec{v} = v \cdot \vec{e}$ (v irányú állandó)

$$\frac{d\vec{p}_i}{dt} = m \frac{d}{dt} \frac{\vec{v}_i}{\sqrt{1 - \frac{v^2}{c^2}}} \stackrel{Hf}{=} \frac{m}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \frac{d\vec{v}_i}{dt}$$

a két egyenletet külön-külön

jelentése később

ami mindig igaz $E = \frac{dF}{dt} + m \cdot a$

Energia:
$$\left[E = \sum_i \vec{p}_i \cdot \vec{v}_i - L = \vec{p} \cdot \vec{v} - L = \frac{m v^2}{\sqrt{1 - \frac{v^2}{c^2}}} + m c^2 \sqrt{1 - \frac{v^2}{c^2}} \right]$$

$$= \frac{m v^2 + m c^2 \left(1 - \frac{v^2}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{v=0 \Rightarrow E = m c^2}$$

E, p relatív energiájához

gesetzt werden $mc^2 \neq \sum_i m_i c^2$

$$mc^2 = \sum_i m_i c^2 + \sum_{i < j} \frac{V_{ij}}{r_{ij}}$$

kombinatorisch

$\Rightarrow$ Energie muss neg!

Hamilton-fn

$$H(q_i, p_i) = p v - L = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 \sqrt{1 + \frac{p^2}{m^2 c^2}}$$

$$v \ll c \Rightarrow p \ll mc$$

$$H = mc^2 \sqrt{1 + \frac{p^2}{m^2 c^2}} = mc^2 \sqrt{1 + \frac{p^2}{m^2 c^2}} \approx mc^2 \left(1 + \frac{p^2}{2 m^2 c^2} \right) = mc^2 + \frac{p^2}{2m}$$

relativistische

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \quad E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{p = \frac{E}{c^2} v}$$

$$v \rightarrow c$$

$$v \rightarrow 0$$

p & E ist $\frac{0}{0}$ abh. von

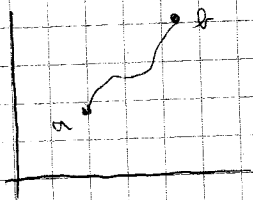
$v=0, v=c$ setzt E ist p in $\frac{0}{0}$ abh. von, da

$$\boxed{p = \frac{E}{c}}$$

reguláris

hatás variációja

$$\delta S = -mc \int_a^b ds$$



$$ds = \sqrt{dx^\mu dx_\mu}$$

$$\delta ds = \frac{dx^\mu \delta dx_\mu}{ds}$$

$$\delta S = -mc \int_a^b \frac{dx^\mu \delta dx_\mu}{ds} = -mc \int_a^b u_\mu \delta dx^\mu =$$

$$= -mc \cdot u_\mu dx^\mu \Big|_a^b + mc \int_a^b dx^\mu \cdot \frac{du_\mu}{ds} ds$$

megszegés: $dx^\mu_a = dx^\mu_b = 0$

$$\int_a^b dx^\mu \cdot \frac{du_\mu}{ds} ds = 0 \quad \forall dx^\mu = 0$$

$$\Rightarrow \frac{du_\mu}{ds} = 0 \Rightarrow u_\mu = \text{const}$$

hatás változatlan $dx^\mu_a = 0$ $dx^\mu_b = dx^\mu$
és megszegés teljesül

$$\delta S = -mc \cdot u_\mu \delta x^\mu$$

Klassikus mechanika: $\delta S = \sum_i p_i \delta x_i - E \cdot \delta t$

$$\frac{\partial S}{\partial t} = -E$$

$$\frac{\partial S}{\partial x_i} = p_i$$

Def. $p_\mu = - \partial_\mu S = - \frac{\partial S}{\partial x^\mu}$ ¹ röggesimpulzus

(szabad részlen: $p^\mu = m \cdot c \cdot u^\mu$)

p^μ : röggesvektor

komponensei: $\left(\frac{E}{c}, \mathbf{p}\right)$

$$p^0 = - \frac{\partial S}{\partial x^0} = - \frac{\partial S}{\partial f(t)} = \frac{E}{c}$$

$$p^i = - \frac{\partial S}{\partial x^i} = + \frac{\partial S}{\partial x^i} = (\mathbf{p})^i \leftarrow \text{komposimpulzus}$$

p^μ megmaradás megijiség

transzformációja: $p^\mu = \Lambda^\mu_\nu p^\nu$

x irányú mozgás esetén

$$p_x = \frac{p_x + \frac{v}{c^2} E}{\sqrt{1 - \frac{v^2}{c^2}}} \quad p_y = p_y \quad p_z = p_z \quad E = \frac{E + v p_x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

szabad 4-es impulzus

$$p^\mu = m \cdot c \cdot u^\mu = \left(\frac{m c}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{m v}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

$\uparrow$
 $\frac{E}{c}$

$\uparrow$
 $\mathbf{p}$

$$u_\mu u^\mu = 1 \Rightarrow \boxed{p_\mu p^\mu = m^2 c^2} \quad \frac{E^2}{c^2} - p^2 = m^2 c^2$$

$$\boxed{E^2 - p^2 c^2 = m^2 c^4}$$

invariancia feltétel

erő regresszora

$$\underline{F} = \frac{d\underline{p}}{dt}$$

$$F^{\mu} = \frac{d\underline{p}^{\mu}}{ds} \left(= \gamma c \cdot \frac{d\underline{p}^{\mu}}{dt} \right)$$

erő komponensei

$$F^{\mu} = \left(\frac{\underline{F} \cdot \underline{v}}{c^2 \sqrt{1 - \frac{v^2}{c^2}}}, \frac{\underline{F}}{c \sqrt{1 - \frac{v^2}{c^2}}} \right)$$

0. komponens a teljesítmény regresszora

Relativitás lendless (legyen $c=1$)

Teljesítmény egy M tömegű részecske lendless

m_1, m_2 tömegűek a M nyugalmi rendszerekben

energiaregresszóra:

$$M = E_1 + E_2 \geq m_1 + m_2$$

lends feltétele $M > m_1 + m_2$

ha nem teljesül, akkor legalább $m_1 + m_2 = M$ energiát
ke kell támasztani

Impulzus regresszóra

$$0 = \underline{p}_1 + \underline{p}_2$$

$$\underline{p}_1 = -\underline{p}_2$$

$$E_1^2 - m_1^2 = E_2^2 - m_2^2$$

$$E_1^2 - \underline{p}_1^2 = m_1^2$$

$$E_2^2 - \underline{p}_2^2 = m_2^2$$

$$\left. \begin{array}{l} E_1^2 - E_2^2 = m_1^2 - m_2^2 \\ E_1 + E_2 = M \\ E_1 - E_2 = \frac{m_1^2 - m_2^2}{M} \end{array} \right\} \Rightarrow \boxed{\begin{array}{l} E_1 = \frac{m_1^2 - m_2^2 + M^2}{2M} \\ E_2 = \frac{m_2^2 - m_1^2 + M^2}{2M} \end{array}}$$

Ugyszer fordítás:

$$\begin{array}{c} m_1 \quad m_2 \\ \longrightarrow \quad \longleftarrow \end{array} \Rightarrow \bullet M$$

m_1 és m_2 tömegű részecskék találkoznak egy M tömegű részecskével

Def: a részecskék analitikus $\sum p = 0$, vagyis ahol a teljes
megimpulzus

$$p^\mu = (E, \mathbf{0}) = (M, \mathbf{0})$$

$$E_1, E_2 \rightarrow M = ?$$

$$p^\mu p_\mu = M^2$$

$$(E_1, \mathbf{p}_1) \quad (E_2, \mathbf{p}_2)$$

$$p^\mu = (E_1 + E_2, \mathbf{p}_1 + \mathbf{p}_2)$$

$$p_1^\mu = E_1^\mu - m_1^\mu$$

$$p_2^\mu = E_2^\mu - m_2^\mu$$

$$|\mathbf{p}_1| = \sqrt{E_1^2 - m_1^2}$$

$$|\mathbf{p}_2| = \sqrt{E_2^2 - m_2^2}$$

$$p^\mu p_\mu = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = E_1^2 - \mathbf{p}_1^2 + E_2^2 - \mathbf{p}_2^2 + 2(E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2)$$

$$= m_1^2 + m_2^2 + 2(E_1 E_2 - \sqrt{E_1^2 - m_1^2} \sqrt{E_2^2 - m_2^2} \cos \varphi) = M^2$$

2. experimenti eset:

a) $\begin{array}{c} \longrightarrow \\ m_1 \\ E_1 \end{array} \quad \bullet \quad \begin{array}{c} \uparrow \vec{p} = 0 \\ E_2 = m_2 \end{array}$

$$\vec{p}^* = (E_1 + m_2, \sqrt{E_1^2 - m_1^2}, 0, 0)$$

$$\begin{aligned} \vec{p}_1 \cdot \vec{p}^* &= M^2 = (E_1 + m_2)^2 - (E_1^2 - m_1^2) = \\ &= E_1^2 + 2E_1 m_2 + m_2^2 - E_1^2 + m_1^2 = m_1^2 + m_2^2 + 2E_1 m_2 \end{aligned}$$

b) $\longrightarrow \longleftarrow$

$$m_1 = m_2 = m$$

$$E_1 = E_2 = E$$

$$\vec{p}_1 = -\vec{p}_2$$

$$\vec{p}^* = (2E, 0, 0, 0) \quad M = 2E$$

mintételek (LHC) $m_1 = m_2 = m \sim 1 \text{ GeV}$

$$E_1 = E = \sqrt{s} \text{ TeV}$$

a) $M = 118,3 \text{ GeV}$

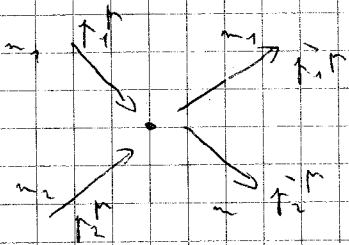
b) $M = 14 \text{ TeV}$

Regel aus 1. Fall

($c=1$)

$$\vec{p}_1 \vec{p}_1 = m_1^2$$

Regeln: räumlich sein / typisch von relativ



Regelung: negativ

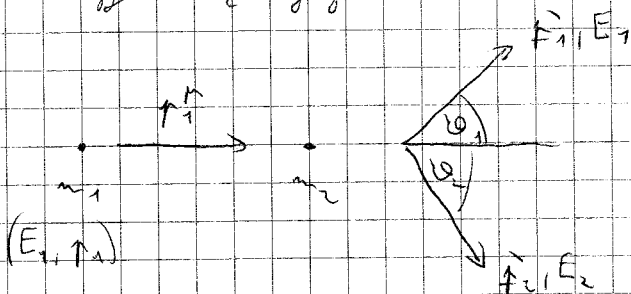
$$\vec{p}_1^M + \vec{p}_2^M = \vec{p}_1^I + \vec{p}_2^I$$

$$\vec{p}_1^M + \vec{p}_2^M - \vec{p}_1^I = \vec{p}_2^I$$

$$2m_1^2 + m_2^2 + 2(\vec{p}_1^M \vec{p}_2^M - \vec{p}_1^M \vec{p}_2^I - \vec{p}_2^M \vec{p}_1^I) = m_2^2$$

$$\begin{aligned} m_1^2 + \vec{p}_1^M \vec{p}_2^M - \vec{p}_1^M \vec{p}_1^I - \vec{p}_2^M \vec{p}_1^I &= 0 \\ 1 \leftrightarrow 2 \quad m_2^2 + \vec{p}_2^M \vec{p}_1^M - \vec{p}_2^M \vec{p}_2^I - \vec{p}_1^M \vec{p}_2^I &= 0 \end{aligned}$$

Legen m_2 ruhebedingung



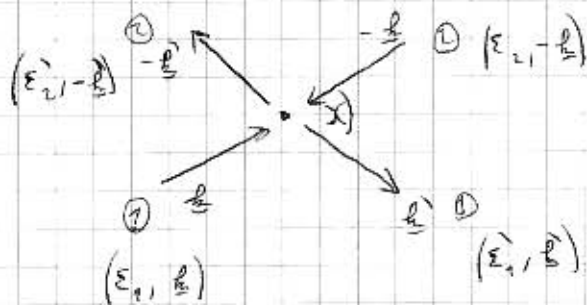
$$\vec{p}_1^M \cdot \vec{p}_2^M = E_1 m_2$$

$$\vec{p}_1^M \cdot \vec{p}_1^I = E_1 E_1 - \vec{p}_1 \vec{p}_1 \cos \theta_1$$

$$\vec{p}_2^M \cdot \vec{p}_1^I = m_2 E_1$$

$$\Rightarrow \cos \theta_1 = \frac{E_1(E_1 + m_2) - E_1 m_2 - m_1^2}{\vec{p}_1 \vec{p}_1}$$

TKP-i rendszer



$$\varepsilon_1^i - k^i = \varepsilon_1^f - k^{i2} (= m_1^2)$$

$$- \varepsilon_2^i - k^i = \varepsilon_2^f - k^{i2} (= m_2^2)$$

$$\varepsilon_1^i - \varepsilon_2^i = \varepsilon_1^f - \varepsilon_2^f$$

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_1^f + \varepsilon_2^f$$

2 szimmetria $\Rightarrow \varepsilon_1 = \varepsilon_1^f \quad \varepsilon_2 = \varepsilon_2^f \Rightarrow |k_1| = |k_2|$

$$m_1^2 + p_1^\mu p_{2\mu} - p_1^\mu p_{1\mu} - p_2^\mu p_{1\mu} = 0$$

$$m_2^2 + p_1^\mu p_{2\mu} - p_2^\mu p_{2\mu} - p_1^\mu p_{2\mu} = 0$$

$$p_1^\mu p_{1\mu} = \varepsilon_1 \varepsilon_1^i - k \cdot k^i = \varepsilon_1 \varepsilon_1^i - k^2 \cos X$$

Első: $p_1^\mu p_{2\mu} = E_1 m_2$

$$p_2^\mu p_{1\mu} = m_2 E_1$$

$$m_1^2 + E_1 m_2 - m_2 E_1^i - \underbrace{\varepsilon_1 \varepsilon_1^i + k^2 \cos X}_{- \varepsilon_1^2 + k^2 \cos X} = 0$$

$$- \varepsilon_1^2 + k^2 \cos X = \underbrace{- \varepsilon_1^2 + k^2}_{- m_1^2} + k^2 (\cos X - 1)$$

$$\cancel{m_1^2} + E_1 m_2 - m_2 E_1^i - \cancel{m_1^2} - k^2 (1 - \cos X) = 0$$

$$E_1^i - E_1 = - \frac{k^2}{m_2} (1 - \cos X)$$

$$p_2^* \cdot p_{2P}^* = m_2 E_2^*$$

$$p_1^* \cdot p_{2P}^* = E_1 E_2^* - p_1 p_2 \cos \vartheta_2$$

$$\Rightarrow \cos \vartheta_2 = \frac{E_2^* (E_1 + m_2) - E_1 m_2 - m_2^2}{p_1 p_2} = \frac{(E_1 + m_2)(E_2^* - m_2)}{p_1 p_2}$$

einzigf. falltatsächl.

$$p_1 = \sqrt{E_1^2 - m_1^2}$$

$$p_1 = \sqrt{E_1^{*2} - m_1^2}$$

$$p_2 = \sqrt{E_2^2 - m_2^2}$$

$$\Rightarrow E_2^* = m_2 \frac{(E_1 + m_2)^2 + (E_1^2 - m_1^2) \cos^2 \vartheta_2}{(E_1 + m_2)^2 - (E_1^2 - m_1^2) \cos^2 \vartheta_2}$$

$m_1 = 0$ Elektronen (Compton streuung)

$$E_1 = p_1 \quad E_1^* = p_1^*$$

$$\cos \vartheta_1 = \frac{E_1^* (E_1 + m_1) - E_1 m_1}{E_1 E_1^*}$$

$$E_1 E_1^* (\cos \vartheta_1 - 1) - E_1^* m_2 = -E_1 m_2$$

$$E_1^* = \frac{E_1}{\frac{E_1}{m_2} (1 - \cos \vartheta_1) + 1}$$

h-t fjerntil hi E, p-vel

$$\uparrow 1^{\mu} r_{2\mu} = E_{12} = \xi_1 \xi_2 + h^2$$

$$Z_1 = \sqrt{\frac{2}{1} + b}$$

$$r = \sqrt{m^2 + h^2}$$

$$\Rightarrow \ell^2 = \frac{m_1^2 (E_1^2 - m_1^2)}{m_1^2 + m_2^2 + 2m_2 E_1}$$

$$\vec{E}_1 = \vec{E}_1 - \frac{m_2 (\vec{E}_1^2 - \vec{u}_1^2)}{u_1^2 + u_2^2 + 2m_2 E_1} (1 - \cos X)$$

E segm. $E_1 + m_2 = E_1' + E_2'$

$$E_2 = m_2 + \frac{m_2(E_1^2 - m_1^2)}{m_1^2 + m_2^2 + 2m_2 E_1} (1 - \cos \chi)$$

ergibt sich:
$$\Delta E = \frac{m_1^2 (E_1^2 - c^2)}{m_1^2 + m_2^2 + 2m_1 E_1} (1 - \cos \chi)$$

$$\Delta E_{me}, \text{ La con } X = -1 \quad X = x$$

$$\Delta E_{\text{max}} = 2 \cdot \frac{m_1 (E_1^2 + m_1^2)}{m_1^2 + m_2^2 + 2m_1 E_1}$$

Als $m_1 \ll m_2, E_1$

$$\frac{\Delta E_{\text{mol}}}{E_1} \approx \frac{Z_{\text{mol}}' E_1}{m_e^2 + Z_{\text{mol}} E_1}$$

for $E_1 \gg m_e$: $\frac{\Delta E_{\text{rel}}}{E_1} \rightarrow 1$

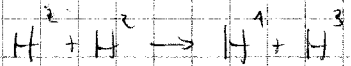
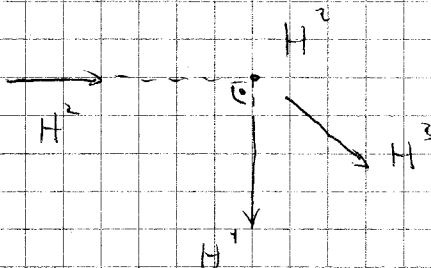
Jika $m_1 \ll m_2$ berarti berat $\frac{\Delta E_{mse}}{E_1}$?

oh: $E \sim p^2$

rel: $E \sim p$

Regelnutzen

kinet



$m_3 = ?$ a H^3 detektálás nélkül

- 4-es imp. megm.

- tömeglej. felt.

En: $m_2 + E_2 = E_1 + E_3$

Imp. (x) $p_2 = 0 + p_3^x$

$$p_3^x = p_2$$

(y) $0 = p_1 + p_3^y$

$$p_3^y = -p_1$$

(z) $0 = 0 + p_3^z$

$$p_3^z = 0$$

tömeglej.

$$E_2^2 - p_2^2 = m_2^2$$

$$E_1^2 - p_1^2 = m_1^2$$

$$E_3^2 - p_3^x^2 - p_3^y^2 - p_3^z^2 = m_3^2$$

$$m_3^2 = (m_2 + E_2 - E_1)^2 - p_2^2 - p_1^2 = m_2^2 + m_2^2 + m_1^2$$

$$+ 2m_2E_2 - 2m_2E_1 - 2E_2E_1 =$$

$$= m_1^2 + 2(m_2 + E_2)(m_2 - E_2) = m_1^2 + 2(2m_2 + T_2)(m_2 - m_1 - T_1)$$

T_1, T_2 kinetische Energien

$$T_1 = E_1 - m_1$$

$$T_2 = E_2 - m_2$$

Imp. momentum

Bewegung $\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$

$\Rightarrow$ betr. negativsteins fortsetzen

Legen x^μ 1 dr. räumliche (u. 1 zeitliche)

u. 4D fortsetzen (fortsetzen u. Lorentz transform.)

$$x'^\mu - x^\mu \text{ lin. fun. von } x^\mu \text{ u. } x^\nu$$

$$x'^\mu - x^\mu = \delta \Omega^{\mu\nu} x_\nu$$

$\delta \Omega^{\mu\nu}$ antisym. wobei invarianz nicht

$$x'^\mu x'_\mu = x^\mu x_\mu$$

$$(x^\mu + \delta \Omega^{\mu\nu} x_\nu) / (x_\mu + \delta \Omega_{\mu\nu} x^\nu) = x^\mu x_\mu$$

$$\delta \Omega^{\mu\nu} x_\mu x_\nu + \delta \Omega_{\mu\nu} x^\mu x^\nu = 0$$

$$x_\mu x_\nu \delta \Omega^{\mu\nu} = 0 \quad \forall x_\mu, x_\nu \text{-re}$$

$\Rightarrow \delta \Omega^{\mu\nu}$ antisymmetrisch

$$\delta \Omega^{\mu\nu} = -\delta \Omega^{\nu\mu}$$

Hasznos megfigyelések:

$$\delta S = - \sum_i \dot{x}_i^\mu \delta x_{i\mu} = - \sum_i \dot{x}_i^\mu \delta \Omega_{\mu\nu} x_i^\nu$$

$$= - \delta \Omega_{\mu\nu} \sum_i \dot{x}_i^\mu x_i^\nu =$$

$$= - \delta \Omega_{\mu\nu} \frac{1}{2} \sum_i (\dot{x}_i^\mu x_i^\nu - \dot{x}_i^\nu x_i^\mu)$$

Impulzus nem: $\frac{\delta S}{\delta \Omega_{\mu\nu}} = \text{val}$

Def: $J^{\mu\nu} = \sum_i x_i^\mu \dot{x}_i^\nu - x_i^\nu \dot{x}_i^\mu$

megmaradó mennyiség (összes komponense)

Elemek

$$J^{\mu\nu} = \begin{pmatrix} 0 & J^{01} & J^{02} & J^{03} \\ -J^{01} & 0 & -J^{12} & J^{13} \\ -J^{02} & J^{12} & 0 & -J^{23} \\ -J^{03} & -J^{13} & J^{23} & 0 \end{pmatrix}$$

$\underline{J}$: 3D imp. nem

mi a másik 3? (most $c \neq 1$)

$$\underline{I} = \sum_i t \mathbf{r}_i = \frac{E_i \mathbf{r}_i}{c^2}$$

$$\sum_i E_i = \text{konst}$$

$$\Rightarrow \frac{I}{\sum_i E_i} = konst = t \cdot \frac{\sum_i p_i}{\sum_i E_i} = \frac{1}{c} \cdot \frac{\sum_i E_i \cdot v_i}{\sum_i E_i}$$

$$c \cdot t \cdot \frac{\sum_i p_i}{\sum_i E_i} = \frac{\sum_i E_i \cdot v_i}{\sum_i E_i} = konst$$

$$t \cdot V = B = konst$$

B helyvektorát pont általós V sebességgel mozg

$$\underline{R} = \underline{R}_0 + \underline{V} t$$

$\underline{R}$ relativisztikus tömegmomentát definiáljuk

$$\underline{R}_{rel} = \frac{\sum_i \underline{E}_i \cdot \underline{r}_i}{\sum_i E_i}$$

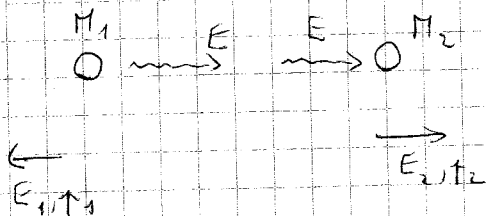
fontos $\underline{R}$ nem egy 4-es, vektor 3-as része

$\Rightarrow$ $\underline{R}$ nem invariáns

0 tömegű részecskék (fotonok) ($c=1$)

$$E = |\mathbf{p}|$$

foton nyugalmi tömege 0, de energiát hordoz (és így tömeget is)



$$E \text{ negn.} \quad M_1 = E_1 + E$$

$$\text{Imp. negn.} \quad 0 = p_1 - E$$

$$M_1^2 = E_1^2 - p_1^2 = (M_1 - E)^2 - E^2 = M_1^2 - 2M_1 E < M_1^2$$

$$\text{Hef.} \quad M_2^2 > M_1^2$$

$$\text{Planck hypothesis: } E = h\nu$$

, alol h universalis 'allade

Van - e 'isthne? Invarians - e esse ösrefüggä?

$$E = h\nu \quad ? \quad (E = |p|)$$

Leggen $K, K' \rightarrow_v$ x tangely nenten

fäng is x tangely nenten tänged

$$p^\mu = (E, E, 0, 0)$$

$$p'^\mu = \Lambda^\mu_\nu p^\nu$$

$$p'^\mu = (E', E', 0, 0)$$

$$E = E' dx + E' dx = E' e^x$$

$$\square \phi = 0 \quad \phi \sim e^{i k^\mu x_\mu} = e^{i(\omega t - kx)}$$

$$k^\mu = (\omega, k)$$

$\downarrow$

$$k^\mu \cdot k_\mu = 0$$

x tangely nenten tänged's kullän: $k^\mu = (\omega, k, 0, 0)$

$$k^\mu k_\mu = 0 \Rightarrow \omega = |k|$$

$$\Rightarrow k^\mu = (\omega, \omega, 0, 0)$$

L^μ transformiert $L^\mu = \Lambda^\mu_\nu L^\nu$

$$\omega = \vec{\omega} \cdot d\vec{x} + \omega' d\vec{x}' = \vec{\omega}' \cdot d\vec{x}'$$

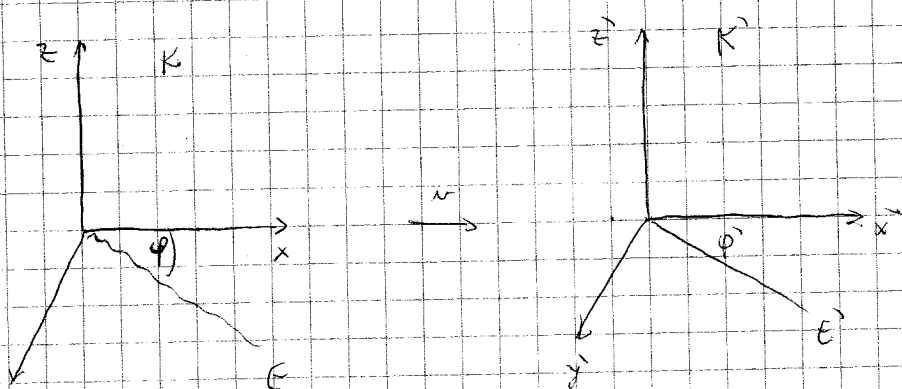
||

$$\vec{\omega} = \vec{\omega}' \cdot e^{\vec{x}}$$

E & $\vec{v}$ zusammen transformierbar

$$\frac{E}{\vec{v}} = k \quad \text{skalar}$$

Doppler - effekt



K-Gen: $p^\mu = (E, p \cos \varphi, p \sin \varphi, 0) \quad E = p$

K'-Gen: $p^{\mu'} = (E', p' \cos \varphi', p' \sin \varphi', 0) \quad E' = p'$

Lorentz - tra.

$$E = E' \cosh \chi + p' \cos \varphi' \sinh \chi$$

$$p \cos \varphi = E' \sinh \chi + p' \cos \varphi' \cosh \chi$$

$$p \sin \varphi = p' \sin \varphi'$$

E transformierte

$$E = \vec{E} (dx + \cos \varphi \cdot tX) = \vec{E} dx (1 + \cos^2 \varphi \cdot tX) =$$

$$= \vec{E} dx \left(1 + \frac{v}{c} \cos \varphi\right)$$

$$E = \frac{1 + \frac{v}{c} \cos \varphi}{\sqrt{1 - \frac{v^2}{c^2}}} E'$$

$$v = \frac{1}{\gamma} \cdot v'$$

$$E \cos \varphi = \vec{E} dx (tX + \cos \varphi)$$

$$E \cos \varphi = \frac{E}{1 + \cos \varphi \cdot tX} (tX + \cos \varphi)$$

$$\cos \varphi = \frac{tX + \cos \varphi}{1 + tX \cdot \cos \varphi} = \frac{\frac{v}{c} + \cos \varphi}{1 + \frac{v}{c} \cos \varphi}$$

Erdschwingungstransformation

rechenregel im Erdsystem

$f(t) d^3x$ die rechte von d^3x transformieren

zeigen Transformierbarkeit f ill. d^3x ?

$$dN = f d^3x \quad \text{invariant}$$

$\rightarrow d^4x$ invariant

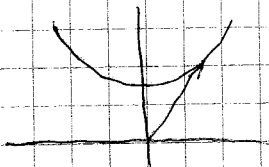
$$x^\mu = \Lambda^\mu_\nu x'^\nu$$

$$\int d^4x g(x) = \int d^4x' \underbrace{\det \Lambda}_{=1} g(x')$$

$$d^4x = d^4x'$$

cool: verstehen in Impulsraum, analytisch

$$p^2 = m^2 \quad \text{es} \quad p_0 = E > 0$$



$$d^4 p \, \delta(p^2 - m^2) \, \Theta(p_0)$$

↓
es ist invariant

$$\delta(p^2 - m^2) = \delta(p_0^2 - \vec{p}^2 - m^2) = \delta(g(p_0))$$

$$g(p_0) = p_0^2 - \vec{p}^2 - m^2$$

$$\delta(p_0) d^4 p \Rightarrow d^3 p$$

$$\delta(g(x)) = \sum_i \frac{\delta(x - x_i)}{\left| \frac{\partial g}{\partial x_i} \right|} \quad g(x_i) = 0$$

$$\delta(p^2 - m^2) = ?$$

$$p_0^2 = \vec{p}^2 + m^2$$

$$\left| \frac{\partial g}{\partial p_0} \right| = |2p_0| = 2E$$

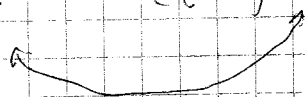
$$p_0 = \pm \sqrt{\vec{p}^2 + m^2}$$

$$\delta(p^2 - m^2) = \frac{\delta(p_0 + E)}{2E} + \frac{\delta(p_0 - E)}{2E}$$

$$E = + \sqrt{\vec{p}^2 + m^2}$$

$$d^4 p \, \delta(p^2 - m^2) \, \Theta(p_0) = d^4 p \left(\frac{\delta(p_0 + E)}{2E} + \frac{\delta(p_0 - E)}{2E} \right) \Theta(p_0)$$

$$= d^4 p \, \frac{\delta(p_0 - E)}{2E} \Rightarrow \frac{d^3 p}{2E}$$



0

Elektrodynamik

+ klassische

e: elektr

$$S = S_r + S_{re} + S_e$$

$$S_r = - \frac{1}{2} \int d\mathbf{s} \quad S_{re} = ?$$

skalare $\phi(x^r)$

$$S_{re} = \frac{1}{2} \int \phi d\mathbf{s}$$

$$\phi = \phi_0 + \Delta\phi(x^r)$$

$$S_{re} = \frac{1}{2} \int \phi_0 d\mathbf{s} + \frac{1}{2} \int \Delta\phi d\mathbf{s}$$

$$\frac{1}{2} \phi_0 \int d\mathbf{s}$$

4-v. Vektore

$$A^r = (\phi, \mathbf{A})$$

$$S_{re} = - Q \int A^r dx_r$$

$$S = S_r + S_{re} = - \frac{1}{2} \int d\mathbf{s} - Q \int A^r dx_r$$

$$= - \frac{1}{2} \int d\mathbf{s} + Q \int \mathbf{A} d^3\mathbf{x} - Q \int \phi dt$$

$$= \int L dt =$$

$$= \int (-m \sqrt{1-v^2} + Q \underline{A} v - Q \cdot \underline{\phi}) dt$$

(dynamik)

$$dx = v dt$$

$$L = -m \sqrt{1-v^2} + Q \underline{A} v - Q \cdot \underline{\phi}$$

tot. (kanon.) impuls

$$\underline{p} = \frac{\partial L}{\partial \underline{v}} = \frac{m \underline{v}}{\sqrt{1-v^2}} + Q \cdot \underline{A} = \underline{p}_{rel} + Q \cdot \underline{A}$$

Ham. fun.

$$H = \underline{v} \frac{\partial L}{\partial \underline{v}} - L = \frac{m v^2}{\sqrt{1-v^2}} + Q \underline{A} v + m \sqrt{1-v^2} - Q \underline{A} v + Q \cdot \underline{\phi} =$$

$$= \frac{m}{\sqrt{1-v^2}} + Q \cdot \underline{\phi}$$

$$(H - Q \cdot \underline{\phi})^2 = \frac{m^2}{1-v^2} = m^2 + \underbrace{\frac{m^2 v^2}{1-v^2}}_{(\underline{p} - Q \underline{A})^2} = m^2 + (\underline{p} - Q \underline{A})^2$$

$$H = \sqrt{(m^2 + (\underline{p} - Q \underline{A})^2)} + Q \cdot \underline{\phi}$$

noether'sche Lsg.

$$\frac{dP_i}{dt} = \frac{\partial L}{\partial x_i}$$

$$\frac{dP_i}{dt} = \frac{d p_{rel,i}}{dt} + Q \left(\frac{\partial A_i}{\partial t} + \frac{\partial A_i}{\partial x_j} v_j \right)$$

$$\frac{\partial L}{\partial x_i} = Q \left(\frac{\partial A_j}{\partial x_i} v_j - \frac{\partial \Phi}{\partial x_i} \right)$$

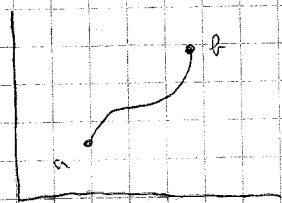
$$\frac{d \uparrow_{rel}}{dt} = Q \left(- \frac{\partial \Phi}{\partial x_i} - \frac{\partial A_i}{\partial t} \right) + Q \left(\frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j} \right) v_j$$

$$\left(\frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j} \right) v_j \underset{\substack{\uparrow \\ \text{all.}}}{=} \varepsilon_{ijl} v_j \varepsilon_{llm} \partial_l A_m = (\underline{v} \times \text{rot } \underline{A})_i$$

$$\frac{d \uparrow_{rel}}{dt} = Q \underbrace{\left(- \nabla \Phi - \frac{\partial \underline{A}}{\partial t} \right)}_{\underline{E}} + Q \underline{v} \times \underbrace{\text{rot } \underline{A}}_{\underline{B}} = Q (\underline{E} + \underline{v} \times \underline{B}) \quad \text{Lorentz = "}$$

Relativistisches 4-es formalismus

$$S = - \int_a^b m ds - \int_a^b Q A_\mu dx^\mu$$



$$\delta S = - \delta \int m ds - \delta \int Q A_\mu dx^\mu$$

$$ds = \sqrt{dx_\mu dx^\mu} \quad \delta ds = \frac{2 dx_\mu \delta(dx^\mu)}{2 \underbrace{\sqrt{dx_\mu dx^\mu}}_{ds}} =$$

$$= \frac{dx_\mu}{ds} \delta dx^\mu = u_\mu \delta dx^\mu$$

$$\delta S = - \int (m u_\mu \delta dx^\mu + Q A_\mu \delta dx^\mu + Q \delta A_\mu dx^\mu)$$

$$\delta S = \delta d$$

$$\delta S = - \int (m u_\mu + Q A_\mu) dx^\mu + Q \delta A_\mu dx^\mu$$

↓
part. int.

$$= - (m u_\mu + Q A_\mu) dx^\mu \Big|_a^b + \int (m du_\mu + Q dA_\mu) dx^\mu - \int Q \delta A_\mu dx^\mu$$

Heute noch ein bisschen

$$\delta S = \int (m du_\mu + Q dA_\mu) dx^\mu - Q \delta A_\mu dx^\mu$$

$$\delta A_\mu = \frac{\partial A_\mu}{\partial x^\nu} \delta x^\nu$$

$$dA_\mu = \frac{\partial A_\mu}{\partial x^\nu} dx^\nu$$

$$du_\mu = \frac{du_\mu}{ds} ds$$

$$dx^\mu = u^\mu ds$$

$$\delta S = \int \left(m \frac{du_\mu}{ds} + Q \left(\frac{\partial A_\mu}{\partial x^\nu} u^\nu \right) - Q \frac{\partial A_\mu}{\partial x^\nu} u^\nu \delta x^\nu \right) ds =$$

$$= \int \left(m \frac{du_\mu}{ds} + Q \frac{\partial A_\mu}{\partial x^\nu} u^\nu - Q \frac{\partial A_\nu}{\partial x^\mu} u^\nu \right) dx^\mu ds$$

$$\forall dx^\mu \quad \delta S = 0$$

$$m \frac{du_\mu}{ds} = Q \underbrace{\left(\frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu} \right)}_{F_{\mu\nu}} u^\nu$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Levi-Civita Tensor antisymmetrisches

Maxwellgleichung: $m \frac{d u^\mu}{d s} = Q F_{\mu\nu} u^\nu$

$$m \cdot a_\mu = Q F_{\mu\nu} u^\nu = f_\mu$$

$F_{\mu\nu}$ Komponenten

$$A^\mu = (\Phi, \underline{A})$$

$$A_\mu = (\Phi, -\underline{A})$$

$$F_{00} = 0$$

$$F_{\alpha\alpha} = 0$$

$$\alpha = 1, 2, 3$$

$$F_{0i} = \frac{\partial}{\partial t}(-A_i) - \frac{\partial}{\partial x_i} A_0 = -\frac{\partial A_i}{\partial t} - \frac{\partial \Phi}{\partial x_i} = E_i$$

$$F_{i0} = -E_i$$

$$F_{ij} = ?$$

$$\text{z.B. } F_{12} = \frac{\partial}{\partial x_1}(-A_2) - \frac{\partial}{\partial x_2}(-A_1) =$$

$$= \frac{\partial A_1}{\partial x_2} - \frac{\partial A_2}{\partial x_1} = -(\text{rot } \underline{A})_3 = -B_3$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$\underline{E}$ Vektor

$\underline{B}$ Vektor

4-er Impuls

4-er Impuls in "4er" Vektoren $\Rightarrow$ vereinfacht die
Berechnung

$$\delta S = - (m u_\mu + Q A_\mu) \delta x^\mu$$

$$\boxed{\pi_\mu = -\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = m \dot{x}_\mu + Q A_\mu}$$

$\underline{E}$ & $\underline{B}$ transformierens relativistisch

$$F_{\mu\nu} = \Lambda_\mu^\alpha \Lambda_\nu^\sigma F_{\alpha\sigma} = (\Lambda \cdot F \cdot \Lambda^T)_{\mu\nu}$$

$$\Lambda_\mu^\nu = \begin{pmatrix} \cosh X & -\sinh X & & \\ -\sinh X & \cosh X & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} = \begin{pmatrix} \cosh X & -\sinh X & 0 & 0 \\ +\sinh X & \cosh X & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\cdot \begin{pmatrix} \cosh X & -\sinh X & 0 & 0 \\ -\sinh X & \cosh X & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_x = E_x \quad E_y = E_y \cosh X + B_z \sinh X$$

$$E_z = E_z \cosh X - B_y \sinh X$$

$$B_x = B_x \quad B_y = B_y \cosh X - E_z \sinh X$$

$$B_z = B_z \cosh X + E_y \sinh X$$

$$c \neq 1 \quad \text{setzen} \quad \underline{B} \rightarrow c \underline{B}$$

$$\frac{v}{c} \ll 1 \quad \text{sh } X \approx 1 \quad \text{sh } X = \frac{v}{c}$$

$$E_x = E'_x \quad E_y = E'_y + v \cdot B'_z \quad E_z = E'_z - v \cdot B'_y$$

$$B_x = B'_x \quad B_y = B'_y \quad B_z = B'_z$$

Térmezők invariánsai

$$F_{\mu\nu} \quad 4\text{-es Tenzor} \quad F_{\mu\nu} F^{\mu\nu} = \text{inv.} \rightarrow \text{skalár}$$

$$\varepsilon^{\mu\nu\sigma\alpha} F_{\mu\nu} F_{\sigma\alpha} = \text{inv.} \rightarrow \text{pseudoskalár}$$

Def.

$$F_{\mu\nu} F^{\mu\nu} = 2(B^2 - E^2)$$

$$\varepsilon^{\mu\nu\sigma\alpha} F_{\mu\nu} F_{\sigma\alpha} = -8(\underline{E} \cdot \underline{B})$$

Kivételmentesek:

- ha egy rendszerben $\underline{E} \perp \underline{B} \Rightarrow \forall \text{ rez. -ben } \underline{E} \perp \underline{B}'$
- ha egy rendszerben $|\underline{E}| = |\underline{B}| \Rightarrow \forall \text{ rez. -ben } |\underline{E}'| = |\underline{B}'|$
- ha legalább az egyik inv. $\neq 0 \Rightarrow \exists$ olyan rez., ahol $\underline{E}' \parallel \underline{B}'$
- ha $\underline{E} \cdot \underline{B} = 0 \Rightarrow \exists$ olyan rez., ahol $\underline{E}' = 0$ v. $\underline{B}' = 0$
($B^2 - E^2$ előjelétől függ)

E.M. törvényeinek

Maxwell - egyenletek (vákuumbeli)

- csak vécsették el a rel. elvvel
- feltételezzük, hogy "helyes", van konvenció alakjuk

$$1) \operatorname{rot} \underline{E} = - \frac{\partial \underline{B}}{\partial t}$$

$$2) \operatorname{div} \underline{B} = 0$$

$$3) \operatorname{div} \underline{E} = \frac{1}{\epsilon_0} \rho$$

$$4) \operatorname{rot} \underline{B} = \frac{1}{c_0} \underline{j} + \frac{\partial \underline{E}}{\partial t}$$

↑

$$\operatorname{rot} \underline{B} = \mu_0 \underline{j} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

$$\mu_0 \epsilon_0 = \frac{1}{c^2} = 1$$

formuliertes ergibt sich: $\underline{B} = \operatorname{rot} \underline{A} \Rightarrow 2)$

$$\underline{E} = - \nabla \phi - \frac{\partial \underline{A}}{\partial t} \Rightarrow 1)$$

$$3), 4) \Rightarrow \operatorname{div} \underline{j} + \frac{\partial \rho}{\partial t} = 0$$

prüfen für 1-4 ergebnisse $F_{\mu\nu}$ -val

$$3: \sum_i \frac{\partial E_i}{\partial x_i} = \frac{1}{\epsilon_0} \rho \Leftrightarrow \sum_{i=1}^3 \partial_i F_{0i} = \frac{1}{\epsilon_0} \rho$$

$$F_{00} = 0 \quad \partial^\mu = (\partial_t, -\partial_i)$$

$$\partial^\mu F_{0\mu} = \partial_t F_{00} - \sum_{i=1}^3 \partial_i F_{0i} = -\frac{1}{\epsilon_0} \rho$$

$$\boxed{\partial^\mu F_{\mu 0} = \frac{1}{\epsilon_0} \rho}$$

4: $\times$ komponente

$$(\operatorname{rot} \underline{B})_x = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \frac{1}{\epsilon_0} j_x + \frac{\partial E_x}{\partial t}$$

$$\frac{\partial F_{21}}{\partial x_2} + \frac{\partial F_{31}}{\partial x_3} = \frac{1}{\epsilon_0} j_1 + \frac{\partial F_{01}}{\partial x_0}$$

$$\frac{\partial F_{01}}{\partial x_0} - \frac{\partial F_{01}}{\partial x_1} - \frac{\partial F_{21}}{\partial x_2} - \frac{\partial F_{31}}{\partial x_3} = -\frac{1}{\epsilon_0} j_1$$

$\underbrace{\hspace{10em}}_0 \quad \quad \quad \searrow \quad \partial^\mu$

$$\partial^\mu F_{\mu 1} = -\frac{1}{\epsilon_0} j_1$$

$$\partial^\mu F_{\mu 2} = -\frac{1}{\epsilon_0} j_2$$

$$\partial^\mu F_{\mu 3} = -\frac{1}{\epsilon_0} j_3$$

$$\partial^\mu F_{\mu 0} = +\frac{1}{\epsilon_0} j_0$$

$$\Rightarrow \boxed{\partial^\mu F_{\mu\nu} = \frac{1}{\epsilon_0} j_\nu}$$

$$j_\nu = (j_0, \mathbf{j})$$

$$j^\nu = (j_0, +\mathbf{j})$$

$$\underbrace{0}_{\text{in}} = \underbrace{\partial^\nu \partial^\mu F_{\mu\nu}}_{\text{antisym}} = \frac{1}{\epsilon_0} \partial^\nu j_\nu \quad \rightarrow \quad \partial^\mu j_\mu = 0$$

kontinuitási egyenlet

1-2. egyenlet $F_{\mu\nu}$ -vel

$$\text{div } \mathbf{B} = 0 \quad \partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} = 0$$

$$\text{rot } \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad \times \text{komp} \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = 0$$

$$\partial_2 F_{03} + \partial_3 F_{02} + \partial_0 F_{32} = 0$$

$$y, z \quad \partial_1 F_{03} + \partial_3 F_{10} + \partial_0 F_{31} = 0$$

$$\partial_1 F_{02} + \partial_2 F_{10} + \partial_0 F_{21} = 0$$

1 db egyenlet

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \quad \forall \alpha \neq \beta \neq \gamma \neq \alpha - \text{ra}$$

Legyen $\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ F dualisa

Def - $\tilde{F}$ kompresszió

$$- \partial_\mu \tilde{F}^{\mu\nu} = 0 \quad \text{elv. a skálár egyenlettel}$$

$$\Rightarrow \boxed{\begin{aligned} \partial^\mu F_{\mu\nu} &= \frac{1}{\varepsilon_0} j_\nu \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0 \end{aligned}} \Leftrightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \quad \alpha \neq \beta \neq \gamma \neq \alpha$$

J. f. l. A_μ , legyen $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\partial_\alpha (\partial_\beta A_\gamma - \partial_\gamma A_\beta) + \partial_\beta (\partial_\gamma A_\alpha - \partial_\alpha A_\gamma) + \partial_\gamma (\partial_\alpha A_\beta - \partial_\beta A_\alpha) = 0$$

$$\partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) = \frac{1}{\varepsilon_0} j_\nu$$

$$\square A_\nu - \partial_\nu (\partial^\mu A_\mu) = \frac{1}{\varepsilon_0} j_\nu$$

A_μ nem egyértelmű

'teljesítmény' $A'_\mu = A_\mu + \partial_\mu f$ f tetszőleges skálár fun.

'név' transformáció

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu = \partial_\mu A_\nu + \cancel{\partial_\mu \partial_\nu f} - \partial_\nu A_\mu - \cancel{\partial_\nu \partial_\mu f} = F_{\mu\nu}$$

Legyen A'_μ olyan, hogy $\partial^\mu A'_\mu = 0$ Lorenz - 'név'

Kérdés: létezik-e olyan f , hogy $\partial^\mu A'_\mu = 0$?

$$\partial^\mu (A_\mu + \partial_\mu f) = 0$$

$$\square f = -\partial^\mu A_\mu$$

$\Rightarrow \nabla$ negativ (pl. Green für induziert)

Maxwell - eigenständig Lorentz - metrik

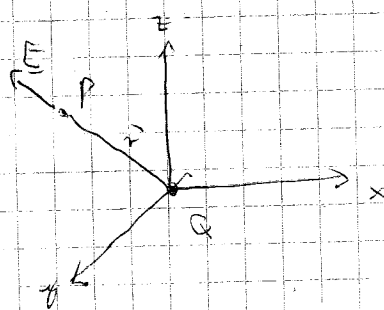
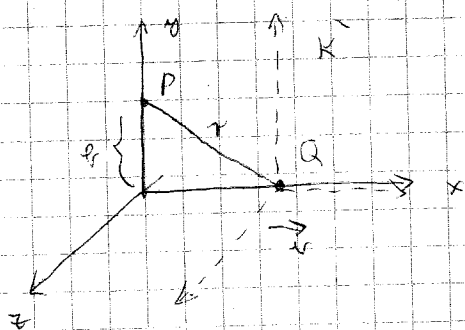
$$\square A_\nu = \frac{1}{\epsilon_0} j_\nu$$

$\Rightarrow \nabla$ negativ

$$\partial^\nu j_\nu = 0 \quad \text{erster} \quad \partial^\nu A_\nu = 0 \quad \text{ist gefüllt}$$

$\Rightarrow \nabla$ Maxwell eigenständig ist negativ

Monge'sches EM Feld



$$r^2 = l^2 + (vt)^2$$

P koordinaten:

K

t

$x = 0$

$y = l$

$z = 0$

K'

$$\vec{t} = t \cosh X - x \sinh X = t \cosh X$$

$$x' = -t \sinh X + x \cosh X = -t \sinh X$$

$$y' = y = l$$

$$z' = z = 0$$

K' -Linienelemente:

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{|\vec{r}|^2}$$

$$r'^2 = x'^2 + y'^2 = l^2 + t^2 \sinh^2 X$$

$$\vec{E}_x = \frac{\vec{x}}{r^3} \cdot (\vec{E}) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\vec{x}}{r^3} = - \frac{Q}{4\pi\epsilon_0} \cdot \frac{t \cdot dX}{(b^2 + t^2 \cdot dX^2)^{3/2}}$$

$$\vec{E}_y = \frac{\vec{y}}{r^3} \cdot (\vec{E}) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\vec{y}}{r^3} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b}{(b^2 + t^2 \cdot dX^2)^{3/2}}$$

$$\vec{E}_z = 0$$

$$\vec{B} = 0$$

$$E_x = \vec{E}_x \cdot dX = - \frac{Q}{4\pi\epsilon_0} \cdot \frac{t \cdot dX}{(b^2 + t^2 \cdot dX^2)^{3/2}} = - \frac{Q}{4\pi\epsilon_0} \cdot \frac{\frac{r}{\sqrt{1-v^2}} t}{\left(b^2 + \frac{t^2 v^2}{1-v^2}\right)^{3/2}}$$

$$E_y = \vec{E}_y \cdot dX + \vec{B}_z \cdot dX = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b \cdot dX}{(\dots)^{3/2}} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\frac{b}{1-v^2}}{(\dots)^{3/2}}$$

$$E_z = \vec{E}_z \cdot dX - \vec{B}_y \cdot dX = 0$$

$$B_x = B_y = 0$$

$$B_z = \vec{B}_z \cdot dX + \vec{E}_y \cdot dX = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b \cdot dX}{(\dots)^{3/2}} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\frac{\sqrt{1-v^2} b}{1-v^2}}{(\dots)^{3/2}}$$

$$r \ll c \quad (v \ll 1) \quad r = r'$$

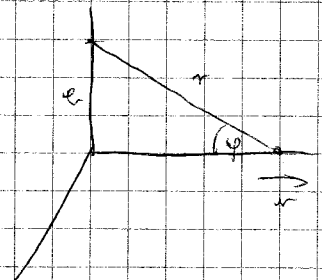
$$B = ?$$

$$B_z = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b v}{r^2}$$

$$r = t \quad \text{wenn } v \ll 1: \quad B_z = \frac{Q}{4\pi} \mu_0 \cdot \frac{b v}{r^2}$$

$$b v = |\vec{v} \times \vec{r}| = b \cdot r \sin \varphi$$

$$\Rightarrow \underline{B} = \frac{Q}{4\pi} \mu_0 \cdot \frac{\vec{v} \times \vec{r}}{r^3} \quad \text{wegen letzter ingesetzten Form}$$



$$K\text{-len } \frac{E_x}{E_y} = - \frac{t}{r} \frac{dx}{dy} = - \frac{t}{r} \Rightarrow E \parallel v$$

polárformátumban

de nem irány (Hf.)

$$\left. \begin{aligned} r &= r \cdot \sin \varphi \\ t r &= r \cdot \cos \varphi \end{aligned} \right\} \Rightarrow |E| \text{ függ } \varphi\text{-től}$$

(22f.) 2. megoldás

$$F_{\mu\nu} = -F_{\nu\mu}$$

$F_{\mu\nu}$ 2. rangszemponttal függhet

$$I) x^\mu := x^\mu_p - x^\mu_q$$

$$II) u^\mu$$

$$F_{\mu\nu} = (x_\mu u_\nu - x_\nu u_\mu) f(x_\mu x^\mu, x_\mu u^\mu, \underbrace{u_\mu u^\mu}_{\substack{\text{skalár} \\ 1}})$$

f -t olyan a részecske határozza meg, ahol

a teljes nyugalomban van: $u_\mu = (1, 0, 0, 0)$

$\Rightarrow F_{\mu\nu}$ a Coulomb-tér

Klasszikus térelmélet

$$x(t) \rightarrow \phi_2(x, t)$$

$$S = \int dt L \rightarrow S = \int d^4x \mathcal{L}$$

$\mathcal{L}$: Lagrange sűrűség

$\downarrow$

$$L = \int d^3x \mathcal{L}$$

skalár

feldtheorie $\mathcal{L}(\Phi_x, \partial_\mu \Phi_x, \dots)$

→ Lagrange-derivativteil von $\mathcal{L}$

→ Koordinatenteil explizit von $\mathcal{L}$ (kin-iso-elliptische Symmetrie)

$$S = \int d^4x \mathcal{L}(\Phi_x, \partial_\mu \Phi_x)$$

Variationsprinzip $\delta S = 0$

feldtheorie, Lagrange $\Phi_x, \partial_\mu \Phi_x \rightarrow 0$ & vergleichen

$$\delta S = \int d^4x \left(\sum_x \frac{\partial \mathcal{L}}{\partial \Phi_x} \delta \Phi_x + \sum_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \delta (\partial_\mu \Phi_x) \right)$$

$$\int d^4x \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \delta (\partial_\mu \Phi_x) = \int d^4x \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \partial_\mu (\delta \Phi_x) =$$

$$= \underbrace{\int d^4x \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \delta \Phi_x \right)}_0 - \int d^4x \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \right) \delta \Phi_x$$

$$\delta S = \sum_x \int d^4x \left(\frac{\partial \mathcal{L}}{\partial \Phi_x} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \right) \delta \Phi_x$$

$$\delta S = 0 \quad \forall \delta \Phi_x$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial \Phi_x} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_x} \right)} \quad \forall x$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \quad \text{relativistisch}$$

variational

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi \quad \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \right) = \partial_\mu \partial^\mu \phi = \square \phi$$

$$\square \phi = -m^2 \phi \quad (\square + m^2) \phi = 0$$

$\mathcal{L}$ ist invariant unter Translationen: $\partial_\mu \mathcal{L} = 0$

$$\tilde{\mathcal{L}} = \mathcal{L} + \partial_\mu f^\mu(\phi, \partial_\mu \phi)$$

ergibt sich die Bedingung, dass $\oint f^\mu dF_\mu = 0$

Energie-Impuls Tensor

kanonisch $\quad T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_\nu} \dot{\phi}_\mu - \mathcal{L}$

$$H = \sum_k \frac{\partial \mathcal{L}}{\partial \dot{\phi}_k} \dot{\phi}_k - \mathcal{L}$$

$\Rightarrow$ kanonische Ableitungen $\quad \dot{\phi}_k \rightarrow \partial_\mu \phi_k$

ergibt sich die Bedingung $\Rightarrow H$ invariant unter (μ, ν)

Def:

$$T^{\mu\nu} = \sum_k \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_k} \partial^\nu \phi_k - \mathcal{L} \cdot \eta^{\mu\nu}$$

Energie-Impuls Tensor

$$T^{00} = \sum_k \frac{\partial \mathcal{L}}{\partial \dot{\phi}_k} \dot{\phi}_k - \mathcal{L} = \text{Energiedichte}$$

$T^{\mu\nu}$ "negatív" négyzet a mozgásegyenlet megoldásán
 alapján az értelemben, hogy $\partial_\mu T^{\mu\nu} = 0$

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= \sum_k \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_k)} \partial^\nu \phi_k \right) - \partial^\nu \mathcal{L} = \\ &= \sum_k \left(\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_k)} \right) \partial^\nu \phi_k + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_k)} \partial_\mu \partial^\nu \phi_k \right) - \partial^\nu \mathcal{L} = \\ &= \sum_k \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \phi_k} \partial^\nu \phi_k + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_k)} \partial^\nu (\partial_\mu \phi_k) \right)}_{\partial^\nu \mathcal{L}} - \partial^\nu \mathcal{L} = 0\end{aligned}$$

mozgásegyenlet

negatív

$$0 = \int_{x_0 = \text{const}} \partial_\mu T^{\mu\nu} d^3x = \partial_0 \int_{x_0 = \text{const}} T^{0\nu} d^3x + \underbrace{\int_{x_0 = \text{const}} \partial_i T^{i\nu} d^3x}_{\substack{\nearrow \\ \text{3-as divergencia}}} =$$

$$= \frac{d}{dt} \int_{x_0 = \text{const}} T^{0\nu} d^3x = 0$$

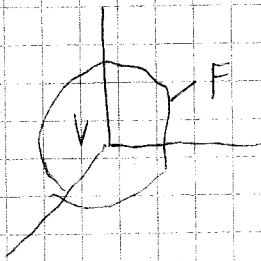
$$\uparrow^\nu = \int_{x_0 = \text{const}} T^{0\nu} d^3x = \text{const} \quad : \text{ teljes negatívimpulzus}$$

$$\uparrow^0 = \int T^{00} d^3x = \int \mathcal{H} d^3x \quad : \text{ teljes energia}$$

T^{0i} : ^{h h}impuls ^{h h}stromung

$$T^{\mu\nu} = \begin{pmatrix} \text{ge} & \text{imp. stromung} \\ | & \\ | & \\ | & \end{pmatrix}$$

Ma: T^{i0} ill T^{ij} ^Pgeletem?



$$0 = \int_{x_0 = \text{limit}}^{x_0 + \text{limit}} \partial_\mu T^{\mu\nu} d^3x = \frac{d}{dt} \int_V T^{0\nu} d^3x +$$

$$+ \int_V \partial_i T^{i\nu} d^3x =$$

$$= \frac{d}{dt} \int_V T^{0\nu} d^3x + \oint_{\partial V} T^{i\nu} dF_i = 0$$

$$\frac{d}{dt} \int_V T^{0\nu} d^3x = - \oint_{\partial V} T^{i\nu} dF_i$$

T^{i0} : ^{h h}reggimpuls ^{h h}stromung

T^{00} : ^{h h}energia ^{h h}stromung $\frac{d}{dt} \int \text{ge} d^3x = - \oint T^{i0} dF_i$

T^{ij} : ^{h h}impuls ^{h h}stromung

Menyir: egentligen $T^{\mu\nu}$?

$$\hat{T}^{\mu\nu} = T^{\mu\nu} + \partial_\beta \Lambda^{\beta\mu\nu} \quad \text{där} \quad \Lambda^{\beta\mu\nu} = -\Lambda^{\mu\beta\nu}$$

alltså

$$\partial_\mu \hat{T}^{\mu\nu} = \partial_\mu T^{\mu\nu} + \partial_\mu \partial_\beta \Lambda^{\beta\mu\nu} = 0$$

enklare definition: $T^{\mu\nu} \neq T^{\nu\mu}$

enklare, bygg $\hat{T}^{\mu\nu} = \hat{T}^{\nu\mu}$

måst kolla på?

Impulsmomentum

enklare: $J^{\mu\nu} = \sum_k (x_k^\mu p_k^\nu - x_k^\nu p_k^\mu)$

not: $\sum_k \rightarrow \int d^4x$

$$p_k^\mu \rightarrow T^{0\mu}$$

$$J^{\mu\nu} = \int d^3x \underbrace{(x^\mu T^{0\nu} - x^\nu T^{0\mu})}_{\text{se på } M^{\alpha\mu\nu} \text{ eller } M^{\nu\mu\alpha}}$$

komponenter

$$M^{\alpha\mu\nu} = x^\mu T^{\alpha\nu} - x^\nu T^{\alpha\mu}$$

$$J^{\mu\nu} = \int d^3x M^{0\mu\nu}$$

alltså, $\partial_\alpha M^{\alpha\mu\nu} = 0 \Rightarrow J^{\mu\nu}$ är konstant

szimmetria $\partial_\alpha M^{\alpha\mu\nu} = 0 \quad -t$

$$\partial_\alpha (x^\mu T^{\alpha\mu} - x^\nu T^{\alpha\nu}) = 0$$

$$\underbrace{\partial_\alpha T^{\alpha\nu}}_0 + x^\mu \underbrace{\partial_\alpha T^{\alpha\mu}}_0 - \underbrace{\partial_\alpha T^{\alpha\mu}}_0 - x^\nu \underbrace{\partial_\alpha T^{\alpha\mu}}_0 = 0$$

$$T^{\mu\nu} - T^{\nu\mu} = 0$$

$$T^{\mu\nu} = T^{\nu\mu} \iff \partial_\alpha M^{\alpha\mu\nu} = 0$$

Ih $\partial_\mu T^{\mu\nu} = 0$ és $T^{\mu\nu} = T^{\nu\mu}$ akkor az olyan anyagok esetében

is le, ahol megmarad az energia, az impulzus és az impulzusmomentum.

EM ter Lagrange sűrűsége

Mi a fundamentális ter? $F_{\mu\nu}$ vagy A_μ ?

$\rightarrow A_\mu$ (nem kell extra kiegészítés)

$\rightarrow$ megadja a fundamentális egyenletet)

$\mathcal{L}(A_\mu, \partial_\nu A_\mu)$ és legyen 'antikiválasztás' (leghalál a kérés)

leletesség $F_{\mu\nu} \quad F^{\mu\nu} \quad \tilde{F}_{\mu\nu} \quad \tilde{F}^{\mu\nu} \quad F_{\mu\nu} \quad \tilde{F}^{\mu\nu}$
 $\nearrow \quad \nearrow$
 közös konstansmód
 sz. teljes megdivergencia

Legyen $\mathcal{L} = a \cdot F_{\mu\nu} F^{\mu\nu}$

Kétszeres a töltés és a mágneses tér

1. töltés: $S_{\text{int}} = -Q \int A_\mu dx^\mu \rightarrow$

$\Leftrightarrow$ átalakítás $S_{\text{int}} = - \int A_\mu j^\mu d^4x$

vagy 2 db kétszeres (mágneses $\rightarrow$ töltés ill. töltés $\rightarrow$ mágnes)

hasonlóan csak 1 (mágneses $\leftrightarrow$ töltés)

$$\mathcal{L} = a F_{\mu\nu} F^{\mu\nu} - A_\mu j^\mu$$

invariancia!

$$A'_\mu = A_\mu + \partial_\mu \phi$$

$$\mathcal{L}' = a F_{\mu\nu} F^{\mu\nu} = A_\mu j^\mu - j^\mu \partial_\mu \phi =$$

$$= \mathcal{L} - \partial_\mu (\phi j^\mu) + \underbrace{j^\mu \partial_\mu \phi}_0$$

$\downarrow$
4-es divergencia

$$\vec{J} = S$$

megjegyzések

$$\mathcal{L} = a (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial^\mu A^\nu - \partial^\nu A^\mu) - A_\mu j^\mu$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} \right) = \frac{\partial \mathcal{L}}{\partial A_\nu} \quad (\phi \leftrightarrow A_\nu)$$

$$\partial_\mu \alpha \cdot 4 (\partial^\mu A^\nu - \partial^\nu A^\mu) = - \partial^\nu$$

$$\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} j^\nu \rightarrow \alpha := - \frac{\epsilon_0}{4}$$

$$\mathcal{L} = - \frac{\epsilon_0}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu j^\mu$$

Energie-impuls-tensor

1) Felder selbst ($j^\mu = 0$)

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\sigma)} \partial^\nu A_\sigma - \eta^{\mu\nu} \mathcal{L} =$$

$$= - \epsilon_0 F^{\mu\alpha} \partial^\nu A_\alpha + \frac{\epsilon_0}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

Komponenten (22f.)

$$T^{00} = \frac{\epsilon_0}{2} (-E^2 + B^2) = \epsilon_0 E \underbrace{\frac{\partial A}{\partial t}}_{= -\nabla\phi} = \frac{\epsilon_0}{2} (E^2 + B^2) + \epsilon_0 E \underbrace{\nabla\phi}_{\nabla(E\phi)}$$

, weil $\nabla E = 0$

da maximal feldfrei

$$T^{0i} = \epsilon_0 (E \times B)_i + \epsilon_0 \nabla(A_i E)$$

$$T^{i0} = \epsilon_0 (E \times B)_i + \epsilon_0 ((\nabla \times \phi B)_i - \frac{\partial}{\partial t} (\phi E_i))$$

man sieht, beiderseits, da

$$\int d^3x T^{00} = \int \frac{\epsilon_0}{2} (E^2 + B^2) d^3x$$

$$\int d^3x T^{0i} = \int \epsilon_0 (\mathbf{E} \times \mathbf{B})_i d^3x$$

Symmetrisierung!

$$\partial^\nu A_\sigma \text{ liefert 'symmetrisch'} F^\nu_\sigma + \partial_\sigma A^\nu$$

$$T^{\mu\nu} = \underbrace{-\epsilon_0 F^{\mu\sigma} F^\nu_\sigma + \frac{\epsilon_0}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}}_{\text{sym.}} - \epsilon_0 F^{\mu\nu} \partial_\sigma A^\sigma \quad \text{tats. div.}$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\sigma)} \partial^\nu A_\sigma - \eta^{\mu\nu} \mathcal{L} = -\epsilon_0 F^{\mu\sigma} \partial^\nu A_\sigma + \frac{\epsilon_0}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

$$-\epsilon_0 F^{\mu\sigma} \partial_\sigma A^\nu = \epsilon_0 F^{\sigma\mu} \partial_\sigma A^\nu = \epsilon_0 \partial_\sigma (F^{\sigma\mu} A^\nu) -$$

$$-\epsilon_0 \partial_\sigma F^{\sigma\mu} A^\nu = \epsilon_0 \partial_\sigma (F^{\sigma\mu} A^\nu)$$

$\parallel$
 0
 $A^{\sigma\mu\nu}$

$$T^{\mu\nu} = \epsilon_0 F^{\mu\sigma} F^\nu_\sigma + \frac{\epsilon_0}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

symmetrisch

$$G^{\mu\nu} = \epsilon_0 \left(F^{\mu\sigma} F^\nu_\sigma + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

$$\Theta^{00} = \frac{\epsilon_0}{2} (E^2 + B^2) = u$$

$$\Theta^{0i} = \epsilon_0 (\mathbf{E} \times \mathbf{B})_i = g_i \quad (c \cdot g_i) \quad \text{Poynting-Vektor}$$

$$\Theta^{i0} = \Theta^{0i}$$

$$\partial^\mu j^\nu = -\epsilon_0 \left[E^\nu E^\mu + B^\nu B^\mu - \frac{1}{2} \delta^{\mu\nu} (E^2 + B^2) \right] = -T^{\mu\nu}_{EM}$$

Maxwell-feld energietensor

$$\Theta^{\mu\nu} = \epsilon_0 \left(F^{\mu\sigma} F_\sigma^\nu + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \quad \begin{array}{l} j^\nu = 0 \\ \partial_\mu F^{\mu\nu} = 0 \end{array}$$

Falsch! $\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} j^\nu \neq 0$

$$\partial_\mu \Theta^{\mu\nu} \neq 0$$

W! $\partial_\mu \Theta^{\mu\nu} = -F^{\nu\sigma} j_\sigma$

$$\begin{aligned} \partial_\mu \Theta^{\mu\nu} &= \epsilon_0 \left[\partial_\mu (F^{\mu\sigma} F_\sigma^\nu) + \frac{1}{4} \partial^\nu (F_{\alpha\beta} F^{\alpha\beta}) \right] = \\ &= \epsilon_0 \left[\underbrace{F_\sigma^\nu \partial_\mu F^{\mu\sigma}}_{= F_\sigma^\nu \frac{1}{\epsilon_0} j^\sigma} + F^{\mu\sigma} \partial_\mu F_\sigma^\nu + \frac{1}{2} F_{\alpha\beta} \partial^\nu F^{\alpha\beta} \right] \end{aligned}$$

$$\begin{aligned} \partial_\mu \Theta^{\mu\nu} + F^{\nu\sigma} j_\sigma &= \epsilon_0 \left[F^{\mu\sigma} \partial_\mu F_\sigma^\nu + \frac{1}{2} F_{\alpha\beta} \partial^\nu F^{\alpha\beta} \right] = \\ &= \epsilon_0 \frac{1}{2} \left[F_{\mu\sigma} \partial^\mu F^{\sigma\nu} + F_{\mu\sigma} \partial^\mu F^{\sigma\nu} + F_{\mu\sigma} \partial^\nu F^{\mu\sigma} \right] = \\ &= \frac{\epsilon_0}{2} F_{\mu\sigma} \left[\partial^\mu F^{\sigma\nu} + \partial^\mu F^{\sigma\nu} + \partial^\nu F^{\mu\sigma} \right] = \\ &= \partial^\sigma F^{\nu\mu} = \partial^\sigma F^{\mu\nu} \end{aligned}$$

$$= \frac{\epsilon_0}{2} F_{\mu\sigma} \underbrace{(\partial^\mu F^{\sigma\nu} + \partial^\sigma F^{\mu\nu})}_{\mu, \sigma\text{-lm symmetrisches}} = 0$$

↙
antisymmetrisches

↘
μ, σ-lm symmetrisches

$$F^{\nu\sigma} g_{\sigma} \quad \text{Lorentz-erw.} \quad \text{summiert}$$

$$Q F^{\nu\sigma} u_{\sigma}$$

$$f^{\nu} = F^{\nu\sigma} g_{\sigma} = (f^E, f^B + f \times B)$$

Lorentz-erw.

$$F = \int f d^3x = \frac{d \text{ "momentum" }}{dt}$$

note:

$$\int \partial_{\mu} \Theta^{\mu\nu} d^3x = \frac{d}{dt} \int \Theta^{0\nu} d^3x + \underbrace{\int \partial_i \Theta^{i\nu} d^3x}_{\oint \Theta^{i\nu} dF_i = 0} = \frac{d}{dt} \uparrow_{\text{new}}$$

$$0 = \int (\partial_{\mu} \Theta^{\mu\nu} + F^{\nu\sigma} g_{\sigma}) d^3x = \int (\partial_{\mu} \Theta^{\mu\nu} + f^{\nu}) d^3x =$$

$$= \frac{d}{dt} \uparrow_{\text{new}} + \frac{d}{dt} \uparrow_{\text{reversed}} = \frac{d}{dt} (\uparrow_{\text{new}} + \uparrow_{\text{reversed}})$$

Relativistisches Quantenmechanik

Schrodinger-Gleichung

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(x) \right] \psi$$

non relativ

$$\hat{E} = \frac{\hat{p}^2}{2m} + V$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t} \quad \hat{p} = -i\hbar \nabla$$

Äquivalenz 'ogg, Logg u. rel. E-p relativ
 tejszultsai (relativ eseten)

$$E^2 = \vec{p}^2 + m^2$$

$$\left(-\hbar^2 \frac{\partial^2}{\partial t^2} + \hbar^2 \Delta - m^2 \right) \psi = 0$$

$$\left(\underbrace{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}}_{\square} + \frac{m^2}{\hbar^2} \right) \psi = 0$$

$$\square = \partial_\mu \partial^\mu$$

$$\left(\square + \frac{m^2}{\hbar^2} \right) \psi = 0$$

Klein-Gordon egyenlet

4-es impulzus operátor: $p^\mu = i\hbar \frac{\partial}{\partial x_\mu} = i\hbar \partial^\mu$

Mik a KG egy. megoldásai?

Keressük a m.o.-t $\psi(x,t) = A e^{-\frac{i}{\hbar} p^\mu x_\mu} = A e^{\frac{i}{\hbar} (E \cdot t - \vec{p} \cdot \vec{x})}$

$$\partial_\mu \partial^\mu \psi = -\frac{1}{\hbar^2} p_\mu p^\mu \psi$$

$$\left(-\frac{1}{\hbar^2} p_\mu p^\mu + \frac{m^2}{\hbar^2} \right) \psi = 0 \Rightarrow p_\mu p^\mu = m^2$$

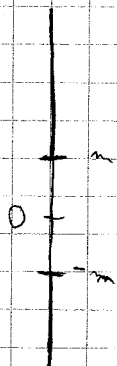
p^μ - nek 3 független komponense van $p^\mu = (E, \vec{p})$
 plusz $\vec{p}$

$$E^2 = \vec{p}^2 + m^2 \quad E = \pm \sqrt{\vec{p}^2 + m^2}$$

$$E \geq m$$

$$E \leq -m$$

wann $E < 0$ negativ & antineutrino



Mit der Dirac-Gleichung

$$\left(\partial_\mu \partial^\mu + \frac{m^2}{\hbar^2} \right) \psi = 0 \quad / \cdot \psi^*$$

$$- \left(\partial_\mu \partial^\mu + \frac{m^2}{\hbar^2} \right) \psi^* = 0 \quad / \cdot \psi$$

$$\psi^* \partial_\mu \partial^\mu \psi - \psi \partial_\mu \partial^\mu \psi^* = 0$$

$$\partial_\mu (\psi^* \partial^\mu \psi - \psi \partial^\mu \psi^*) = 0$$

$$j^\mu = \frac{i\hbar}{2m} (\psi^* \partial^\mu \psi - \psi \partial^\mu \psi^*)$$

↑

norm. Strom

j^μ : normierter Strom

$$j^\mu = (\rho, \mathbf{j})$$

$$\int \rho d^3x = \text{const}$$

$$\rho = \frac{i\hbar}{2m} (\psi^* \dot{\psi} - \dot{\psi} \psi^*)$$

$K \leq 0$ ergibt "normierter"

$t = 0$ -> $\psi, \dot{\psi}$ "totales"

$\rho \leq 0$ ist "Belastung"

Mu a j^μ jelentése?

Itt a KG (Dirac) egyenlet csak töltött részecskéket ír le

$$\Rightarrow \partial_\mu j^\mu = 0 : \text{töltésmegmaradás}$$

$g > 0$ és $g < 0$ is lehetséges
részecske antirészecske

$$j^\mu = \frac{ie\hbar}{2m} (\psi^* \partial^\mu \psi - \psi \partial^\mu \psi^*)$$

$$g = \frac{ie\hbar}{2m} (\psi^* \cdot \dot{\psi} - \dot{\psi} \cdot \psi^*)$$

Köröszték szélhullám megoldása

$$\psi = A e^{-\frac{i}{\hbar} \vec{p} \cdot \vec{r} + iEt} = A e^{\frac{i}{\hbar} (\vec{p} \cdot \vec{r} - Et)}$$

$$E = \pm \sqrt{p^2 + m^2}$$

$$\psi_{\pm} = A_{\pm} e^{\frac{i}{\hbar} (\vec{p} \cdot \vec{r} \mp |E_{\pm}| t)}$$

Számoljuk ki $g_{\pm}$ -t:

$$g_{\pm} = \pm \frac{e|E_{\pm}|}{m} \cdot \psi_{\pm}^* \psi_{\pm}$$

$$g_{+} = \frac{e|E_{+}|}{m} \psi_{+}^* \psi_{+} > 0$$

$$g_{-} = -\frac{e|E_{-}|}{m} \psi_{-}^* \psi_{-} < 0$$

Teljes ψ_{+} + c töltött részecskét ír le

ψ_{-} - c töltött részecskét ír le

relativisztikus nem marad meg a részecske

$\rightarrow \infty$ sebességre foh $\rightarrow$ kvantumszámok

KG egyenlet klasszikus megközelítés

0 spinű töltött részecskék in la

$\rightarrow$ másodrendű $\Psi \rightarrow \hat{\Psi}$

Durac egyenlet

relativisztikus elmozdító egyenlet

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

kovariancia $\Rightarrow \hat{H}$ tartalmazza első deriváltakat

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-i\hbar \left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} \right) + \beta mc^2 \right] \Psi = \hat{H} \Psi$$

α, β nem látható mágisch (vagy a kovariancia

vagy $E^2 = p^2 c^2 + m^2 c^4$ miatt)

Legyen Ψ töltés komponensű

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{pmatrix} \quad \alpha, \beta \text{ mátrixok}$$

azt is szeretnénk $\mathcal{G} = \Psi^* \Psi = \sum_i \Psi_i^* \cdot \Psi_i \geq 0$

$$\mathcal{G} = \mathcal{G}^\dagger \quad \text{ahol} \quad \partial_\mu \mathcal{G}^\mu = 0$$

Feladat:
$$i\hbar \frac{\partial \Psi_i}{\partial t} = \sum_j \left[-i\hbar \left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} \right) \alpha_{ij} + \beta_{ij} mc^2 \right] \Psi_j$$

a) $E^2 = p^2 c^2 + m^2 c^4$

b) $\mathcal{G} = \Psi^* \Psi = \mathcal{G}^\dagger$, ahol $\partial_\mu \mathcal{G}^\mu = 0$

c) Lorentz - kovariancia

a) $\Rightarrow \Psi_i$ teigweise a. KG eigenfunkt. $\left\{ \frac{\partial}{\partial x_i} \right.$ von hermit. u. invertiert

$\Rightarrow$ KG $\forall$ kommutierend

$$\left(\partial_\mu \partial^\mu + \frac{m^2}{\hbar^2} \right) \Psi_i = 0$$

miss ableiten $-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = (-\hbar^2 \nabla^2 + m^2) \Psi_i$

missen

$$-\hbar^2 \frac{\partial^2 \Psi_i}{\partial t^2} = i\hbar \frac{\partial}{\partial t} \left(i\hbar \frac{\partial \Psi_i}{\partial t} \right) = i\hbar \frac{\partial}{\partial t} \left(\sum_j \left[-i\hbar \alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3} + \beta_{ij} m \right] \Psi_j \right)$$

$$-\hbar^2 \frac{\partial^2 \Psi_i}{\partial t^2} = -\hbar^2 \sum_{a,1} \frac{\alpha_a \alpha_a + \alpha_a \alpha_a}{2} \frac{\partial^2 \Psi}{\partial x_a \partial x_a} =$$

$$-i\hbar m \sum_a (\alpha_a \beta + \beta \alpha_a) \frac{\partial \Psi}{\partial x_a} + \beta^2 m^2 \Psi$$

$$\{\alpha_a, \alpha_a\} = 2\delta_{a,a} \mathbb{1}$$

$$\{\alpha_a, \beta\} = 0$$

$$\alpha_a^2 = \beta^2 = 1$$

ist adjungiert, folgt $\hat{H} = \hat{H}^\dagger$, also

$$\alpha_a^\dagger = \alpha_a, \quad \beta^\dagger = \beta$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-i\hbar \sum_i \alpha_i \partial_i + \beta m \right) \Psi$$

$$E^2 = p^2 + m^2$$

$$\{\alpha_n, \alpha_m\} = 2\delta_{nm} \cdot 1$$

$$\{\alpha_n, \beta\} = 0$$

$$\alpha_n^2 = \beta^2 = 1 \quad \alpha_n^+ = \alpha_n \quad \beta^+ = \beta$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix} \quad N \geq 4 \text{ hell}$$

Egg beliebigen $N=4$ werden

$$\alpha_n = \begin{pmatrix} 0 & \sigma_n \\ \sigma_n & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\alpha_1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} \quad \alpha_2 = \begin{pmatrix} & & & -i \\ & & i & \\ & -i & & \\ i & & & \end{pmatrix}$$

$$\alpha_3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{pmatrix} \quad \beta = \begin{pmatrix} +1 & & & \\ & +1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$(1) \quad i\hbar \frac{\partial \Psi}{\partial t} = \left(-i\hbar \sum_i \cancel{\partial_i} \mathcal{L}_i \partial_i + \beta \cdot m \right) \Psi$$

$$(2) \quad -i\hbar \frac{\partial \Psi^+}{\partial t} = +i\hbar \sum_i \underbrace{\partial_i \Psi^+}_{\mathcal{L}_i} \underbrace{\mathcal{L}_i^+}_{\beta} + m \underbrace{\Psi^+}_{\beta} \beta$$

$$\Psi^+ \cdot (1) - (2) \cdot \Psi$$

$$i\hbar \left(\Psi^+ \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi^+}{\partial t} \Psi \right) = -i\hbar \sum_i \left(\Psi^+ \mathcal{L}_i \partial_i \Psi + \partial_i \Psi^+ \mathcal{L}_i \Psi \right) + \cancel{\Psi^+ \beta m \Psi} - \cancel{\Psi^+ \beta m \Psi}$$

$$i\hbar \frac{\partial}{\partial t} (\Psi^+ \Psi) = -i\hbar \sum_i \partial_i (\Psi^+ \mathcal{L}_i \Psi)$$

$$\frac{\partial g}{\partial t} + \text{div } \vec{g} = 0 \quad g = \Psi^+ \Psi = \sum_i \Psi_i^* \Psi_i \geq 0$$

$$g_i = \Psi^+ \mathcal{L}_i \Psi$$

Kontroll: $(g, \vec{g})$ reigenvektoren abstr.-e?

Diese eigenlet 4-er als

$$\frac{\partial}{\partial t} \quad \partial_i \quad \Rightarrow \quad \partial_\mu \quad \alpha^\mu \partial_\mu$$

$$\text{Legen } \gamma^0 = \beta \quad \gamma^i = \beta \cdot \mathcal{L}_i$$

$$\boxed{(i\hbar \gamma^\mu \partial_\mu + m) \Psi = 0}$$

problem: γ^μ von bloßes vektoren transformiert

$$\cancel{\gamma^\mu = \Lambda^\mu_\nu \gamma^\nu} \quad \gamma^0 \text{ von lokal}$$

Dírac - egyenlet kovarianciája

$$\Lambda: K' \rightarrow K \quad \text{Lorentz-transfo}$$

$$x^\mu = \Lambda^\mu_\nu x'^\nu$$

Kérdés: Hogyan transformálódik a Ψ ?

$$\Psi(x) = R(\Lambda) \Psi'(\underbrace{\Lambda^{-1}x}_{\tilde{x}})$$

$$(i\hbar \gamma^\mu \partial_\mu - m) \Psi(x) = 0$$

$$(i\hbar \gamma'^\mu \partial'_\mu - m) \Psi'(x) = 0$$

$$\boxed{\gamma'^\mu = \gamma^\mu}$$

$$\partial_\mu = \Lambda_\mu^\nu \partial'_\nu$$

$$(i\hbar \gamma^\mu \Lambda_\mu^\nu \partial'_\nu - m) R(\Lambda) \Psi'(\tilde{x}) = 0 \quad / R^{-1}(\Lambda) \cdot \text{balra!}$$

$$i\hbar \underbrace{\Lambda_\mu^\nu R^{-1} \gamma^\mu R(\Lambda)}_{\gamma'^\nu \partial'_\nu} \partial'_\nu \Psi' - m \Psi' = 0$$

$$\boxed{\gamma'^\nu = \Lambda_\mu^\nu R^{-1} \gamma^\mu R}$$

határozzuk meg R -et

Hely: $\boxed{R \gamma^\nu R^{-1} = \Lambda_\mu^\nu \gamma^\mu}$

$$\boxed{\Lambda^\mu_\nu \gamma'^\nu = R^{-1} \gamma^\mu R}$$

Belichtete, lang fortgesetzte R unter $[R, \gamma_0] = 0$
 betrachten, lassen

$$R^{-1} = \gamma_0 R^+ \gamma_0$$

$$L_i = \gamma_0 \gamma_i$$

4-es sein:

$$j^M = \Psi^+ \gamma^0 \gamma^M \Psi$$

Dies konjugiert: $\bar{\Psi} = \Psi^+ \gamma^0$

$$j^M = \bar{\Psi} \gamma^M \Psi$$

$$j^M = \Psi^+ \gamma^0 \gamma^M \Psi = \Psi^+ R^+ (\Lambda) \gamma^0 \gamma^M R (\Lambda) \Psi =$$

$$= \Psi^{1+} \gamma^0 \underbrace{R^{-1} (\Lambda) \gamma^M R (\Lambda)}_{\Lambda^M_\nu \gamma^\nu} \Psi = \Lambda^M_\nu \underbrace{\Psi^+ \gamma^0 \gamma^\nu \Psi}_{j^\nu} = \Lambda^M_\nu j^\nu$$

$$R^+ \gamma^0 = \gamma^0 R^{-1}$$

Hilf: $\bar{\Psi} \Psi = \Psi^+ \gamma^0 \Psi$ Skalar

Diese ergibt sich durch negativ

2. Strategie: 1) Einsetzen

2) ng. reibereiben $\rightarrow$ lost

$$i \not{D} \frac{\partial \Psi}{\partial t} = \hat{H} \Psi = \left(\sum_i L_i \hat{\pi}_i + m \beta \right) \Psi \quad \hat{\pi}_i = -i \hbar \partial_i$$

Stationarius regellos

$$\Psi(x, \frac{t}{\tau}) = \Psi(x) e^{-\frac{i}{2} \varepsilon t} \quad \text{also: regellos}$$

$$\hat{H} \Psi = \varepsilon \Psi$$

setzen $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \varphi \\ X \end{pmatrix}$

$$\varphi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$X = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

$$\varepsilon \begin{pmatrix} \varphi \\ X \end{pmatrix} = \sum_i \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \hat{r}_i \begin{pmatrix} \varphi \\ X \end{pmatrix} + m \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \varphi \\ X \end{pmatrix}$$

$$\begin{cases} \varepsilon \cdot \varphi = \sum_i \sigma_i \hat{r}_i X + m \varphi \\ \varepsilon \cdot X = \sum_i \sigma_i \hat{r}_i \varphi - m X \end{cases}$$

Kernsich φ, X - t. umhüllen sollen

$$\begin{pmatrix} \varphi \\ X \end{pmatrix} = \begin{pmatrix} \varphi_0 \\ X_0 \end{pmatrix} e^{\frac{i}{2} \varepsilon t}$$

$$\varphi, X \rightarrow \varphi_0, X_0 \quad \hat{r}_i \rightarrow r_i$$

$$\begin{cases} (\varepsilon - m) \varphi_0 - \sum_i \sigma_i r_i X_0 = 0 \\ - \sum_i \sigma_i r_i \varphi_0 + (\varepsilon + m) X_0 = 0 \end{cases}$$

homogenes lineares Gleichungssystem

$$A \begin{pmatrix} \varphi_0 \\ X_0 \end{pmatrix} = 0 \quad \det A = 0$$

$$(\varepsilon + m)(\varepsilon - m) - \underbrace{\left(\sum_i \sigma_i \uparrow_i \right) \left(\sum_j \sigma_j \uparrow_j \right)}_{\uparrow^2} = 0$$

Ansatz: $(\alpha A)(\alpha B) = A \cdot B + i \alpha (A \times B)$ 222

$$\varepsilon^2 - m^2 - \uparrow^2 = 0 \quad \varepsilon = \pm \sqrt{\uparrow^2 + m^2} = \lambda \cdot E_{\uparrow}$$

$$E_{\uparrow} = \sqrt{\uparrow^2 + m^2} \quad \lambda = \pm 1$$

$$\uparrow_{\phi} = \frac{\sum_i \sigma_i \uparrow_i}{m + \varepsilon} \quad \phi_0 = \frac{\sum_i \sigma_i \uparrow_i}{m + \lambda E_{\uparrow}} \quad \phi_0$$

Ansatz $\phi_0 \equiv N u = N \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ by normalis, by

N : normalis

$$u^+ u = u_1^* u_1 + u_2^* u_2 = 1$$

Ansatz:

$$\psi_{\uparrow, \lambda}(x, t) = N \cdot \begin{pmatrix} u \\ \frac{\sigma \uparrow}{m + \lambda E_{\uparrow}} u \end{pmatrix} \cdot e^{\frac{i}{\hbar} (\uparrow x - \lambda E_{\uparrow} t)}$$

normalis

$$\int \psi_{\uparrow, \lambda}^+(x) \psi_{\uparrow, \lambda}(x) d^3x = \delta_{\lambda \lambda} \cdot \delta(\uparrow - \uparrow)$$

$$\Rightarrow N = \frac{1}{(\sqrt{2\pi\hbar})^3} \sqrt{\frac{m + \lambda E_{\uparrow}}{2\lambda E_{\uparrow}}}$$

$\lambda = \pm 1$ normalis $\leftrightarrow$ antinormalis

u-lan az elmozdítás $\rightarrow$ spin
helicitás

Spin operátor

$$S_z = \frac{\hbar}{2} \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$$

Helicitás

$$\hat{\Lambda}_S = \sum_i \hat{S}_i \cdot \frac{\hat{p}_i}{|\hat{p}|}$$

all: $\hat{\Lambda}_S$ kommutál $\hat{H}$ -vel és $\hat{p}$ -vel

1) $\hat{S} \cdot \hat{p}$

2) $\hat{p}^2 = m^2$

Kísérlet $(\hat{H}, \hat{\Lambda}_S, \hat{p})$ sajátállapotok

megadjuk a részecske a z tengely mentén

$$\hat{\Lambda} = \hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & & 0 \\ & -1 & \\ 0 & & 1 \\ & & & -1 \end{pmatrix}$$

$$\hat{p} = (0, 0, p)$$

s.d. $\pm \frac{\hbar}{2}$

az

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

+ -

$$\psi_{\uparrow, \lambda, +\frac{1}{2}} = N \cdot \begin{pmatrix} 1 \\ 0 \\ \frac{\lambda}{m+\lambda} \uparrow \\ m+\lambda E_{\uparrow} \\ 0 \end{pmatrix} e^{\frac{i}{\hbar} (\uparrow z - \lambda E_{\uparrow} t)}$$

$$\sigma_z = \frac{\hbar}{2} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\psi_{\uparrow, \lambda, -\frac{1}{2}} = N \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{\lambda}{m+\lambda} \uparrow \\ m+\lambda E_{\uparrow} \end{pmatrix} e^{\frac{i}{\hbar} (\uparrow z - \lambda E_{\uparrow} t)}$$

$$\int \psi_{\uparrow, \lambda, 0}^+ \psi_{\uparrow, \lambda, 0} d^3x = \int_{\lambda, \lambda} \delta_{\uparrow, \uparrow} \delta(\uparrow - \uparrow)$$

Klein-Gordon

legen e teilchen in einem E.M. feld

erweitert: kanonisch 4-es impuls $p^\mu = p_{\text{mechan}}^\mu + e A^\mu$

Hamilton-fn $H = \sqrt{m^2 + (\vec{p} - e \vec{A})^2} + e \phi$

$$p^\mu \rightarrow -i \hbar \partial^\mu$$

skalare teilchen erweiteren $\partial^\mu \rightarrow \partial^\mu - \frac{ie}{\hbar} A^\mu$

$$[i \hbar \partial^\mu (\partial_\mu - \frac{ie}{\hbar} A_\mu) + m] \psi = 0$$

$$\hat{H} = \sum_i \alpha_i (\hat{p}_i - e A_i) + \beta m + e \phi$$

normalisiert und ϕ ist er ist gabelnnetzt $V(-) = e \phi(r)$

$$\hat{H} = \hat{H}_0 + V(r)$$

all: $\hat{H}_0$ kommutál a $\hat{J} = \hat{L} + \hat{S}$ teljes imp. momentummal
és a $\hat{P}$ momentummal

$V(r)$ szimmetriát miatt szintén kommutál $\hat{J}$ -vel és $\hat{P}$ -vel

$$\hat{J} = \hat{r} \times \hat{p} + \frac{\hbar}{2} \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$$

Mi a $\hat{P}$?

hogyan transformálódik Ψ

$$x \rightarrow -x$$

$$t \rightarrow t$$

trafo?

$$\Lambda^\mu_\nu = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x^\mu = \Lambda^\mu_\nu x'^\nu$$

$$\Psi(x) = R(\Lambda) \Psi'(\Lambda^{-1}x)$$

$$\sigma^0 = \hat{P}^{-1} \sigma_0 \hat{P}$$

$$\Lambda^\mu_\nu \sigma^\nu = R^{-1}(\Lambda) \sigma^\mu R(\Lambda)$$

$$\sigma^i = -\hat{P}^{-1} \sigma^i \hat{P}$$

$$\hat{P} \sigma^0 = \sigma^0 \hat{P}$$

$$\text{az. } \hat{P} = a \sigma_0$$

$$\hat{P} \sigma^i = -\sigma^i \hat{P}$$

$$|\hat{P} \Psi| = |\Psi| \quad |\alpha| = 1 \quad \alpha = e^{i\varphi}$$

egyszerűség kedvéért $\varphi = 0, \alpha = 1 \quad \hat{P} = \sigma_0$

$$\Psi(x, t) = \sigma_0 \Psi'(-x, t)$$

$(\hat{H}, \hat{J}, \hat{L})$ loois sijaistallot

$\hat{J} \rightarrow \hat{J}^2, \hat{J}_z$ -t jätetään

Välillä määritellään $V(r)$ potentiaal

kuvaus

$$\begin{aligned} j & \quad \hat{J}^2 = j(j+1) \cdot \hbar^2 \\ m & \quad \hat{J}_z = m \cdot \hbar \\ l & \quad \hat{L}^2 = l(l+1) \cdot \hbar^2 \end{aligned}$$

$$\hat{J} = \hat{L} + \hat{S}$$

$$\psi_{jm} = \begin{pmatrix} \varphi_{jlm} \\ \chi_{jlm} \end{pmatrix}$$

$\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$ parissa sijaistallot $\sigma^0 \Psi(-x, t) = \pm \Psi(x, t)$

$\varphi: \text{para} \rightarrow X$ funktio

$$\varphi(-x, t) = \pm \varphi(x, t)$$

$$\chi(-x, t) = \pm \chi(x, t)$$

φ, χ parissa ellei

t sijaistallot näet on aluksi lu. odja. naja:

$$\Omega_{jm} = \sum_{m_1, m_2} \left(l \frac{1}{2} j \mid m_1, m_2 \mid m \right) Y_{l, m_1} X_{\frac{1}{2}, m_2}$$

$\hat{L}$ Clebsch-Gordan k.

$$X_{\frac{1}{2}, \frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad X_{\frac{1}{2}, -\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ω parissa l-lol j:n

$$\hat{P} \Omega_{jm} = (-1)^l \Omega_{jm}$$

$\varphi \rightarrow \chi$ parissa aluksi $\Rightarrow l \neq l'$

määrä $j = l \pm \frac{1}{2} \Rightarrow j = l' \pm \frac{1}{2}$

Egyik helyen $+\epsilon$, másik helyen $-\epsilon$ kell választani.

$$\text{összesen} \Rightarrow \boxed{2j = l + l'}$$

tetszőleges

$$\varphi_{j\ell m}(x) = i g(r) R_{j\ell m}\left(\frac{r}{r_0}\right)$$

$$X_{j\ell m}(x) = -f(r) R_{j\ell m}\left(\frac{r}{r_0}\right)$$

$$\hat{L}^2 \hat{L}_z X + m \varphi + V(r) \varphi = E \cdot \varphi$$

$$\hat{L}^2 \hat{L}_z \varphi - m X + V(r) X = E \cdot X$$

$$\hat{L}^2 \hat{L}_z \varphi_{j\ell m} = \left[-\hbar^2 \frac{d g}{d r} - \frac{g(r)}{r} \hbar^2 l(l+k) \right] R_{j\ell m}$$

$$\hat{L}^2 \hat{L}_z X_{j\ell m} = \left[-i \hbar^2 \frac{d f}{d r} - i \frac{f(r)}{r} \hbar^2 l(l-k) \right] R_{j\ell m}$$

$$k = \mp \left(j + \frac{1}{2} \right) = \begin{cases} -(j + \frac{1}{2}) & , j = l + \frac{1}{2} \\ +(j + \frac{1}{2}) & , j = l - \frac{1}{2} \end{cases}$$

k mindig $l \pm \frac{1}{2}$ esetén

spin - helyre áll

radialis egyenletek: $G = r g$ $F = r f$ segítségével

$$\hbar^2 \frac{d G}{d r} + \hbar^2 \frac{k}{r} G(r) - (E + m - V) F(r) = 0$$

$$\hbar^2 \frac{d F}{d r} - \hbar^2 \frac{k}{r} F(r) + (E - m - V) G(r) = 0$$

setzen $V(r) = - \frac{Z \cdot e^2}{r}$

$$\frac{dG}{dr} = - \frac{\kappa}{r} G(r) + \left[\frac{E + m}{r} + \frac{Z \cdot \alpha}{r} \right] F(r)$$

$$\frac{dF}{dr} = + \frac{\kappa}{r} F(r) - \left[\frac{E + m}{r} + \frac{Z \cdot \alpha}{r} \right] G(r)$$

$$L = \frac{e^2}{r} = \frac{e^2}{\hbar c} = \frac{1}{137} \quad \text{feinsstrukturkonstante 'all'}$$

notieren: 1. $r \rightarrow 0$ linear

2. $r \rightarrow \infty$ linear

3. $r \rightarrow 0, r \rightarrow \infty$ Verhalten der Wellenfunktion im Limes

1) $r \rightarrow 0$

$$\frac{dG}{dr} = - \frac{\kappa}{r} G + \frac{Z \cdot \alpha}{r} F$$

$$\frac{dF}{dr} = + \frac{\kappa}{r} F - \frac{Z \cdot \alpha}{r} G$$

$$F(r) = a \cdot r^x$$

$$G(r) = b \cdot r^x$$

$$x = \pm \sqrt{\kappa^2 - (Z \cdot \alpha)^2}$$

Ψ normalisierbar

Coulomb-energie ist kleiner als Ruheenergie

$$\kappa^2 \geq 1$$

$$Z \cdot \alpha \geq 1 \quad Z > 137$$

$$\Rightarrow x = \pm \sqrt{\kappa^2 - (Z \cdot \alpha)^2}$$

$$\kappa = \mp \left(j + \frac{1}{2} \right)$$

3) $r \rightarrow \infty$

$$\rho = 2 \lambda r$$

$$\lambda = \frac{\sqrt{m^2 c^2 - E^2}}{\hbar c}$$

$$s \rightarrow \infty \quad \frac{dG}{ds} = \frac{E+m}{2E\lambda} F(s)$$

$$\frac{dF}{ds} = - \frac{E-m}{2E\lambda} G(s)$$

$$\frac{d^2 G}{ds^2} = \frac{E^2 - m^2}{4E^2\lambda^2} G(s) \quad G(s) \sim e^{\pm \frac{s}{\lambda}} \quad -: \text{ "normalizable"}$$

$$F(s), G(s) \sim e^{-\frac{s}{\lambda}}$$

$$G(s) = \sqrt{m+E} e^{-\frac{s}{\lambda}} s^{\frac{1}{2}} \cdot (\phi_1(s) + \phi_2(s))$$

$$F(s) = \sqrt{m-E} e^{-\frac{s}{\lambda}} s^{\frac{1}{2}} \cdot (\phi_1(s) - \phi_2(s))$$

$$\phi_1 = \sum_{n=0}^{\infty} \alpha_n s^n$$

$$\phi_2 = \sum_{n=0}^{\infty} \beta_n s^n$$

α_n, β_n "bestimmen"

$$\alpha_n = \frac{(1-n)(2-n) \cdots (m-n)}{n! (2\lambda+1) \cdots (2\lambda+n)} \alpha_0$$

$$\beta_n = (-1)^n \frac{n(n-1) \cdots (n-m+1)}{n! (2\lambda+1) \cdots (2\lambda+n)} \beta_0$$

$$n = \frac{2dE}{2\lambda} - \frac{1}{2}$$

"normalisierbar" $\Rightarrow \phi_1, \phi_2$ "diskret" "normiert"

$$n \text{ "ganzzahlig"} \quad n = 0, 1, 2, \dots$$

Angabe $n = n' + |K| = n' + j + \frac{1}{2}$ ¹ $\text{L\u00f6sungsternchen}$

$n = 1, 2, \dots$

$$\Rightarrow E = m \left[1 + \frac{(Z\alpha)^2}{n - j - \frac{1}{2} + \sqrt{(j + \frac{1}{2})^2 - (Z\alpha)^2}} \right]^{-\frac{1}{2}}$$

Sommerfeld - feine
Strukturkorrektur

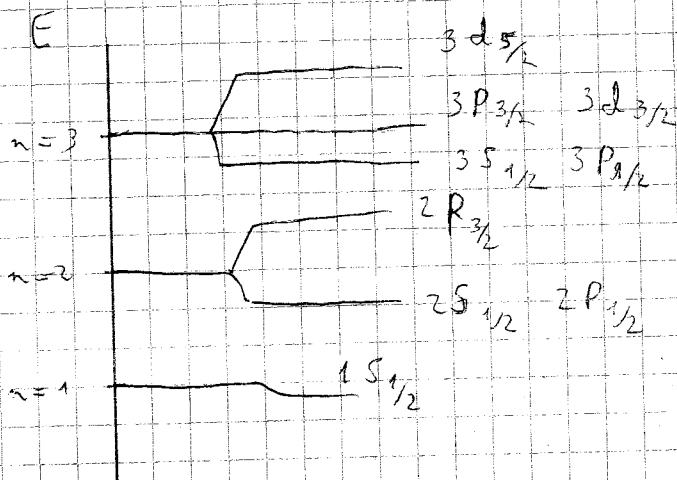
E totalenergie $m c^2$ ruheenergie

E n-teil ist j-teil f\u00fcr l-teil neu

F\u00fcr $Z\alpha$ -re annehmen:

$$\frac{E - m c^2}{m c^2} = - (Z\alpha)^2 \left\{ \frac{1}{2n^2} + \frac{(Z\alpha)^2}{2n^3} \left(\frac{1}{j + \frac{1}{2}} - \frac{3}{4n} \right) \right\}$$

feinstrukturkorrektur



feinstruktur

j-teil f\u00fcr

l-teil neu

l-teil neu f\u00fcr

Landau shift (G.E.D.)