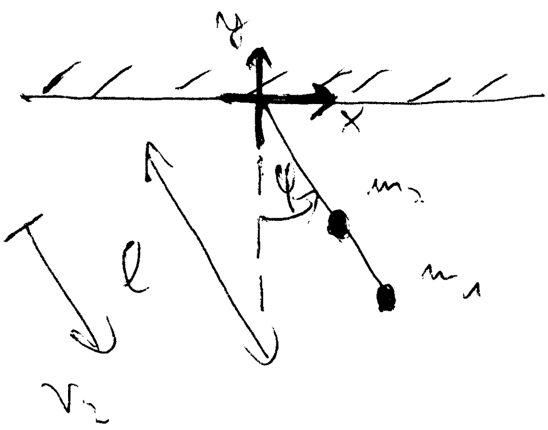
EMB/ZH2Pót/1

1.



1.) x_1, y_1, x_2, y_2

2.) • az 1-es test origótól mért távolsága állandó

• a kettes test az origóhoz képest ugyanolyan irányban van, mint az 1-es

3.) φ, r_2 2p.

4.) $x_1 = l\sin\varphi$

$y_1 = -l\cos\varphi$

$x_2 = r_2\sin\varphi$

$y_2 = -r_2\cos\varphi$

$\dot{x}_1 = l\cos\varphi\,\dot{\varphi}$

$\dot{y}_1 = l\sin\varphi\,\dot{\varphi}$

$\dot{x}_2 = \dot{r}_2\sin\varphi + r_2\cos\varphi\,\dot{\varphi}$

$\dot{y}_2 = -\dot{r}_2\cos\varphi + r_2\sin\varphi\,\dot{\varphi}$ 4p.

5.) $L = T - V$

$T = \frac{1}{2}m_1(\dot{x}_1^2+\dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2+\dot{y}_2^2) =$ [muszáj így lennie]

$= \frac{1}{2}m_1 l^2\dot{\varphi}^2 + \frac{1}{2}m_2(\dot{r}_2^2 + r_2^2\dot{\varphi}^2)$ 1p.

$V = m_1 g y_1 + m_2 g y_2 =$

$= -m_1 g l\cos\varphi - m_2 g r_2\cos\varphi$ 1p.

6.) $\frac{\partial L}{\partial\varphi} = m_1 g l\cdot(-\sin\varphi) + m_2 g r_2\cdot(-\sin\varphi)$

$\frac{\partial L}{\partial\dot{\varphi}} = m_1 l^2\dot{\varphi} + m_2 r_2^2\dot{\varphi}$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m_1 \ell^2 \ddot{\varphi} + 2 m_2 r_2 \dot{r}_2 \dot{\varphi} + m_2 r_2^2 \ddot{\varphi}$$

$\rightarrow$

$$m_1 \ell^2 \ddot{\varphi} + 2 m_2 r_2 \dot{r}_2 \dot{\varphi} + m_2 r_2^2 \ddot{\varphi} =$$

$$= - m_1 g \ell \sin\varphi - m_2 g r_2 \sin\varphi$$

$$\frac{\partial L}{\partial r_2} = m_2 r_2 \dot{\varphi}^2 + m_2 g \cos\varphi$$

$$\frac{\partial L}{\partial \dot{r}_2} = m_2 \dot{r}_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_2}\right) = m_2 \ddot{r}_2$$

$\rightarrow$

$$\cancel{m_2} \ddot{r}_2 = \cancel{m_2} r_2 \dot{\varphi}^2 + \cancel{m_2} g \cos\varphi$$

5 p.

2.

$$V_{eff}^{(1)}(r) = \frac{N_1^2}{2m_1 r^2} + \alpha r^{1/2}$$

$$V_{eff}^{(2)}(r) = \frac{N_2^2}{2m_2 r^2} + \alpha r^{1/2}$$

$$V_{eff}^{(1)\prime}(r) = -\frac{N_1^2}{m_1 \cdot r^3} + \frac{1}{2}\alpha r^{-1/2}$$

$$V_{eff}^{(2)\prime}(r) = -\frac{N_2^2}{m_2 r^3} + \frac{1}{2}\alpha r^{-1/2}$$

$$V_{eff}^{(1)\prime}(r)\Big|_{r=r_1^*} = -\frac{N_1^2}{m_1 \cdot {r_1^*}^3} + \frac{1}{2}\alpha {r_1^*}^{-1/2} = 0 \quad *$$

$$V_{eff}^{(2)\prime}(r)\Big|_{r=r_2^*} = -\frac{N_2^2}{m_2 {r_2^*}^3} + \frac{1}{2}\alpha {r_2^*}^{-1/2} = 0$$

1p.

$r_2^* = 4r_1^*$, $N_2 = \sqrt{32}\, N_1$

$$\Rightarrow \quad -\frac{\cancel{32} N_1^2}{m_2 \cdot \cancel{64} {r_1^*}^3} + \frac{1}{2}\alpha \cdot \frac{1}{\cancel{\sqrt{4}}} {r_1^*}^{-1/2} = 0$$

$$-\frac{N_1^2}{m_2 {r_1^*}^3} + \frac{1}{2}\alpha {r_1^*}^{-1/2} = 0 \quad **$$

!!!

1p.

$$(*) \Rightarrow \quad \frac{1}{2}\alpha r_1^{*\,-1/2} = \frac{N_1^2}{m_1 r_1^{*3}}$$

$$\rightarrow (**):$$

$$-\frac{N_1^2}{m_2 r_1^{*3}} + \frac{N_1^2}{m_1 r_1^{*3}} = 0$$

$$\Rightarrow \quad \boxed{m_2 = m_1}$$

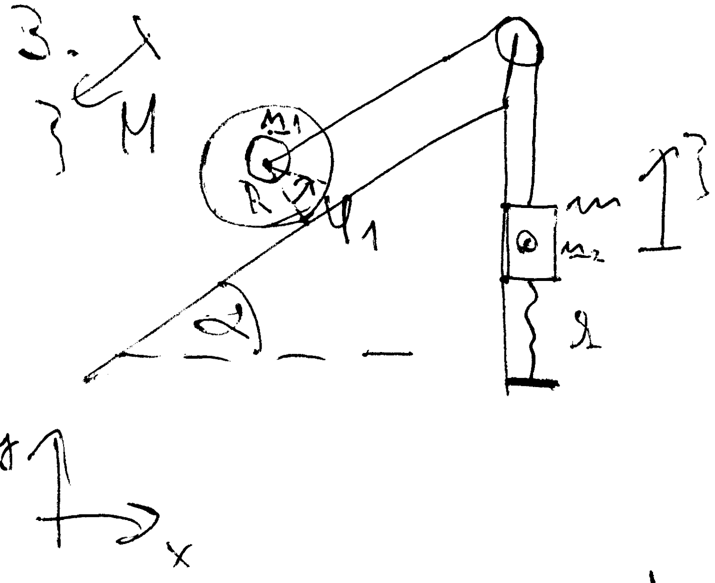
4p.

$$\boxed{\Omega_2^*} = \frac{N_2}{m_2 r_2^{*2}} = \frac{\sqrt{32}\,N_1}{m_1 \cdot 16\, r_1^{*2}} = \frac{2^{5/2}}{2^4}\frac{N_1}{m_1 r_1^{*2}} =$$

$$= 2^{-3/2}\frac{N_1}{m_1 r_1^{*2}} = \boxed{\frac{1}{\sqrt{8}}\Omega_1^*}$$

2p.

3.



1.) $x_1, y_1, \varphi_1, x_2, y_2, \varphi_2$

2.)
- lejtő!
- tiszta gördülés
- kötél
- $x_2 = $ állandó!
- $\varphi_2 = $ állandó!

3.) s

1p.

4.)
$x_1 = c_1 - s\cos\alpha$
$y_1 = d_1 - s\sin\alpha$
$\varphi_1 = s/R$ (tiszta gördülés)

$x_2 = c_2$
$y_2 = d_2 + s$
$\varphi_2 = \Phi_2 = $ állandó

$\dot{x}_1 = -\dot{s}\cos\alpha$
$\dot{y}_1 = -\dot{s}\sin\alpha$
$\dot{\varphi}_1 = \dot{s}/R$
$\dot{x}_2 = 0$
$\dot{y}_2 = \dot{s}$
$\dot{\varphi}_2 = 0$

3p.

5.) $L = T - V$

$$T = \frac{1}{2}M(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}\Theta_{m_1}\dot{\varphi}_1^2 + \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}\Theta_{m_2}\dot{\varphi}_2^2 =$$

$$= \frac{1}{2}M\dot{s}^2 + \frac{1}{2}\cdot\frac{1}{2}M\cancel{R^2}\frac{\dot{s}^2}{\cancel{R^2}} + \frac{1}{2}m\dot{s}^2 + 0 =$$

$\Theta_{m_1} = \frac{1}{2}MR^2$

$$= \left(\frac{3}{4}M + \frac{1}{2}m\right)\dot{s}^2$$

1p.

$$V = Mgz_1 + mgz_2 + \frac{1}{2} k \xi^2 =$$

$$= -Mg\sin\alpha\, \xi + mg\xi + \frac{1}{2} k \xi^2 + \text{konstans}$$

1p.

6.) $$\frac{\partial L}{\partial \xi} = Mg\sin\alpha - mg - k\xi$$

$$\frac{\partial L}{\partial \dot{\xi}} = \left(\frac{3}{2} M + m\right)\dot{\xi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right) = \left(\frac{3}{2} M + m\right)\ddot{\xi}$$

$$\rightarrow \quad \left(\frac{3}{2} M + m\right)\ddot{\xi} = Mg\sin\alpha - mg - k\xi$$

1p.