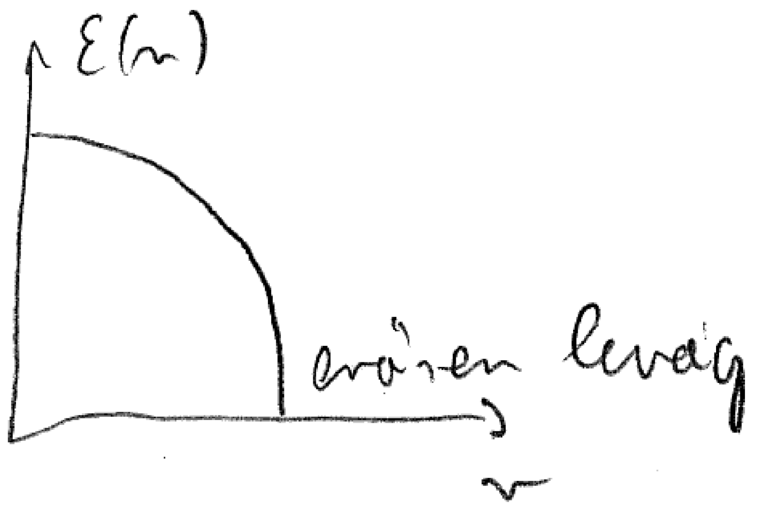
$B(r) = B_e\, e^{-7,67\left[\left(\frac{r}{r_e}\right)^{1/4}-1\right]}$ r_e: ahol levág B, $B(r_e) = B_e$

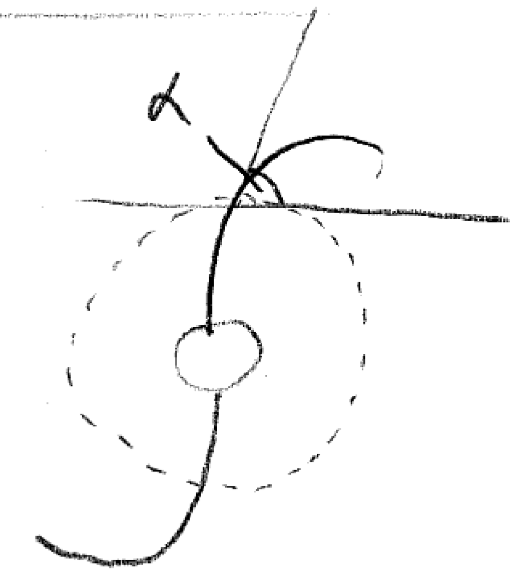
$\mathcal{E}(r) = \frac{L\cdot a}{4\pi}\,\frac{1}{r^2(a+r)^2}$ L: luminozitás, „a" távolság dim-jú paramétere az eloszlásnak skálaparaméter

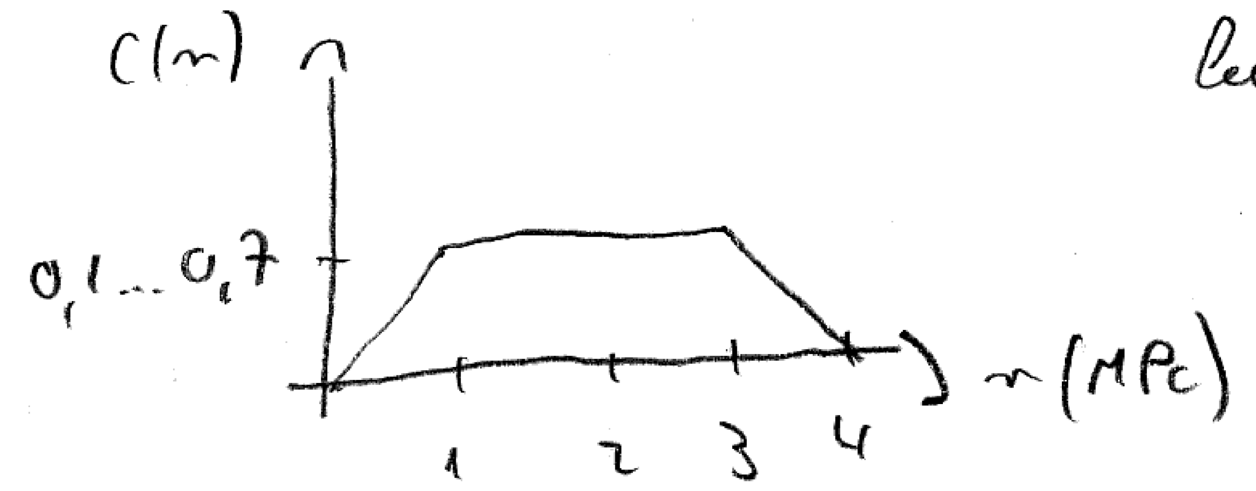


$B(r,\beta) = B_0\cdot e^{-\alpha r}\left[1 + C(r)\sin\left(m\left(\beta - \frac{\ln r}{\mathrm{tg}\,\alpha} - \varphi_0\right)\right)\right]$

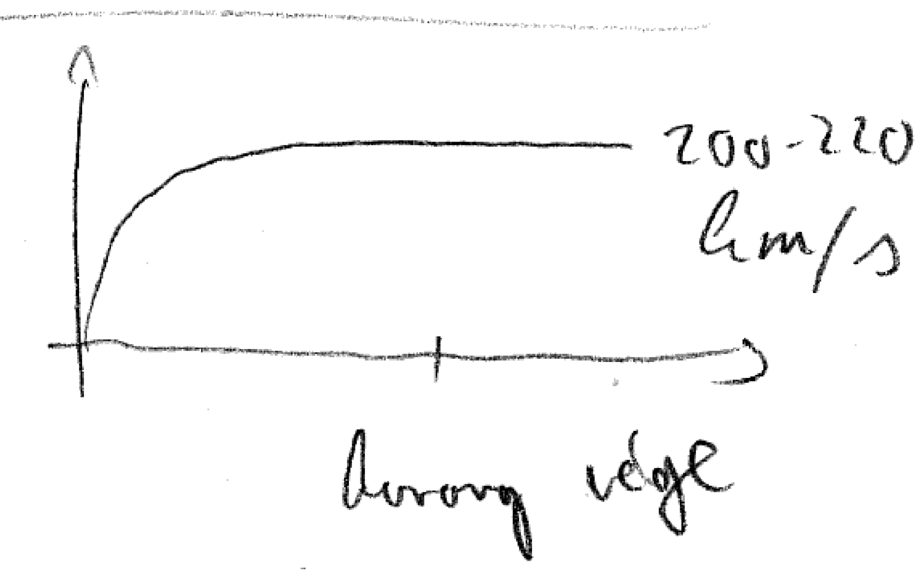
kontrasztja a karnak multiplicitás

α: nyílásszög, állandó

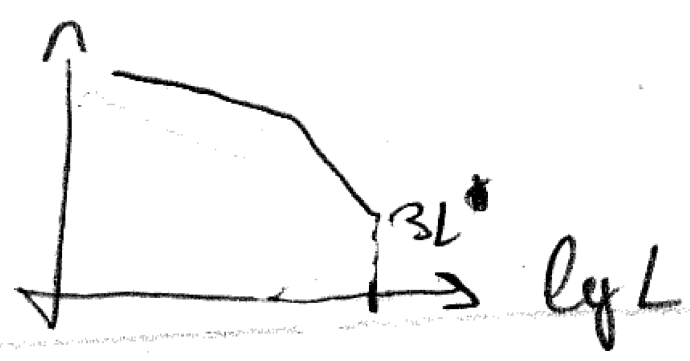




Halo tartalmazza az anyag 90%-át, mert rotációs görbe állandó



$\Phi(L) = \frac{N_0}{L^*}\left(\frac{L}{L^*}\right)^{\alpha} e^{-\frac{L}{L^*}}$, $L^* = 34\cdot 10^{9} L_\odot$



Lyman-α-erdő: észlelt, H spektrumától balra vonalak, integrálja arányos a felhővel

$\Phi(L)$ kimérése: látszólagos fényesség és távolságból kiszámolható L, számláljuk, $L^{-\frac{3}{2}}$-vel súlyozunk

Galaxisok ütközése: $F_\perp = \frac{Gm^2}{b^2}\,\frac{1}{\left(1+\left(\frac{x}{b}\right)^2\right)^{3/2}}$, $\Delta v_\perp = \int F_\perp \underbrace{\frac{dx}{v}}_{dt} = \frac{2Gm}{bv}$

$b+db$ impakt paraméterrel $\frac{N}{\pi R^2}\, b\, 2\pi\, db = \frac{2N}{R^2}\, b\, db$ csillag van, $\Delta v_\perp$ össz $= 0$ ha homogén

$$d(v_\perp^2) = \left(\frac{2Gm}{bv}\right)^2 \frac{2N}{R^2}\, b\, db$$

$\Delta(v_\perp^2) = \int_{b_{min}}^{b_{max}} d(v_\perp^2)$ $b_{min} := \frac{2Gm}{v^2}$, ahol $\frac{1}{2}mv^2 = \frac{Gm\cdot N\cdot m}{R}$, $b_{max} = R$

$\Delta(v_\perp^2) = 8\frac{Gm}{R}\ln\frac{b_{max}}{b_{min}}$ $\frac{b_{max}}{b_{min}} = N/2$

$\frac{\Delta(v_\perp^2)}{v^2} \approx \frac{10\ln N}{N}$, $n_{relax} := \frac{1}{\frac{\Delta(v_\perp)^2}{v^2}} = \frac{0,1\,N}{\ln N}$

$$\frac{\partial f}{\partial t} + \frac{\partial f \dot{\underline{x}}}{\partial \underline{x}} \frac{\partial f \dot{\underline{v}}}{\partial \underline{v}} = 0 \qquad \frac{\partial \dot{x}}{\partial \underline{x}} = 0 \qquad \frac{\partial \dot{\underline{v}}}{\partial \underline{v}} = 0 \Rightarrow \frac{\partial f}{\partial t} + \underline{v}\,\nabla f - \overbrace{\nabla\phi}^{a}\cdot\nabla_v f = 0$$

Jeans I: $\int \bullet\, d^3v$, $\int \frac{\partial}{\partial v_i} f\, d^3v = 0$, $\rho = \int f\, d^3v$, $\bar{v}_i = \frac{\int v_i f\, d^3v}{\int f d^3 v}$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \bar{v}_i}{\partial x_i} = 0$$

Jeans II: $\int \bullet\, v_j\, d^3v$, $\int v_j \frac{\partial f}{\partial v_i} d^3v = -\int \delta_{ij} f\, d^3v = -\delta_{ij}\rho$, $v_i(x_i,t)$

$$\frac{\partial \rho v_j}{\partial t} + \frac{\partial \rho\, \overline{v_i v_j}}{\partial x_i} + \rho \frac{\partial \phi}{\partial x_j} = 0 \qquad \overline{v_i v_j} = \frac{\int v_i v_j f d^3v}{\int f\, d^3v}$$

Jeans III: $\mathrm{II} - \bar{v}_j \cdot \mathrm{I}$, $\sigma_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$

$$\rho \frac{\partial \bar{v}_j}{\partial t} + \bar{v}_i \frac{\partial \rho \bar{v}_j}{\partial x_i} = -\frac{\partial \rho \sigma_{ij}^2}{\partial x_i} - \rho \frac{\partial \phi}{\partial x_j}$$

Hengerkoordinátában: $\frac{\partial f}{\partial t} + \dot{r}\frac{\partial f}{\partial r} + \dot{\varphi}\frac{\partial f}{\partial \varphi} + \dot{z}\frac{\partial f}{\partial z} + \dot{v}_r \frac{\partial f}{\partial v_r} + \dot{v}_\varphi \frac{\partial f}{\partial v_\varphi} + \dot{v}_z \frac{\partial f}{\partial v_z}$

$\dot{r} = v_r$

$\dot{z} = v_z$

$\dot{\varphi} = \frac{v_\varphi}{r}$

$$\int \bullet\, v_z\, d^3v = \frac{\partial(\rho v_z)}{\partial t} + \frac{\partial \rho\, \overline{v_r v_z}}{\partial r} + \frac{\partial \rho\, \overline{v_z^2}}{\partial z} + \frac{\rho v_r v_z}{r} + \rho\frac{\partial \phi}{\partial z}$$

$\dot{v}_r = r\dot{\varphi}^2 - \frac{\partial \phi}{\partial r}$

$$0 \approx \frac{\partial \rho\, \overline{v_z^2}}{\partial z} + \rho\frac{\partial \phi}{\partial z} \qquad \frac{\partial^2 \phi}{\partial z^2} = 4\pi G\rho$$

$\dot{v}_\varphi = -\frac{1}{r}\frac{\partial \phi}{\partial \varphi} - \frac{2 v_r v_\varphi}{r}$

$$\frac{\partial}{\partial z}\left(-\frac{1}{\rho}\frac{\partial \rho\, \overline{v_z^2}}{\partial z}\right) = 4\pi G \rho$$

$\dot{v}_z = -\frac{\partial \phi}{\partial z}$

$$\int \rho \mathrm{II}\, x_a\, d^3x = \underbrace{\int x_a \frac{\partial}{\partial t}(\rho \bar{v}_j)\, d^3x}_{} + \underbrace{\int x_a \frac{\partial}{\partial x_i}(\rho \overline{v_i v_j})\, d^3x}_{} + \underbrace{\int \rho x_a \frac{\partial \phi}{\partial x_j}\, d^3x}_{=-W_{aj}} = 0$$

$$\frac{\partial}{\partial t}\int \frac{\rho x_a \bar{v}_j + \rho x_j \bar{v}_a}{2}\, d^3x = \qquad -\int \rho\, \overline{v_a v_j}\, d^3x = -\int (\underbrace{\rho \sigma_{aj}^2}_{-\Pi_{ja}} + \underbrace{\rho \bar{v}_i \bar{v}_j}_{-2T_{ja}})\, d^3x = -2K_{aj}$$

$$= \frac{1}{2}\frac{\partial}{\partial t}\int \rho \bar{v}_i \frac{\partial}{\partial x_i}(x_j x_a)\, d^3x = -\frac{1}{2}\frac{\partial}{\partial t}\int -\frac{\partial \rho}{\partial t} x_j x_a\, d^3x =$$

$$= \frac{1}{2}\frac{\partial^2}{\partial t^2} I_{ja} \qquad \frac{1}{2}\frac{d^2}{dt^2} I_{ja} = 2T_{ja} + \Pi_{ja} + W_{ja}, \text{ stac esetben } 2K_{ja} + W_{ja} = 0$$

$$\frac{1}{2}\frac{\partial^2 I_{jk}}{\partial t^2} = 2T_{jk} + \Pi_{jk} + W_{jk}$$

stacionárius, $x-y$ szimmetria esetén $\quad 2T_{xx} = \frac{1}{2}\int \rho\, \bar{v}_\varphi^2\, d^3x$

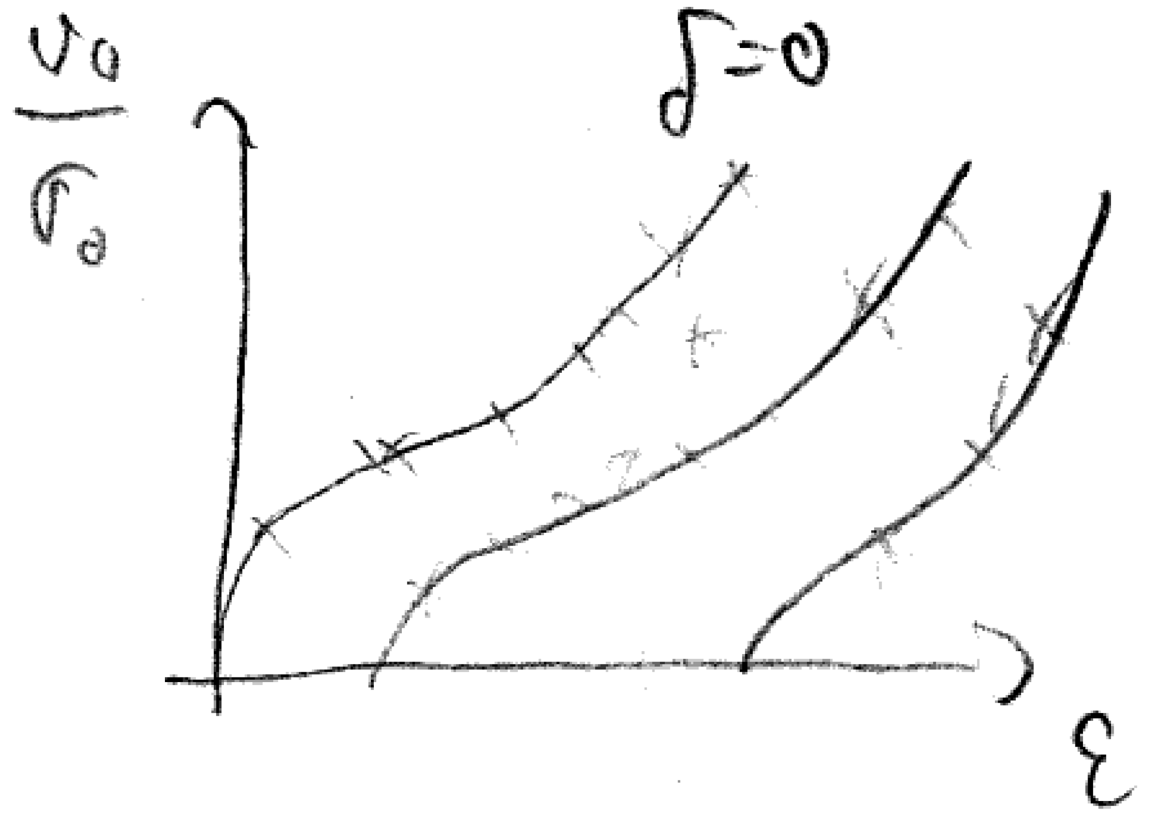
$$v_0^2 := \frac{\int \rho\, \bar{v}_\varphi^2\, d^3x}{\int \rho\, d^3x}\qquad M$$

$$\frac{1}{2} M v_0^2 := \qquad M\sigma_0^2$$

$$\left.\begin{aligned} 0 &= 2T_{xx} + \Pi_{xx} + W_{xx} \\ 0 &= 2T_{zz} + \Pi_{zz} + W_{zz} \end{aligned}\right\}\quad \frac{W_{xx}}{W_{zz}} = \frac{2T_{xx} + \Pi_{xx}}{\underbrace{2T_{zz}}_{0} + \underbrace{\Pi_{zz}}_{M\sigma_0^2(1-\delta)}} = \frac{\frac{1}{2} v_0^2 + \sigma_0^2}{\sigma_0^2 (1-\delta)} \Rightarrow \frac{v_0^2}{\sigma_0^2} = 2\,\frac{W_{xx}}{W_{zz}}(1-\delta) - 2$$

σ_0, v_0 mérhető (vöröseltolódás a spirálkar oldalainál, spektrum szélessége és eltolódás mértéke)

$\frac{W_{xx}}{W_{zz}}$ geometriából számolható $\Rightarrow$ δ jön



kis galaxisok csak baloldalt vannak $\Rightarrow$ forgástól lapultak

nagy galaxisoknál anizotrópia is van

$$\nabla g = -\nabla^2 \phi = -4\pi G \rho = \frac{\partial g_z}{\partial z} + \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r g_r)}_{-2\frac{1}{r}\frac{\partial v_c}{\partial r} = -2(A^2-B^2)}$$

$$g_r = -\frac{v_c^2}{r}$$

$$A := \frac{1}{2}\left(\frac{v_c}{r} - \frac{\partial v_c}{\partial r}\right)$$

$$B := -\frac{1}{2}\left(\frac{v_c}{r} + \frac{\partial v_c}{\partial r}\right)$$

$$\frac{\partial g_z}{\partial z} = -4\pi G \rho + 2(A^2 - B^2)$$

Merev teste $A = 0$

$B = -\omega$

Galaxisra a megoldás harmonikus oszcillátor $\left(\begin{array}{l} g_z = 0 \text{ ha } z = 0 \\ g_z \sim z \text{ ha } z \text{ kicsi} \end{array}\right)$

Galaxisra $A = -B = \frac{1}{2}\omega(r)$

$$-\frac{\partial g_z}{\partial z} = \omega^2 = 4\pi g \rho \,, \quad t = 10 \text{ Mév}$$

$$\rho_{Halo} = \rho_0 \left(\frac{r_0}{r}\right)^2 \,, \quad M_r = \int_0^r \rho_0 \left(\frac{r_0}{r}\right)^2 d^3 r = \int \rho_0 4\pi r^2 \frac{r_0^2}{r^2} dr = \rho_0 4\pi r_0^2 r \frac{r^2}{r^2} = \rho_H 4\pi r^3$$

$$\left.\begin{array}{l} g_z = -4\pi \rho_H G \cdot z \\ g_r = -\frac{G M_r}{r^2} \end{array}\right\} \quad \frac{g_z}{g_r} = \frac{z}{r}$$

$$\frac{\partial g_z}{\partial z} = -4\pi G \rho_D \qquad g_z = -4\pi G \int_0^z \rho_D \, dz = -2\pi G \int_{-z}^{z} \rho_D \, dz = -2\pi G\, \Sigma(z)$$

$$\frac{\partial(\overbrace{\sigma_z^2 \rho}^{\text{nyomás}})}{\partial z} = \rho g_z = -2\pi G\, \Sigma(z)\, \rho \qquad \sigma_z^2 \text{ független } z\text{-től}$$

$$\sigma_z^2 \frac{\partial}{\partial z} \ln \rho = -2\pi G\, \Sigma(z)$$

kis z-re $\rho_D(z) = \rho_0$

$$\sigma_z^2 \frac{\partial}{\partial z} \ln \rho = -2\pi G (2 z \rho_0)$$

$$\sigma_z^2 \ln \frac{\rho}{\rho_0} = -4\pi G \rho_0 \frac{z^2}{2}$$

$$\rho = \rho_0 e^{-\frac{z^2}{2 z_0^2}} \qquad z_0^2 = \frac{\sigma_z^2}{4\pi G \rho_0}$$

Nagy z-re $\Sigma(z)$ állandó

$$\sigma_z^2 \frac{\partial \ln \rho}{\partial z} = -2\pi G \Sigma_D$$

$$\rho = \rho_0 e^{-\frac{z}{H}} \qquad H = \frac{\sigma_z^2}{2\pi G \Sigma_D}$$

$$H = \frac{z_0}{\sqrt{2}}$$

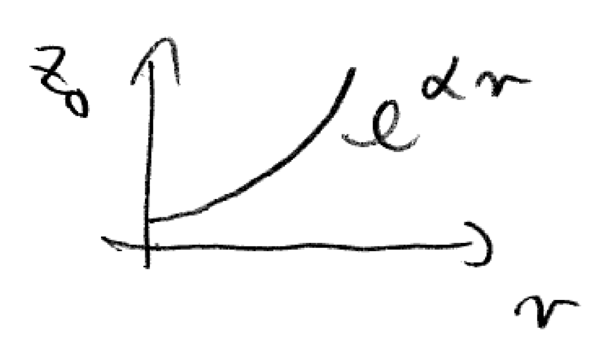
$$\rho(z) = \rho_0 \operatorname{sech}^2 \frac{z}{\sqrt{2 z_0^2}}$$

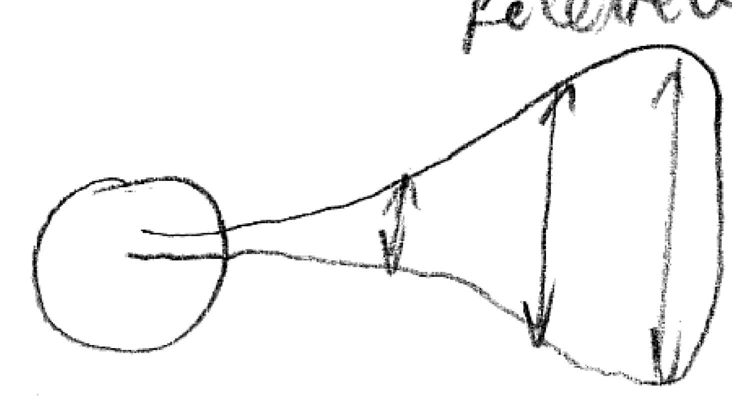
$$z_0^2 = \frac{\sigma_z^2}{4\pi G (\rho_D + \rho_H)} \qquad \rho_0 = \rho_D + \rho_H \qquad \rho_D \cdot \beta \cdot z_0 = \Sigma_D$$

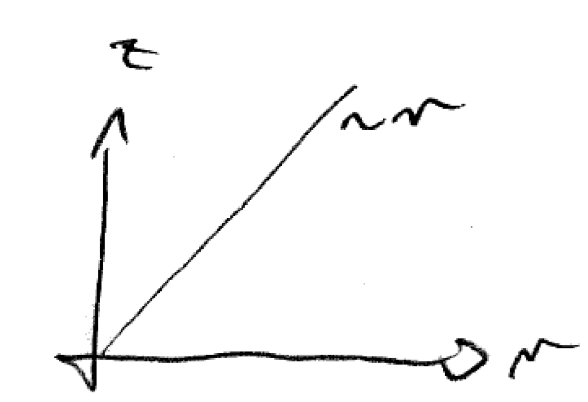
$$4\pi G \rho_H = \omega^2$$

$$z_0^2 = \frac{\sigma_z^2}{\frac{4\pi G \Sigma_D}{\beta z_0} + \omega^2} \Rightarrow z_0 = \frac{4\pi G \Sigma_D}{2\beta \omega^2} \left(1 + \sqrt{1 + \underbrace{\left(\frac{2\beta\sigma\omega}{4\pi G \Sigma} \right)^2}_{Q_z^2}} \right) = \frac{\sigma_z}{\omega} \frac{Q_z}{\sqrt{1 + Q_z^2} + 1}$$

Kicsi Q_z: $z_0 = \frac{\beta \sigma_z^2}{4\pi G \Sigma_D}$, ekkor ρ_0 dominál







Nagy Q_z: $z_0 = \frac{\sigma_z}{\omega} = \frac{\sigma_z r}{v_c}$

$r = r_0(1+\xi)$ $\qquad \varphi = \Omega_0 t + \eta$ $\qquad \Omega_0 = \frac{v_{c0}}{r_0}$

$j = r_0^2\Omega_0 = r^2\dot\varphi = r_0^2(1+\xi)^2(\Omega_0+\dot\eta) = r_0^2 + 2r_0\xi\Omega_0 + r_0^2\Omega_0 \Rightarrow \dot\eta = -2\xi\Omega_0$

$\dot v_r = r\dot\varphi^2 + f \Rightarrow f = \ddot r - r\dot\varphi^2 = r_0\ddot\xi - \frac{j^2}{r^3}$

$f = -\frac{v_c^2}{r} = -\frac{v_{c0}^2}{r_0} - \frac{d}{dr}\frac{v_c^2}{r}\Big|_{r=r_0}\Delta x = -\frac{v_{c0}^2}{r_0} - r_0\xi\left[\overbrace{\frac{2v_{c0}}{r_0}}^{\Omega_0}\overbrace{\frac{dv_c}{dr}\Big|_{r=r_0}}^{v_{c0}'} - \overbrace{\frac{v_{c0}^2}{r^2}}^{\Omega_0^2}\right]$

$\ddot\xi - \frac{j^2}{r_0 r^3} + \overbrace{\frac{v_{c0}^2}{r_0^2}}^{\Omega_0^2} + 2\xi\Omega_0 v_{c0}' - \xi\Omega_0^2 = 0$

$\frac{j^2}{r_0 r^3} = \frac{r_0^4\Omega_0^2}{r_0 r^3} = \Omega_0^2\frac{r_0^3}{r^3} = \Omega_0^2\frac{\cancel{r_0^3}}{\cancel{r_0^3}(1+\xi)^3}$

$\Omega_0^2\left(\frac{-1}{(1+\xi)^3} + 1 - \xi\right) = \Omega_0^2\left(-(1-3\xi) + 1 - \xi\right) =$

$= 2\Omega_0^2\xi$

$\ddot\xi + 2\Omega_0^2\xi + 2\Omega_0 v_{c0}'\xi = 0$

$\ddot\xi + 2\Omega_0\overbrace{\left[\Omega_0 + v_{c0}'\right]}^{-2B}\cdot\xi = 0$

$\ddot\xi - 4\Omega_0 B\xi = 0 \qquad k^2 := -4\Omega_0 B$

$\ddot\xi + k^2\xi = 0 \longrightarrow$

Galaxisra $B = \frac{\Omega_0}{2} \Rightarrow k^2 = 2\Omega^2 \Rightarrow k = \sqrt{2}\,\Omega$

Naprendszerre $v \sim \frac{1}{\sqrt{r}}$, $v' \sim \frac{1}{r^{3/2}} = \frac{\sqrt{r}}{r} = \frac{v}{r} = \Omega$

$\Downarrow$

$v_{c0}' = -\frac{\Omega_0}{2} \Rightarrow B = \frac{\Omega_0}{4}$

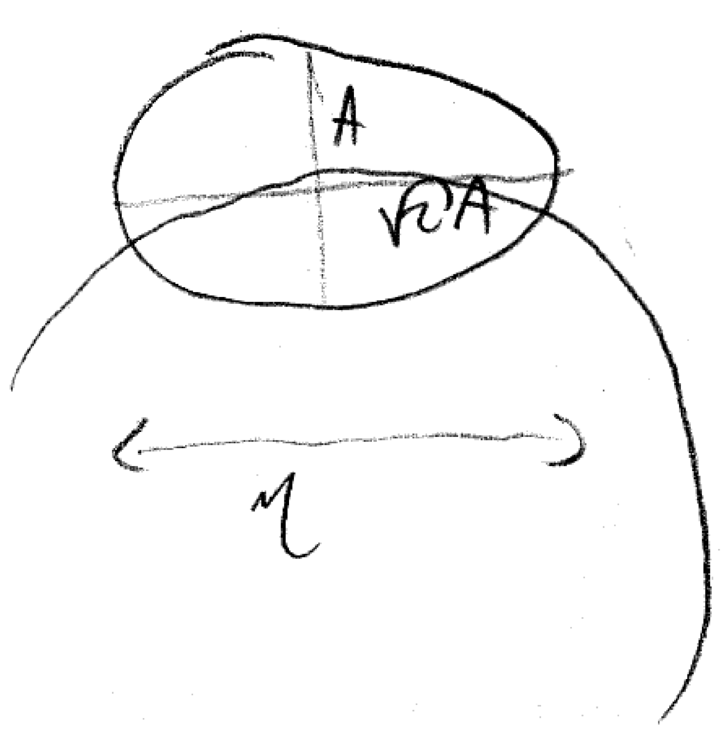
$k^2 = \Omega_0^2$

Az epiciklus egy galaxisban:

$\xi = A\cos kt$

$\dot\eta = -2\xi\Omega_0 = -2A\Omega_0\cos(kt)$

$\eta = -2A\frac{\Omega_0}{k}\sin kt = -\sqrt{2}A\sin kt$



Tejútban

$\sigma_r = 40$ km/s

$\sigma_\varphi = 28$ km/s $\neq \sigma_r\cdot\sqrt{2}$

De csak látszólag, $2\sigma_\varphi = \sigma_r\sqrt{2}$ ✓

$$\Sigma := \Sigma_0 + \underbrace{\Sigma_{\text{spirálkar}}}_{= H(R,t)\, e^{im\varphi}\, e^{i f(R,t)}}$$

Laplace-Poisson egyenlet, ütközésmentes Boltzmann-egyenlet, epiciklus approximáció felhasználásával

$\kappa^2 = -4B\Omega$ epiciklus frekvencia

$\kappa = \pm m(\Omega - \Omega_p) \Leftrightarrow \Omega = \Omega_p \pm \frac{\kappa}{m}$ Lindblad rezonancia feltétele

+ kontinuitási egyenlet

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G \Sigma \cdot |k| + k^2 v_s^2$$

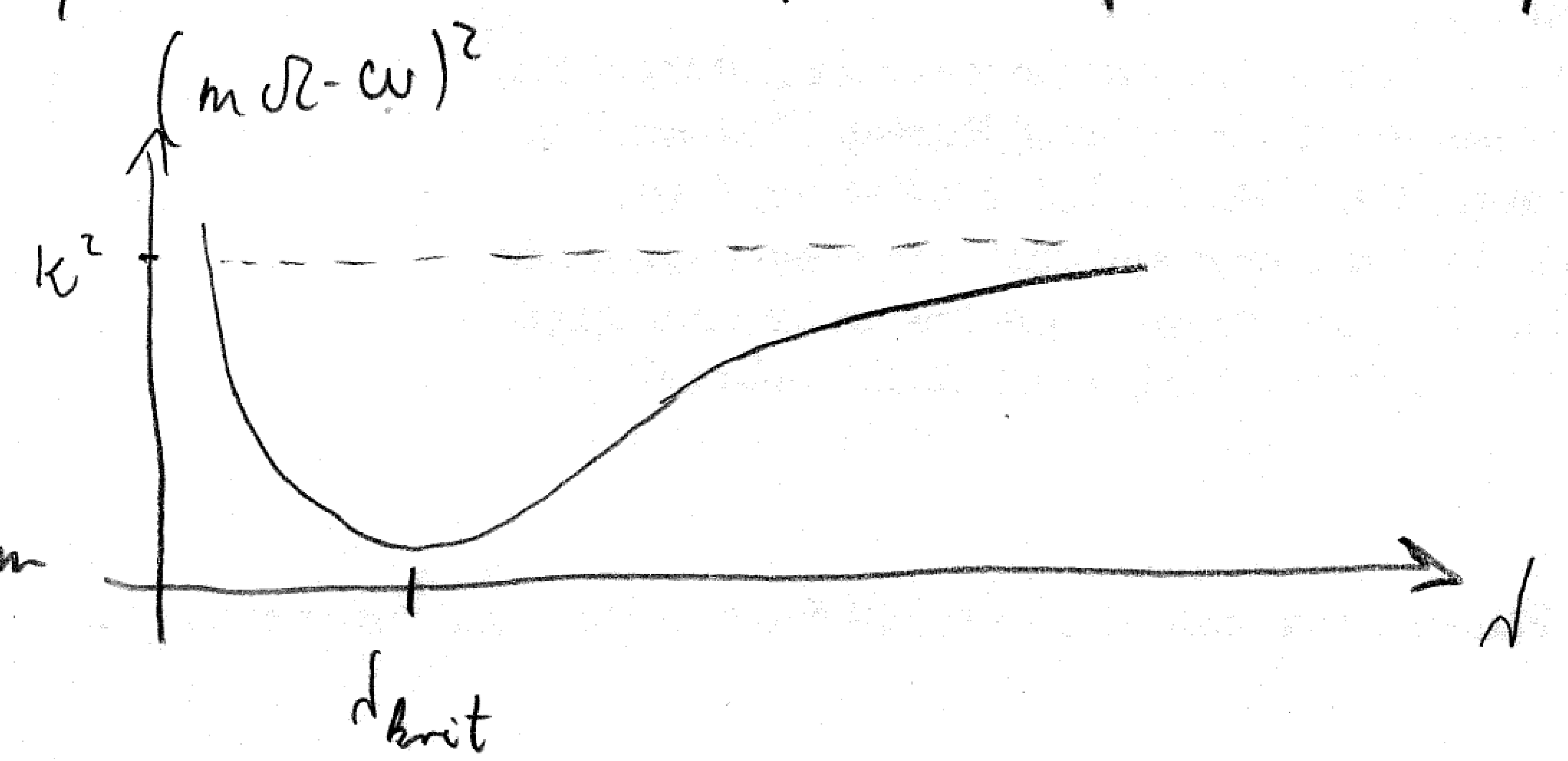
Toomre-Q feltétel

$Q =: \frac{v_s \kappa}{\pi G \Sigma} > 1$ esetén létezik megoldás, stabil a rendszer. Ez gázra volt jó.

Csillagokat tartalmazóra: $Q = \frac{\sigma_R \kappa}{3 \cdot G \cdot \Sigma}$

kb π, valójában

és kb π

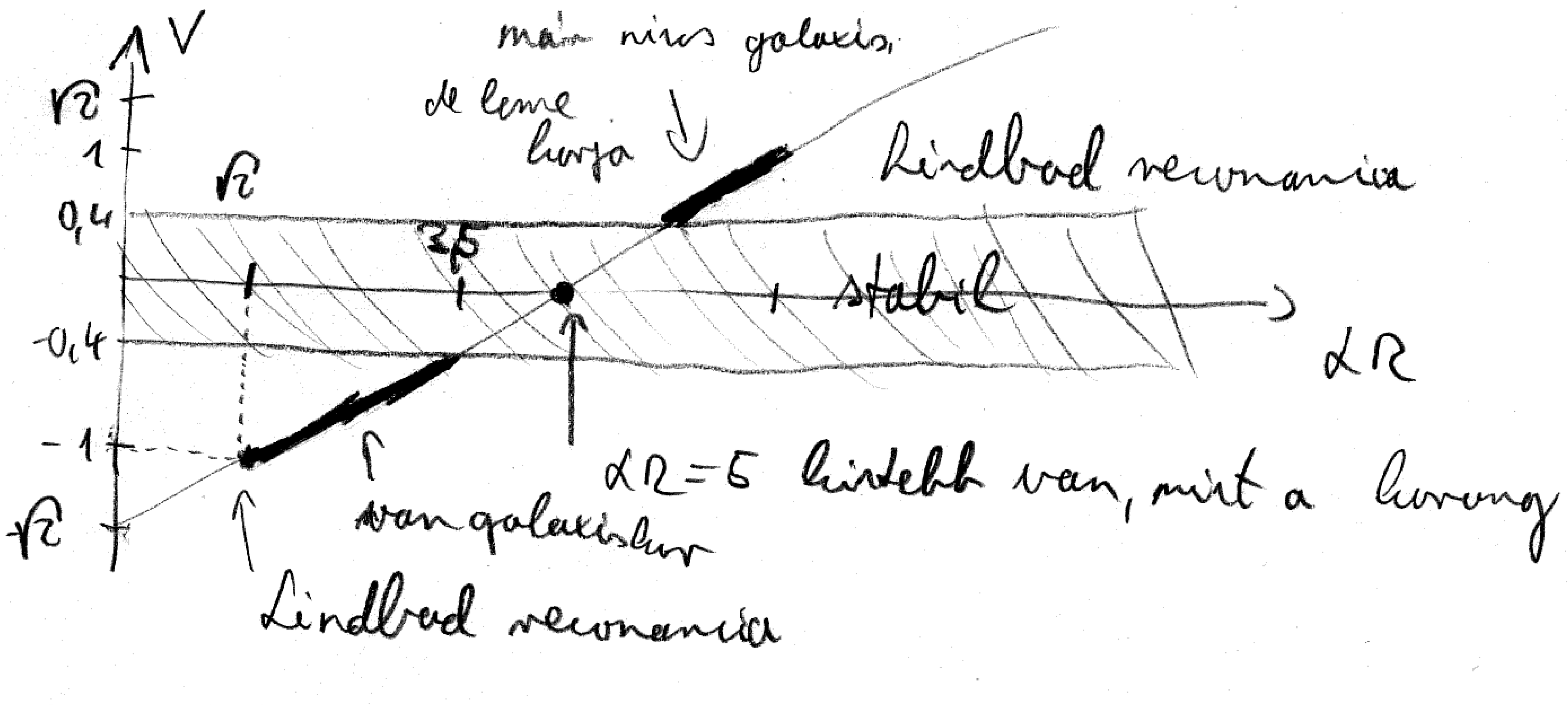


$\leftarrow k$

$Q = 1{,}7 > 1$ a mi galaxisunkra, vagyis nem lehetne spirálkar

$\lambda_{krit} \approx 10$ kpc

Diszperziót osszuk κ^2-tel



$$V = \sqrt{2}\left(\frac{R}{R_{c_0}} - 1\right)$$