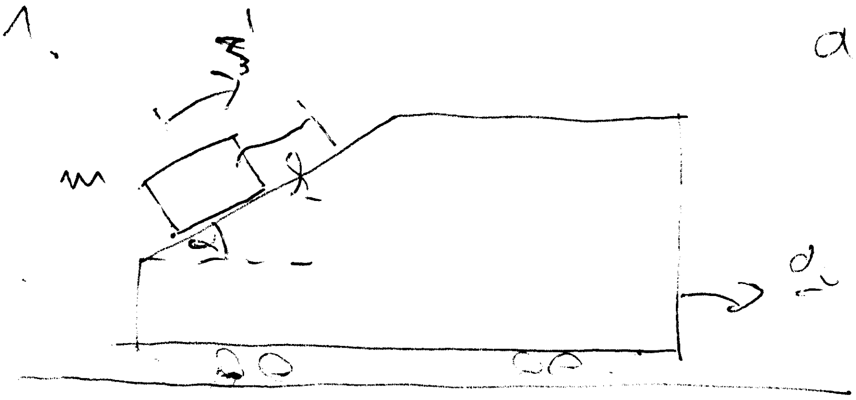
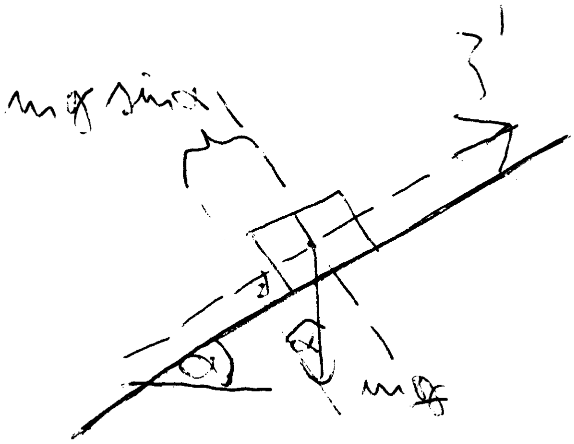
1.



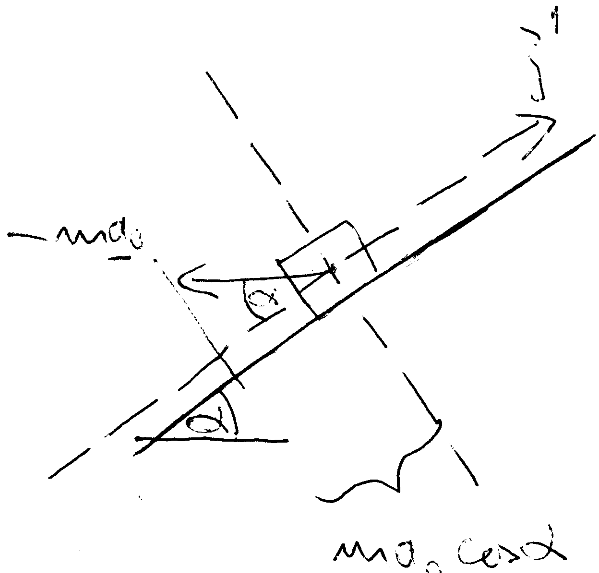
a) $m\ddot{\underline{r}}' = \underline{F}_{mg} + m\underline{g} + \underline{N} - m\underline{a}_0$

$(m\ddot{\underline{r}}')_{\parallel} = m\ddot{\xi}'$

$(\underline{F}_{mg})_{\parallel} = -k\xi'$



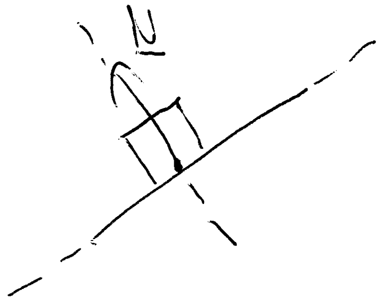
$(m\underline{g})_{\parallel} = -mg\sin\alpha$



$(-m\underline{a}_0)_{\parallel} = -ma_0\cos\alpha$

2p. MIN

$(\underline{N})_{\parallel} = 0$



$\rightarrow m\ddot{\xi}' = -k\xi' - \underset{mg\sin\alpha}{\cancel{ma_0\cos\alpha}} - ma_0\cos\alpha$ 2p.

1p.

b) $\ddot{\xi}' = 0 \Rightarrow 0 = -k\xi'^{*} - mg\sin\alpha - ma_0\cos\alpha$

$\Rightarrow \xi'^{*} = -\frac{m(g\sin\alpha + a_0\cos\alpha)}{k}$

2.

$v_{fx} = 0$

$v_{fy}(x) = v_0 \left(\frac{x}{l}\right)^n$

$t=0$: $x=0,\ y=0,\ v_x=0,\ v_y=0$

$\Downarrow \qquad \Downarrow$

$v_{relx} = 0 \qquad v_{rely} = 0$

$a_{relx} = a_1$

$a_{rely} = a_2$

Pálya: $y(x) = \frac{a_2}{a_1} x + C x^{5/2}$

$n = ?$

$v_{relx}(t) = v_{relx}(t_0) + \int_{t_0}^{t} a_{relx}(t')\,dt' = 0 + \int_0^t a_1\,dt' = a_1 t$

$v_x = v_{fx} + v_{relx} = 0 + a_1 t = a_1 t$

$x(t) = x(t_0) + \int_{t_0}^{t} v_x(t')\,dt' = 0 + \int_0^t a_1 t'\,dt' = \frac{1}{2} a_1 t^2$ (*)

2p.

$v_{rely}(t) = v_{rely}(t_0) + \int_{t_0}^{t} a_{rely}(t')\,dt' = 0 + \int_0^t a_2\,dt' = a_2 t$

$v_y = v_{fy} + v_{rely} = v_0 \left(\frac{x}{l}\right)^n + a_2 t$ (*)

$y(t) = y(t_0) + \int_{t_0}^{t} v_y(t')\,dt' = 0 + \int_0^t \left(v_0 \left(\frac{x(t')}{l}\right)^n + a_2 t' \right) dt' =$

$$= \int_0^t \left(v_0 \cdot \left(\frac{\frac{1}{2}a_1 t'^2}{\ell}\right)^n + a_2 t' \right) dt' =$$

$$= \cancel{\frac{v_0 a_1^n}{2\ell}} \quad v_0 \left(\frac{a_1}{2\ell}\right)^n \cdot \int_0^t t'^{2n}\, dt' + a_2 \cdot \int_0^t t'\, dt' =$$

$$= v_0 \left(\frac{a_1}{2\ell}\right)^n \frac{t^{2n+1}}{2n+1} + \frac{1}{2} a_2 t^2$$

5 p. MIN

$$(*) \Rightarrow \quad t = \sqrt{\frac{2x}{a_1}}$$

$$\Rightarrow y(x) \equiv y(t(x)) = \frac{v_0}{2n+1}\left(\frac{a_1}{2\ell}\right)^n \cdot \left(\frac{2x}{a_1}\right)^{\frac{2n+1}{2}} +$$

$$+ \frac{1}{2} a_2 \cdot \frac{2x}{a_1} = \frac{v_0}{2n+1}\left(\frac{a_1}{2\ell}\right)^n \cdot \left(\frac{2}{a_1}\right)^{\frac{2n+1}{2}} \cdot x^{\frac{2n+1}{2}} + \frac{a_2}{a_1} x$$

3 p.

$$y(x) \overset{!}{=} \frac{a_2}{a_1} x + C x^{5/2}$$

$$\Rightarrow \frac{2n+1}{2} = \frac{5}{2}$$

$$2n+1 = 5$$

$$\boxed{n = 2}$$

2 p.

3.

$$V'(x) = ax^2 + bx + c$$

$$V'(x)\Big|_{x=x^*} = a{x^*}^2 + bx^* + c \overset{!}{=} 0$$

$$\longrightarrow \quad x^*_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad 1p.$$

$x_1^* = x_2^* \in \mathbb{R}$ akkor és csak akkor, ha

$$\boxed{b^2 - 4ac = 0}$$

Ezen feltétel mellett lesz pontosan 1db egyensúlyi pont.

3p.

Ez a következő:

$$x^* = -\frac{b}{2a} \quad 1p.$$

$$V''(x) = 2ax + b$$

$$V''(x)\Big|_{x=x^*} = 2a\cdot\left(-\frac{b}{2a}\right) + b = 0$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{V''(x)\big|_{x=x^*}}{m}} = 0} \quad 2p.$$

$$V(x = x^*) = \frac{1}{3}a\left(-\frac{b}{2a}\right)^3 + \frac{1}{2}b\left(-\frac{b}{2a}\right)^2 + c\left(-\frac{b}{2a}\right) + d =$$

$$= -\frac{1}{24}\frac{b^3}{a^2} + \frac{1}{8}\frac{b^3}{a^2} - \frac{bc}{2a} + d = \frac{b^3}{12a^2} - \frac{bc}{2a} + d \overset{!}{\leq} E \quad 3p.$$

Behelyettesítve az 1 egyensúlyi pont feltételét:

$$\hookrightarrow c = \frac{b^2}{4a}$$

$$V(x=x^*) = \frac{b^3}{12a^2} - \frac{b \cdot \frac{b^2}{4a}}{2a} + d =$$

$$= \frac{b^3}{12a^2} - \frac{b^3}{8a^2} + d = -\frac{b^3}{24a^2} + d$$

$$\Rightarrow \boxed{E \overset{!}{\geq} -\frac{b^3}{24a^2} + d}$$

+1p.