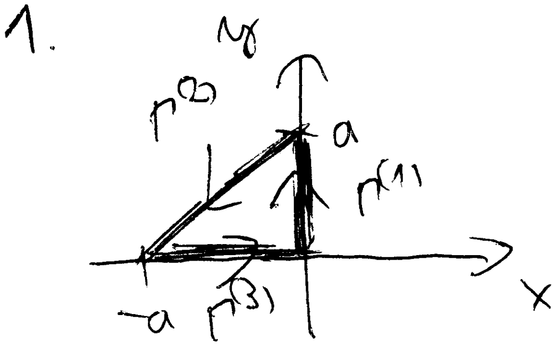
1.



$$\Psi_\Gamma = \Psi_{\Gamma^{(1)}} + \Psi_{\Gamma^{(2)}} + \Psi_{\Gamma^{(3)}}$$

$\Gamma^{(1)}$: $l = a$

$x(s) = 0$

$y(s) = s$

$\underline{T} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\rightarrow \Psi_{\Gamma^{(1)}} = \int_0^a C\cdot(0 - s)\, \underbrace{\underline{e}_x \underline{T}}_{(1\,0)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0}\, ds = 0$$

$\Gamma^{(2)}$: $l = \sqrt{2}a$

$x(s) = -\frac{1}{\sqrt{2}} s$

$y(s) = a - \frac{1}{\sqrt{2}} s$

$\underline{T} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\rightarrow \Psi_{\Gamma^{(2)}} = \int_0^{\sqrt{2}a} C\cdot\left(\cancel{-\frac{1}{\sqrt{2}} s} - a + \cancel{\frac{1}{\sqrt{2}} s}\right) \underbrace{\underline{e}_x \underline{T}}_{-\frac{1}{\sqrt{2}}\cdot(1\,0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = -\frac{1}{\sqrt{2}}}\, ds =$$

$$= \int_0^{\sqrt{2}a} C\cdot(-a)\cdot\left(-\frac{1}{\sqrt{2}}\right) ds = \frac{1}{\sqrt{2}} Ca \int_0^{\sqrt{2}a} 1\, ds = \frac{1}{\cancel{\sqrt{2}}} Ca \cancel{\sqrt{2}} a = Ca^2$$

$\Gamma^{(3)}$: $\ell = a$

$x = -a + s$

$y = 0$

$\underline{T} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\Rightarrow W_{\Gamma^{(3)}} = \int_0^a C \cdot (-a + s - 0)\, \underbrace{\underline{e}_x \underline{T}}_{(1\ 0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1}\, ds =$$

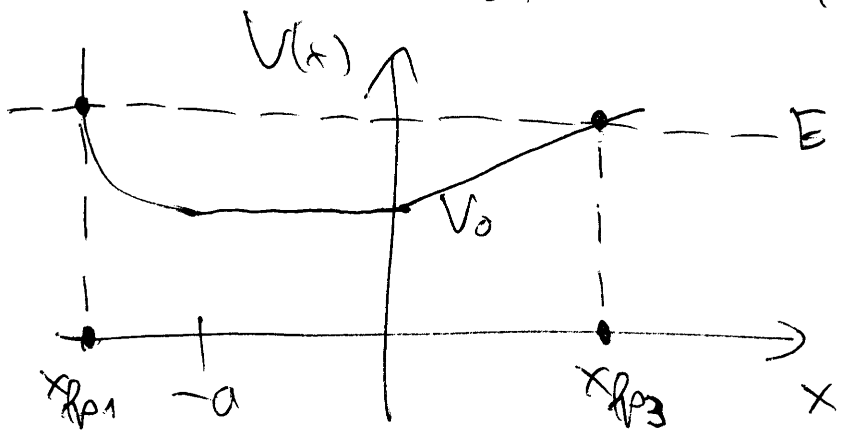
$$= \int_0^a C \cdot (s - a)\, ds = C \cdot \left[\frac{s^2}{2} - a \cdot s\right]_0^a = C \cdot \left(\frac{a^2}{2} - a^2\right) =$$

$$= -\frac{1}{2} C a^2$$

$$W_\Gamma = 0 + Ca^2 - \frac{1}{2}Ca^2 = \underline{\underline{\frac{1}{2}Ca^2}} \neq 0 \Rightarrow$$ a [illegible] nem lehet konzervatív.

zárt görbe mentén vett vonalintegrál

2. $$V(x)=\begin{cases} V_0+\frac{1}{2}k(x+a)^2 & \text{, ha } x\leq -a \text{ (I. tartomány)}\\ V_0 & \text{, ha } -a<x<0 \text{ (II. tartomány)}\\ V_0+\alpha x & \text{, ha } x\geq 0 \text{ (III. tartomány)}\end{cases}$$



Mivel a potenciál minimuma V_0, $E>V_0$ esetén valósulhat meg mozgás.

• fp. I.: $V_0+\frac{1}{2}k(x_{fp}+a)^2=E$

$$\Rightarrow x_{fp_{1,2}}=-a\mp\sqrt{\frac{2(E-V_0)}{k}}$$

$x_{fp_2}>-a \Rightarrow$ nem valósul meg, $x_{fp1}<-a$ ✓

II.: $V_0\neq E \Rightarrow \nexists$ fp.

III.: $V_0+\alpha x_{fp}=E$

$$\Rightarrow x_{fp3}=\frac{E-V_0}{\alpha}>0$$ ✓

• traj.-egy. I.: $V_0+\frac{1}{2}k(x+a)^2+\frac{1}{2}m\dot{x}^2=E$

$$\frac{1}{2}k(x+a)^2+\frac{1}{2}m\dot{x}^2=\underbrace{E-V_0}_{\text{konstans},\ >0}$$

$\rightarrow$ ellipszis

II.:

$$V_0 + \frac{1}{2} m \dot{x}^2 = E$$

$$\dot{x} = \pm\sqrt{\frac{2(E-V_0)}{m}} = \text{konst.}$$

$\rightarrow$ 2 egyenes

III.:

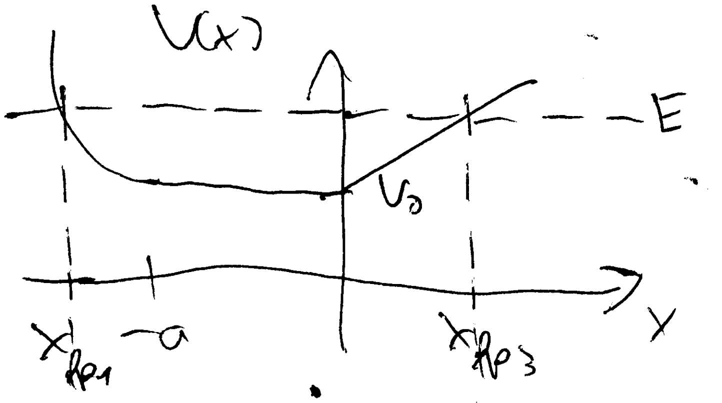
$$V_0 + \alpha x + \frac{1}{2} m \dot{x}^2 = E$$

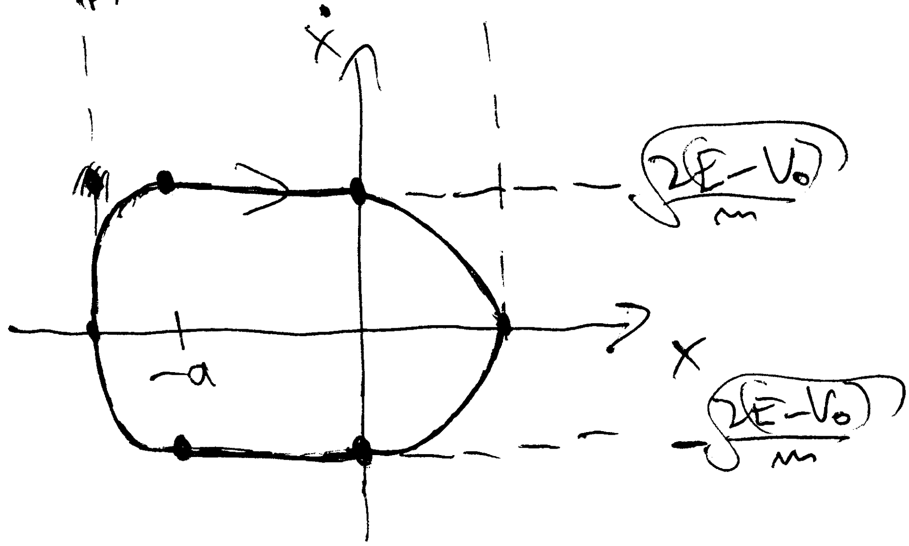
$$\alpha x + \frac{1}{2} m \dot{x}^2 = E - V_0$$

$\rightarrow$ fekvő parabola

$$\dot{x}(x=-a) \stackrel{\text{I.}}{=} \pm\sqrt{\frac{2}{m}\left(E - V_0 - \frac{1}{2}k\cdot\underbrace{(-a+a)^2}_{=0}\right)} = \pm\sqrt{\frac{2(E-V_0)}{m}}$$

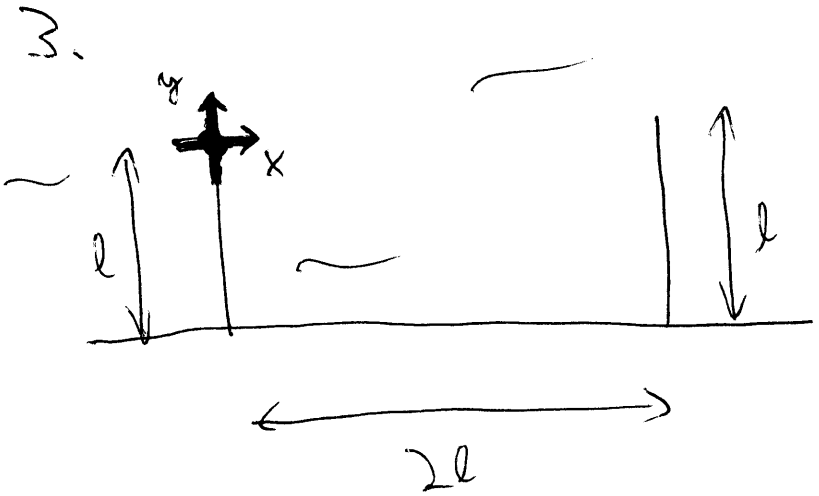
$$\dot{x}(x=0) \stackrel{\text{III.}}{=} \pm\sqrt{\frac{2}{m}\left(E - V_0 - \alpha\cdot 0\right)} = \pm\sqrt{\frac{2(E-V_0)}{m}}$$





Igen, a mozgás periodikus, mert az ábrán szereplő trajektória tetszőleges pontjából a tömegpont újra és újra visszatér, és onnantól kezdve önmagát ismétli a mozgás.

3.



$v_{fx} = 0$

$v_{fy} = v_0 \cos(\omega t)$

$a_{rel\,x} = \frac{a_0}{l} x \, tg\varphi$

$a_{rel\,y} = -\frac{1}{2} \frac{dv_{fy}}{dt}$

$\underline{v}(t) = ?$

$x(t=T) \overset{!}{=} 2l, \; y(t=T) \overset{!}{=} 0$

$$tg\varphi = \frac{y}{x} \Rightarrow a_{rel\,x} = \frac{a_0}{l} \cdot \cancel{x} \cdot \frac{y}{\cancel{x}} = \frac{a_0}{l} \cdot y$$

$$v_{rel\,y}(t) = v_{rel\,y}(t_0) + \int_{t_0}^{t} a_{rel\,y}(t')\,dt' = 0 + \int_0^t \left(-\frac{1}{2}\frac{dv_{fy}(t')}{dt'}\right)dt' =$$

$$= -\frac{1}{2}\int_0^t \frac{dv_{fy}(t')}{dt'}\,dt' = -\frac{1}{2}\cdot\left[v_{fy}(t')\right]_0^t =$$

$$= -\frac{1}{2}\cdot\left(v_0\cos(\omega t) - v_0\right)$$

$$v_y = v_{fy} + v_{rel\,y} = v_0\cos(\omega t) - \frac{1}{2}v_0\cos(\omega t) + \frac{1}{2}v_0 =$$

$$= \frac{1}{2}v_0\left(\cos(\omega t) + 1\right)$$

$$y(t) = y(t_0) + \int_{t_0}^{t} v_y(t')\,dt' = 0 + \int_0^t \frac{1}{2} v_0 (\cos(\omega t') + 1)\,dt' =$$

$$= \frac{1}{2} v_0 \left[\sin(\omega t') \cdot \frac{1}{\omega} + t'\right]_0^t =$$

$$= \frac{1}{2} v_0 \left(\frac{1}{\omega} \sin(\omega t) + t\right)$$

$$v_{rel_x}(t) = v_{rel_x}(t_0) + \int_{t_0}^{t} a_{rel_x}(t')\,dt' =$$

$$= 0 + \int_0^t \frac{a_0}{l} \cdot y(t')\,dt' = \frac{a_0}{l} \cdot \int_0^t \frac{1}{2} v_0 \left(\frac{1}{\omega} \sin(\omega t') + t'\right) dt' =$$

$$= \frac{1}{2} \frac{a_0 v_0}{l} \cdot \left[-\frac{1}{\omega^2} \cos(\omega t') + \frac{1}{2} t'^2\right]_0^t =$$

$$= \frac{1}{2} \frac{a_0 v_0}{l} \cdot \left(-\frac{1}{\omega^2} (\cos(\omega t) - 1) + \frac{1}{2} t^2\right) =$$

$$= \frac{1}{2} \frac{a_0 v_0}{l} \cdot \left(\frac{1}{\omega^2} (1 - \cos(\omega t)) + \frac{1}{2} t^2\right)$$

$$v_x = v_{fx} + v_{rel_x} = 0 + v_{rel_x} = v_{rel_x} =$$

$$= \frac{1}{2} \frac{a_0 v_0}{l} \cdot \left(\frac{1}{\omega^2} (1 - \cos(\omega t)) + \frac{1}{2} t^2\right)$$

$$x(t) = x(t_0) + \int_{t_0}^{t} v_x(t')\,dt' =$$

$$= 0 + \int_0^t \frac{1}{2}\frac{a_0 v_0}{\ell}\cdot\left(\frac{1}{\omega^2}\left(1-\cos(\omega t')\right) + \frac{1}{2}t'^2\right)dt' =$$

$$= \frac{1}{2}\frac{a_0 v_0}{\ell}\cdot\left[\frac{t'}{\omega^2} - \frac{1}{\omega^3}\sin(\omega t') + \frac{1}{6}t'^3\right]_0^t =$$

$$= \frac{1}{2}\frac{a_0 v_0}{\ell}\cdot\left(-\frac{1}{\omega^3}\sin(\omega t) + \frac{1}{\omega^2}t + \frac{1}{6}t^3\right)$$

$$\frac{1}{2}v_0\cdot\left(\frac{1}{\omega}\sin(\omega T) + T\right) \stackrel{!}{=} 0$$

$$\frac{1}{2}\frac{a_0 v_0}{\ell}\cdot\left(-\frac{1}{\omega^3}\sin(\omega T) + \frac{1}{\omega^2}T + \frac{1}{6}T^3\right) \stackrel{!}{=} 2\ell$$

$$\rightarrow \frac{1}{\omega}\sin(\omega T) = -T$$

$$\rightarrow \frac{1}{2}\frac{a_0 v_0}{\ell}\cdot\left(2\cdot\frac{1}{\omega^2}\cdot T + \frac{1}{6}T^3\right) = 2\ell$$

$\rightarrow$ T <u>zárt alakban</u> kifejezhető $\rightarrow \frac{1}{\omega}\sin(\omega T) = -T$ egy transzcendens egyenlet a feladat paramétereivel

DE: Nincs megoldása! $\frac{\sin\alpha}{\alpha} \neq -1$ semmilyen α-ra!

$\frac{\sin\alpha}{\alpha} > -1\ \forall\alpha \in \mathbb{R}$, és ezért $y(t) = \frac{1}{2}v\left(\frac{1}{\omega}\sin(\omega t) + t\right) > 0$ $\forall t > 0$-ra! A szonda <u>mindig</u> messzebb lesz a parttól, mint a zátony csúcsa!