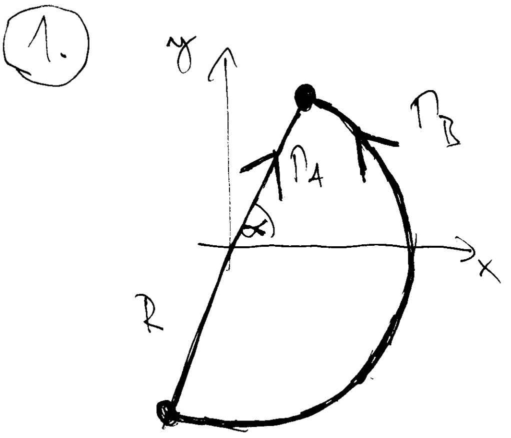
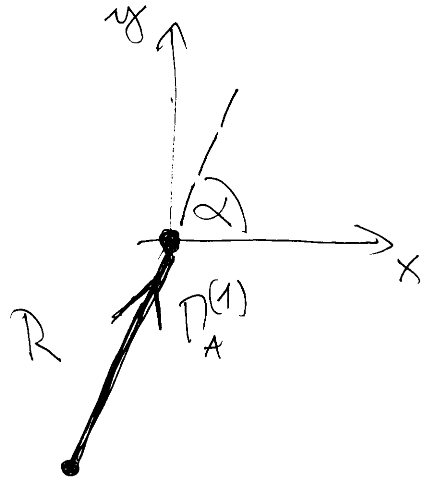
(1.)



$\underline{F}(\underline{r}) = -k\underline{r}$

$W_{\Gamma_A} = ?$

$W_{\Gamma_B} \overset{?}{=} 0$



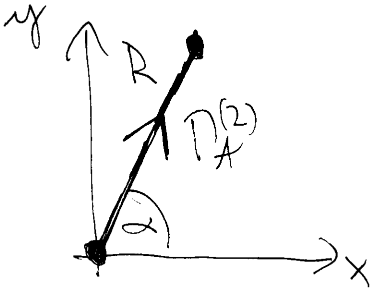
$\Gamma_A^{(1)}$: $l = R$, $r(s) = R - s$, $\varphi(s) = \alpha + \pi$,

$\underline{T}(s) = -\underline{e}_r$

$$\int_{\Gamma_A^{(1)}} \underline{F}(\underline{r})\, d\underline{r} = \int_0^R -k\, \underline{r}(s)\, \underline{e}_r (-1)\, ds =$$

$$= k \int_0^R r(s) \underbrace{\underline{e}_r \underline{e}_r}_{=1} ds = k \int_0^R (R - s)\, ds =$$

$$= k \left[Rs - \frac{s^2}{2} \right]_0^R = k \left(R^2 - \frac{R^2}{2} \right) = \frac{1}{2} k R^2$$



$\Gamma_A^{(2)}$: $l = R$, $r(s) = s$, $\varphi(s) = \alpha$, $\underline{T}(s) = \underline{e}_r$

$$\int_{\Gamma_A^{(2)}} \underline{F}(\underline{r})\, d\underline{r} = \int_0^R -k\, \underline{r}(s)\, \underline{e}_r\, ds = -k \int_0^R r(s) \underbrace{\underline{e}_r \underline{e}_r}_{=1} ds =$$

$= $ [illegible] $-k \int_0^R s\, ds = -k \left[\frac{s^2}{2} \right]_0^R = -\frac{1}{2} k R^2$

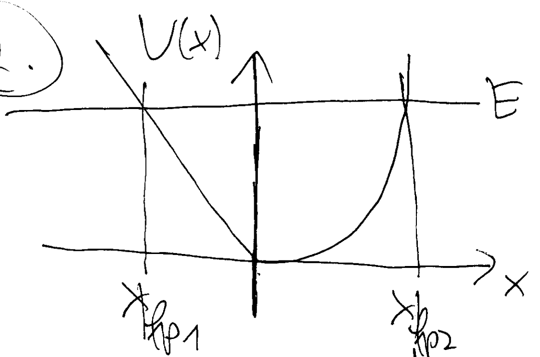
ZH1K/2

$$W_{\Pi_A} = \int_{\Pi_A^{(1)}} \underline{F}(\underline{r})\, d\underline{r} + \int_{\Pi_A^{(2)}} \underline{F}(\underline{r})\, d\underline{r} = \frac{1}{2} k R^2 - \frac{1}{2} k R^2 = 0$$

$$\Pi_B : \underline{T} = \underline{e}_\varphi \,, \quad \underline{F}(\underline{r}) \cdot \underline{T} \sim \underline{e}_r \underline{e}_\varphi = 0 \Rightarrow W_{\Pi_B} = 0$$

Két görbe mentén, melyek kezdő- és végpontja azonos, megegyezik a munkavégzés $\Rightarrow$ **lehet** konzervatív.

2.



$$V(x) = \begin{cases} -kax, & \text{ha } x \leq 0 \quad \text{I.} \\ \frac{1}{2} k x^2, & \text{ha } x > 0 \quad \text{II.} \end{cases}$$

$k, a > 0$

$$|x_{fp1}| \overset{!}{=} |x_{fp2}|$$

Az ábra alapján: csak $E \geq 0$ lehetséges. Az $E = 0$ triviális esetben nincs mozgás, ezzel, az egyszerűség kedvéért, nem foglalkozunk.

• I. $E = V(x_{fp})$

$E = -kax_{fp1}$

$x_{fp1} = -\frac{E}{ka} < 0 \Rightarrow x_{fp1} \in \text{I.}$

II. $E = V(x_{fp})$

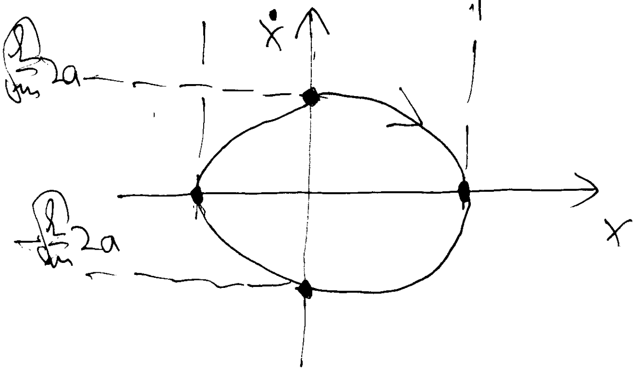
$E = \frac{1}{2} k x_{fp}^2$

$x_{fp2,3} = \pm\sqrt{\frac{2E}{k}}$ → $x_{fp3} < 0 \Rightarrow x_{fp3} \notin \text{II.}$; → $x_{fp2} = \sqrt{\frac{2E}{k}} \in \text{II.}$

→ $-x_{fp1} = x_{fp2}$

$\frac{E}{ka} = \sqrt{\frac{2E}{k}} \xrightarrow{E>0} \sqrt{E} = ka\sqrt{\frac{2}{k}}$

$$\boxed{E = k^2 a^2 \cdot \frac{2}{k} = 2ka^2}$$



• I. $2ka^2 = \frac{1}{2} m \dot{x}^2 - kax$

fekvő parabola

II. $\underbrace{2ka^2}_{>0} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$

ellipszis

• $\dot{x}(x=0) = \pm\sqrt{\frac{2}{m}\left(E - V(x=0)\right)} =$

$= \pm\sqrt{\frac{2}{m}\left(2ka^2 + ka \cdot 0\right)} =$

$= \pm\sqrt{\frac{k}{m}} \cdot 2a$

(3.)

$$v_{fr} = \frac{u_0}{R}(R-r)$$

$$v_{f\varphi} = \frac{u_0}{R^2} r^2$$

$t=0$: $x=R,\ y=0 \rightarrow r=R,\ \varphi=0$

$r(t) = R - u_0 t$

$v_{rel\,\varphi} = v_{rel\,r}$

$\varphi\left(r=\frac{R}{2}\right) = ?$

$$\varphi(t) = \varphi(t_0) + \int_{t_0}^{t} \dot{\varphi}(t')\,dt' = \varphi(t_0) + \int_{t_0}^{t} \frac{v_\varphi(t')}{r(t')}\,dt' =$$

$\dot{r}(t) = -u_0$

$$= 0 + \int_0^t \frac{v_{rel\varphi}(t') + v_{f\varphi}(t')}{r(t')}\,dt' = \int_0^t \frac{\dot{r}(t') - v_{fr}(t') + v_{f\varphi}(t')}{r(t')}\,dt' = (*)$$

$$v_{rel\varphi} = v_{rel\,r} = v_r - v_{fr} = \dot{r} - v_{fr}$$

$$(*) = \int_0^t \frac{-u_0 - \frac{u_0}{R}(R - r(t')) + \frac{u_0}{R^2} r(t')^2}{r(t')}\,dt' = \int_0^t \left(\frac{-2u_0}{r(t')} + \frac{u_0}{R} + \frac{u_0}{R^2} r(t')\right) dt' =$$

$$= \int_0^t \left(\frac{-2u_0}{R - u_0 t'} + \frac{u_0}{R} + \frac{u_0}{R^2}(R - u_0 t')\right) dt' = \left[2\ln(R - u_0 t') + \frac{u_0}{R} t' \cdot 2 - \frac{u_0^2}{R^2}\frac{t'^2}{2}\right]_0^t =$$

$$= 2\ln\frac{R - u_0 t}{R} + 2\frac{u_0}{R} t - \frac{1}{2}\cdot\frac{u_0^2}{R^2} t^2$$

$$t(r) = \frac{r-R}{-u_0} = \frac{R-r}{u_0}$$

$$\varphi(r) = \varphi(t(r)) = 2\ln\frac{r}{R} + 2\frac{u_0}{R}\cdot\frac{R-r}{u_0} - \frac{1}{2}\cdot\frac{u_0^2}{R^2}\cdot\frac{(R-r)^2}{u_0^2}$$

$$\varphi\left(r=\frac{R}{2}\right) = 2\ln\frac{1}{2} + 2\cdot\frac{R-R/2}{R} - \frac{1}{2}\cdot\frac{(R-R/2)^2}{R^2} =$$

$$= 2\ln\frac{1}{2} + 2\cdot\frac{R}{2}\cdot\frac{1}{R} - \frac{1}{2}\cdot\frac{R^2}{4}\cdot\frac{1}{R^2} = 2\ln\frac{1}{2} + 1 - \frac{1}{8} =$$

$$= \frac{7}{8} + 2\cdot\ln\frac{1}{2} \approx \frac{7}{8} + 2\cdot(-0{,}693) < \frac{7}{8} + 2\cdot(-0{,}6) = \frac{7}{8} - 2\cdot\frac{6}{10} =$$

$$= \frac{7}{8} - \frac{6}{5} < 0$$

$\Rightarrow$ **Negatív** irányban sodródik el.