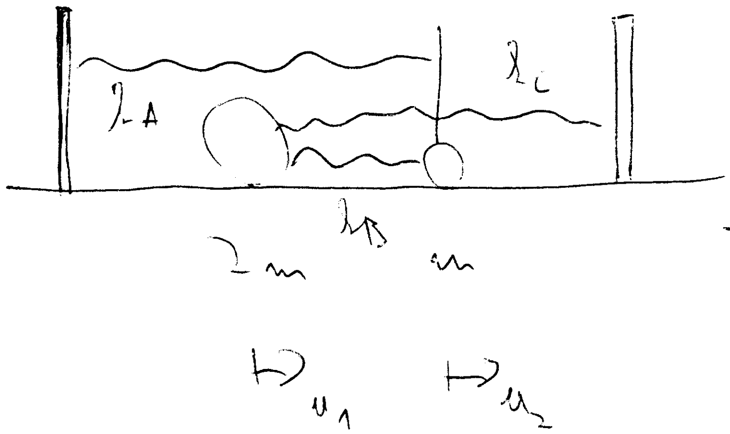
1.



$$2m\ddot{u}_1 = -\lambda_C u_1 - \lambda_B(u_1 - u_2)$$
$$m\ddot{u}_2 = -\lambda_A u_2 - \lambda_B(u_2 - u_1)$$

$$2m\ddot{u}_1 = -(\lambda_B + \lambda_C)u_1 + \lambda_B u_2$$
$$m\ddot{u}_2 = \lambda_B u_1 - (\lambda_A + \lambda_B)u_2$$

Próbafv. $u_1(t) = A_1 e^{i\omega t}$, $u_2(t) = A_2 e^{i\omega t}$

$\rightarrow \ddot{u}_1(t) = -\omega^2 A_1 e^{i\omega t}$, $\ddot{u}_2(t) = -\omega^2 A_2 e^{i\omega t}$

$\rightarrow$

$$-\omega^2 A_1 = -\frac{\lambda_B + \lambda_C}{2m} A_1 + \frac{\lambda_B}{2m} A_2$$
$$-\omega^2 A_2 = \frac{\lambda_B}{m} A_1 - \frac{\lambda_A + \lambda_B}{m} A_2$$

$$\begin{pmatrix} -\frac{\lambda_B + \lambda_C}{2m} & \frac{\lambda_B}{2m} \\ \frac{\lambda_B}{m} & -\frac{\lambda_A + \lambda_B}{m} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \underbrace{-\omega^2}_{=:\lambda} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$\rightarrow \begin{vmatrix} -\frac{\lambda_B + \lambda_C}{2m} - \lambda & \frac{\lambda_B}{2m} \\ \frac{\lambda_B}{m} & -\frac{\lambda_A + \lambda_B}{m} - \lambda \end{vmatrix} = \lambda^2 + \frac{2\lambda_A + 3\lambda_B + \lambda_C}{2m}\lambda + \frac{(\lambda_A + \lambda_B)(\lambda_B + \lambda_C) - \lambda_B^2}{2m^2} \overset{!}{=} 0$$

$$\rightarrow (-\omega^2)_{I,II} = \left(-\frac{2\lambda_A + 3\lambda_B + \lambda_C}{2m} \pm \sqrt{\frac{(2\lambda_A + 3\lambda_B + \lambda_C)^2}{4m^2} - 2\frac{\lambda_A\lambda_B + \lambda_A\lambda_C + \lambda_B\lambda_C}{m^2}}\right) \Big/ 2$$

2. $V(r) = V_0 \sin(\lambda r)$

$V_0, \lambda > 0$

$N = \sqrt{8\pi^3 \frac{m V_0}{\lambda^2}}$

$$V_{eff}(r) = \frac{N^2}{2mr^2} + V(r) = \frac{N^2}{2mr^2} + V_0 \sin(\lambda r)$$

$$V_{eff}'(r) = -\frac{N^2}{mr^3} + \lambda V_0 \cos(\lambda r)$$

$$V_{eff}'(r)\Big|_{r=r^*} = -\frac{N^2}{m r^{*3}} + \lambda V_0 \cos(\lambda r^*) \overset{!}{=} 0$$

$$\rightarrow \quad 1 \overset{!}{=} \frac{m r^{*3}}{N^2} \lambda V_0 \cos(\lambda r^*) =$$

$$= \cancel{m} r^{*3} \frac{1}{(2\pi)^3} \frac{\lambda^2}{\cancel{m} V_0} \lambda \cancel{V_0} \cos(\lambda r^*) =$$

$$= \underbrace{\left(\frac{\lambda r^*}{2\pi}\right)^3}_{\geq 1} \underbrace{\cos(\lambda r^*)}_{\leq 1}$$

A minimális érték tehát 1: $\frac{\lambda r^*}{2\pi} = 1$.

Kielégíti-e ez az érték

a fenti egyenlőséget? $\Rightarrow$

Ha $\frac{kr^*}{2\pi}=1$:

$$\left(\frac{kr^*}{2\pi}\right)^3 \cos(kr^*) = 1\cdot\cos(2\pi) = 1$$

Igen, kielégíti.

Tehát $\frac{kr^*}{2\pi}=1$, azaz $\boxed{r^* = \frac{2\pi}{k}}$.

$$V''_{eff}(r) = 3\frac{N^2}{mr^4} - k^2 V_0 \sin(kr)$$

$$V''_{eff}(r)\Big|_{r=r^*} = 3\frac{N^2}{mr^{*4}} - k^2 V_0 \underbrace{\sin(2\pi)}_{=0} = 3\frac{N^2}{mr^{*4}} > 0$$

$\boxed{\text{Igen, stabil}}$

Mechanikai energia: $r(t)=r^*$, $\dot r(t)=0$

$$\Downarrow$$

$$\boxed{E} = \frac{1}{2}m\dot r^2 + V_{eff}(r) = V_{eff}(r^*) =$$

$$= \frac{N^2}{2mr^{*2}} + V_0\sin(kr^*) =$$

$$= \frac{1}{2m}\cdot\frac{k^2}{4\pi^2}\cdot 8\pi^3\frac{mV_0}{k^2} + 0 =$$

$$= \boxed{\pi\cdot V_0}$$

$$E = \frac{N^2}{2mr^{*2}}$$

Legyen $E' = 2E$, $r = r^*$ egy adott időpillanatban.

$$E' = \frac{1}{2} m v_r^2 + \underbrace{\frac{1}{2} m v_\varphi^2}_{\Downarrow} + \underbrace{V(r^*)}_{=0} \overset{!}{=} 2E$$

$$= \frac{N^2}{2 m r^{*2}} = E$$

(az előző oldalról)

$$\longrightarrow \quad \frac{1}{2} m v_r^2 + E = 2E$$

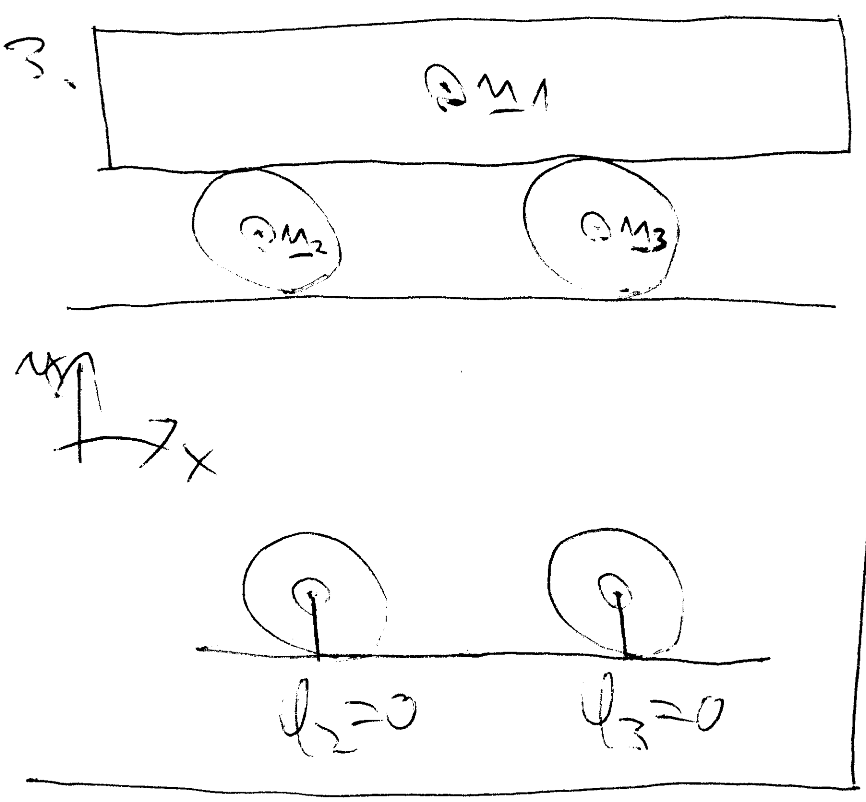
$$\frac{1}{2} m v_r^2 = E$$

Miközben:

$$\frac{1}{2} m v_\varphi^2 = E \text{ szintén!}$$

$$\Rightarrow \boxed{|v_r| = |v_\varphi|}$$

3.



1.) x_1, y_1, φ_1
x_2, y_2, φ_2
x_3, y_3, φ_3

2.) • y_1, y_2 és y_3 konst.
• tiszta görd.
• $\varphi_1 = 0$

3.) x_1

4.)

$x_1 = x_1$
$y_1 = d_1$ $\quad \varphi_1 = 0$ (kényszer)

$x_2 = -R\varphi_2 + c_2$ (tiszta gördülés a talajjal)

$\downarrow$ konstans

Hogyan fejezhető ki x_1 segítségével?
(Erre úgyis szükségünk lesz.)

→ Gondolkozzunk fordított irányban!

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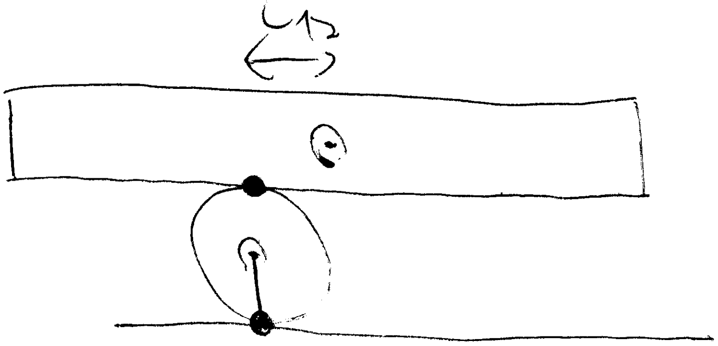
ZH2Pót / 6

Ha $\varphi_2 = 0$:



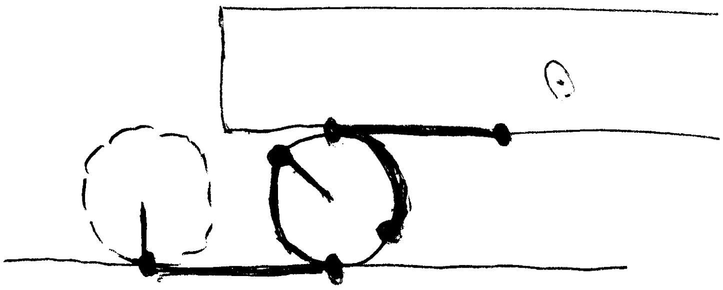
Ilyenkor:

$x_2 = C_2$

$x_1 = x_2 + c_{12} =$

$= C_2 + c_{12}$

Ha $\varphi_2 \neq 0$ ($\varphi_2 < 0$ az ábrán):



Minden kivastagított ív hossza megegyezik!

Azaz:

$x_2 = -R\varphi_2 + C_2$

$x_1 = x_2 - R\varphi_2 + c_{12}$

mert ehhez viszonyítunk

és a téglatest felületén lévő vastag szakasz hossza

$x_1 = -2R\varphi_2 + c_{12} + C_2$

$\varphi_2 = C_2' - \frac{x_1}{2R}$

$x_2 = -R \cdot \left(C' - \frac{x_1}{2R} \right) =$

$= C_2'' + \frac{x_1}{2}$

ahol C_2' és C_2'' konstansok

ZH2Pót/7

Tehát:

$x_1 = x_1$ $\qquad\qquad$ $\dot{x}_1 = \dot{x}_1$

$y_1 = d_1$ $\qquad\qquad$ $\dot{y}_1 = 0$

$\varphi_1 = 0$ $\qquad\qquad$ $\dot{\varphi}_1 = 0$

$x_2 = C_2'' + \frac{x_1}{2}$ $\qquad\qquad$ $\dot{x}_2 = +\frac{\dot{x}_1}{2}$

$y_2 = d_2$ (leolvasható az ábráról) $\qquad\qquad$ $\dot{y}_2 = 0$

$\varphi_2 = C_2' - \frac{x_1}{2R}$ $\qquad\qquad$ $\dot{\varphi}_2 = -\frac{\dot{x}_1}{2R}$

és hasonlóan (ugyanis semmilyen eltérés nincs a két henger között a konstansokon kívül):

$x_3 = C_3'' + \frac{x_1}{2}$ $\qquad\qquad$ $\dot{x}_3 = +\frac{\dot{x}_1}{2}$

$y_3 = d_3$ $\qquad\qquad$ $\dot{y}_3 = 0$

$\varphi_3 = C_3' - \frac{x_1}{2R}$ $\qquad\qquad$ $\dot{\varphi}_3 = -\frac{\dot{x}_1}{2R}$

5.) $L = T - V$ $\qquad$ $\Theta_{m_2} = \Theta_{m_3} = \frac{1}{2}MR^2$

$$T = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}\Theta_{m_1}\dot{\varphi}_1^2 + \frac{1}{2}M(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}\Theta_{m_2}\dot{\varphi}_2^2 +$$

$$+ \frac{1}{2}M(\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2}\Theta_{m_3}\dot{\varphi}_3^2 =$$

$$= \frac{1}{2}m\dot{x}_1^2 + 0 + \left(\frac{1}{2}M\frac{\dot{x}_1^2}{4} + \frac{1}{2}\cdot\frac{1}{2}MR^2\frac{\dot{x}_1^2}{4R^2}\right)\cdot 2 =$$

$$= \left(\frac{1}{2}m + \frac{3}{8}M\right)\dot{x}_1^2$$

$$V = mgy_1 + Mgy_2 + Mgy_3 = C = \text{konstans}$$

$$L = \left(\frac{1}{2}m + \frac{3}{8}M\right)\dot{x}_1^2 - C$$

6.)

$$\frac{\partial L}{\partial x_1} = 0$$

$$\frac{\partial L}{\partial \dot{x}_1} = \left(m + \frac{3}{4}M\right)\dot{x}_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \left(m + \frac{3}{4}M\right)\ddot{x}_1$$

$$\Rightarrow \boxed{\left(m + \frac{3}{4}M\right)\dot{x}_1 = \text{állandó}}$$

$$\rightarrow \left(m + \frac{3}{4}M\right)\ddot{x}_1 = 0$$

A rendszer teljes vízszintes impulzus-komponense:

$$P_x = m\dot{x}_1 + M\dot{x}_2 + M\dot{x}_3 =$$

$$= m\dot{x}_1 + M\frac{\dot{x}_1}{2} + M\frac{\dot{x}_1}{2} = (m+M)\dot{x}_1 \neq P_{x_1} = \left(m_1 + \frac{3}{4}M\right)\dot{x}_1$$

Ebből levonhatjuk a következtetést, hogy ilyenkor (és általában a talajon való tiszta gördülés esetén) nem érvényes az impulzusmegmaradás. Ennek is oka a talajjal való kölcsönhatás.