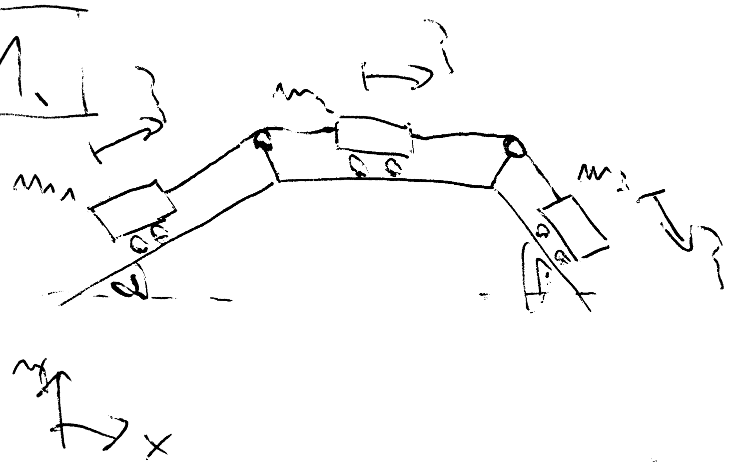
1.



1.) $x_1, y_1, x_2, y_2, x_3, y_3$

2.) •• lejtő,
• y_2 = állandó
•• kötél

3.) ξ

4.) $x_1 = c_1 + \xi\cos\alpha$
$y_1 = d_1 + \xi\sin\alpha$
$x_2 = c_2 + \xi$
$y_2 = d_2$
$x_3 = c_3 + \xi\cos\beta$
$y_3 = d_3 - \xi\sin\beta$

$\dot{x}_1 = \dot{\xi}\cos\alpha$
$\dot{y}_1 = \dot{\xi}\sin\alpha$
$\dot{x}_2 = \dot{\xi}$
$\dot{y}_2 = 0$
$\dot{x}_3 = \dot{\xi}\cos\beta$
$\dot{y}_3 = -\dot{\xi}\sin\beta$

3p.

5) $L = T - V$

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}m_3(\dot{x}_3^2 + \dot{y}_3^2) = \frac{1}{2}m_1\dot{\xi}^2 + \frac{1}{2}m_2\dot{\xi}^2 + \frac{1}{2}m_3\dot{\xi}^2 = \frac{1}{2}(m_1 + m_2 + m_3)\dot{\xi}^2$$

1p.

$$V = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 = m_1 g\xi\sin\alpha - m_3 g\xi\sin\beta + \text{konstans} = (m_1\sin\alpha - m_3\sin\beta)g\cdot\xi + \text{konstans}$$

1p.

6.) $$\frac{\partial L}{\partial \xi} = -(m_1\sin\alpha - m_3\sin\beta)\cdot g$$

$$\frac{\partial L}{\partial \dot{\xi}} = (m_1 + m_2 + m_3)\cdot\dot{\xi} \cancel{=p_\xi}$$

felismerjük:

$$\left|\frac{\partial L}{\partial \dot{\xi}}\right| = p_\xi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right) = (m_1 + m_2 + m_3)\ddot{\xi}$$

$\rightarrow$

$$(m_1+m_2+m_3)\ddot{\xi} = (m_3 \sin\beta - m_1 \sin\alpha)\cdot g$$ 1p.

$p_\xi = \left|\frac{\partial L}{\partial \dot{\xi}}\right|$ = állandó akkor és csak akkor,

ha $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right)=0$, azaz az Euler–Lagrange-

egyenlet szerint: $\frac{\partial L}{\partial \xi}=0$.

$\rightarrow$

$$-(m_1 \sin\alpha - m_3 \sin\beta)\cdot g \overset{!}{=} 0$$

$$m_1 \sin\alpha + m_3 \sin\beta \overset{!}{=} 0$$

Ezen feltétel mellett marad meg p_ξ a kialakuló mozgás során.

3 p.

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$\boxed{2.}$

$$V_{eff}(r) = \frac{N^2}{2mr^2} + \gamma\frac{1}{r^2} - \sigma\frac{1}{r^4} = \left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^2} - \sigma\frac{1}{r^4}$$

$$V'_{eff}(r) = -2\left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^3} + 4\sigma\frac{1}{r^5}$$

$$V'_{eff}(r)\Big|_{r=r^*} = -2\left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^{*3}} + 4\sigma\frac{1}{r^{*5}} = 0 \quad \text{1p.} \quad (\mathrm{I.})$$

Másrészt:

$$E = \frac{1}{2}m\dot{r}^2 + V_{eff}(r) = \frac{1}{2}m\dot{r}^2 + \left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^2} - \sigma\frac{1}{r^4} \quad \text{1p.}$$

Ha $r(t) = r^* = \text{állandó}: \quad \dot{r} = 0$

$$\to \quad E = 0 + \left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^{*2}} - \sigma\frac{1}{r^{*4}} \quad \text{3p.}$$

$(\mathrm{II.})$

(I.) és (II.) 2 egyenlet r^*-ra és N-re.

$$(\mathrm{II.}) \quad \left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^{*2}} = E + \sigma\frac{1}{r^{*4}}$$

$$\to (\mathrm{I.}) \quad -2\left(E + \sigma\frac{1}{r^{*4}}\right)\frac{1}{r^*} + 4\sigma\frac{1}{r^{*5}} = 0$$

$$-2E\frac{1}{r^*} - 2\sigma\frac{1}{r^{*5}} + 4\sigma\frac{1}{r^{*5}} = 0$$

$$\cancel{2}Er^{*4} = \cancel{2}\sigma$$

$$r^* = \sqrt[4]{\frac{\sigma}{E}} \quad \text{2p.} \quad (*)$$

Tehát csak $E > 0$ mellett lehetséges körpálya

$$V''_{eff}(r) = 6\left(\frac{N^2}{2m}+\gamma\right)\frac{1}{r^4} - 20\sigma\frac{1}{r^6}$$

$$V''_{eff}(r)\Big|_{r=r^*} = 6\left(\frac{N^2}{2m}+\gamma\right)\cdot\frac{1}{r^{*2}}\cdot\frac{1}{r^{*2}} - 20\sigma\frac{1}{r^{*6}} =$$

1p.

$$= 6\cdot\left(E+\sigma\frac{1}{r^{*4}}\right)\cdot\frac{1}{r^{*2}} - 20\sigma\frac{1}{r^{*6}} =$$

$\uparrow$ II

$$= 6\cdot(E+E)\cdot\sqrt{\frac{E}{\sigma}} - 20\cdot E\sqrt{\frac{E}{\sigma}} =$$

$\uparrow$ $*$

$$= (12-20)E\cdot\sqrt{\frac{E}{\sigma}} = -8E\sqrt{\frac{E}{\sigma}}$$ 2p.

$-8<0$, $*$ szerint $E>0$, és $\sqrt{\frac{E}{\sigma}}>0$,

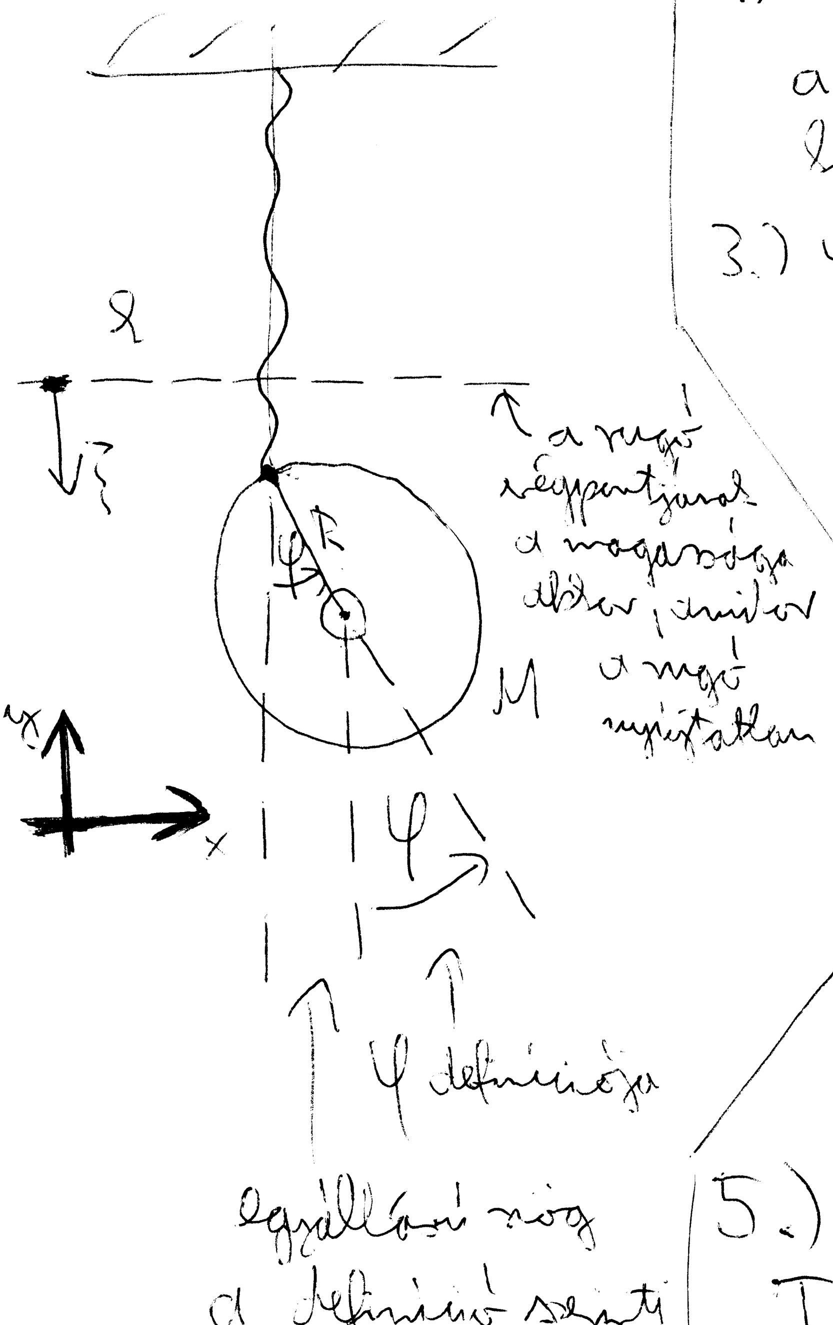
ezért:

$$-8E\cdot\sqrt{\frac{E}{\sigma}} < 0$$

A körpálya instabil.

2p.

3.



1.) $x;\ y;\ \varphi$

2.) φ egyértelműen meghatározza a rögzítési pont vízszintes (és a rögzítési ponthoz viszonyított függőleges) koordinátáját

2p.

3.) $\varphi,\ \xi$ ⟹ a rugó végpontjának a rugó nyújtatlan helyzetétől lefelé mért elmozdulása

4.)
$$x = c + R\sin\varphi$$
$$y = d - \xi - R\cos\varphi$$
$$\varphi = \varphi$$
$$\dot{x} = R\cos\varphi\,\dot{\varphi}$$
$$\dot{y} = -\dot{\xi} + R\sin\varphi\,\dot{\varphi}$$
$$\dot{\varphi} = \dot{\varphi}$$

3p.

5.) $L = T - V$

$$T = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}\Theta_s\dot{\varphi}^2 = \frac{1}{2}M(\cancel{R^2\cos^2\varphi\,\dot{\varphi}^2} + \dot{\xi}^2 - 2\dot{\xi}R\sin\varphi\,\dot{\varphi} + \cancel{R^2\sin^2\varphi\,\dot{\varphi}^2}) + \frac{1}{4}MR^2\dot{\varphi}^2$$

1p.

$$V = Mgy + \frac{1}{2}k\xi^2 = Mgd - Mg\xi - MgR\cos\varphi + \frac{1}{2}k\xi^2$$

1p.