Elektrodinamika gyakorlat

2 ZH lesz

Elmfiz példatár 2.

Elméleti fizikai feladatok Béhérny

Gálfi

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Simonyi Károly: Elméleti villamosságtan

$$\boxed{\begin{aligned} \operatorname{div} \underline{B} &= 0 \\ \operatorname{rot} \underline{E} + \frac{\partial \underline{B}}{\partial t} &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} \operatorname{div} \underline{D} &= \rho \\ \operatorname{rot} \underline{H} - \frac{\partial \underline{D}}{\partial t} &= \underline{j} \end{aligned}}$$

$$\underline{D} = \varepsilon \cdot \varepsilon_0 \underline{E}$$

$$\underline{H} = \frac{1}{\mu \cdot \mu_0} \underline{B}$$

Egyváltozós függvényre difegyenlet:

$$\left.\begin{aligned} &\left[a\left(\frac{d}{dt}\right)^n + b\left(\frac{d}{dt}\right)^{n-1} + \ldots + k\left(\frac{d}{dt}\right)^1 + \ell\right] u(t) = 0 \\ &\Updownarrow \\ &a \dddot{u} + b \ddot{u} + \ldots + \ell \dot{u} + u = 0 \Leftrightarrow L\left(\frac{d}{dt}\right) u(t) = 0 \end{aligned}\right\} \text{homogén}$$

inhomogén: $L\left(\frac{d}{dt}\right) u(t) = f(t)$

Legyen u_1 ennek megoldása: $L u_1 = f$ és $L u_2 = f_2$

u_0 a homogénnek: $L u_0 = 0$

Ekkor $u_1 + u_0$ megoldása az inhomogénnek $L(u_1 + u_0) = L u_1 + L u_0 = f$

A homogén egyenlet megoldásai lineáris teret alkotnak, az inhomogénre $L(u_1 + u_2) = f + f_2$

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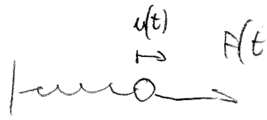
$$L\left(\frac{d}{dt}\right) := a\left(\frac{d}{dt}\right)^n + b\left(\frac{d}{dt}\right)^{n-1} + \ldots + k\left(\frac{d}{dt}\right)^1 + \ell$$

$L\left(\frac{d}{dt}\right) u(t) = f(t)$ kérdés: milyen $u(t)$ elégíti ki ezt?



$$m \ddot{u} = -k u - c \dot{u} + F(t)$$

$$\left[m \frac{d^2}{dt^2} + c \frac{d}{dt} + k\right] u(t) = F(t)$$

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① $f(t) = \sum_n c_n \varphi_n(t)$ $\qquad f(t) = \int_{-\infty}^{\infty} d\tau\, f(\tau)\, \delta(t-\tau)$

② $L\left(\frac{d}{dt}\right) \xi_n(t) := \varphi_n(t)$ legyen $\xi_n(t)$ ilyen $\qquad L\left(\frac{d}{dt}\right) G(t) = \delta(t)$

kauzális

③ $u(t) = \sum_n c_n \xi_n(t)$ $\qquad u(t) = \int_{-\infty}^{\infty} d\tau\, f(\tau)\, G(t-\tau) = \int_{-\infty}^{t} d\tau\, f(\tau)\, G(t-\tau)$

Ez volt időre.

Térre is megy, elektrosztatikában pl.:

$\operatorname{rot} \underline{E} = 0 \Rightarrow \underline{E} = -\operatorname{grad}\phi$

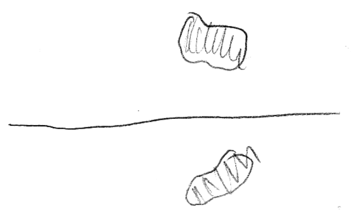
$\operatorname{div} \underline{E} = \rho / \varepsilon_0 \rightarrow \operatorname{div}\operatorname{grad}\phi = -\rho/\varepsilon_0 \iff \Delta\phi = -\rho/\varepsilon_0$

$u(t) \sim \phi(\underline{r}) = \phi(x, y, z)$

$L\left(\frac{d}{dt}\right) \sim \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$f(t) \sim \rho(\underline{r}) = \int d^3 r'\, \rho(\underline{r}')\, \delta(\underline{r} - \underline{r}')$

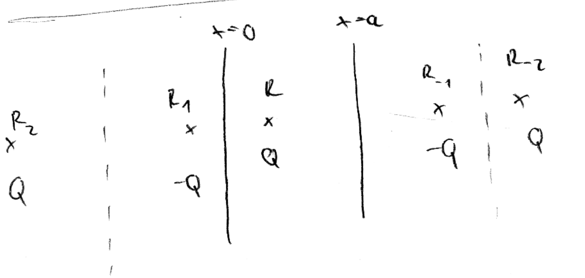
$\phi(\underline{r}) = \int d^3 r'\, \rho(\underline{r}')\, G(\underline{r}, \underline{r}')$



Fühörfelhő

$\rho(x, y, z)$

$$\phi(x,y,z) = \frac{1}{4\pi\varepsilon} \int_{-\infty}^{\infty} dX \int_{-\infty}^{\infty} dY \int_{0}^{\infty} dZ\, \rho(X,Y,Z) \cdot \left(\frac{1}{\sqrt{(x-X)^2 + (y-Y)^2 + (z-Z)^2}} - \frac{1}{\sqrt{(x-X)^2 + (y-Y)^2 + (z+Z)^2}} \right)$$



$\underline{R}_1 = (-x, y, z)$ $\qquad \underline{R}_{-1} = (-x + 2a, y, z)$

$\underline{R}_2 = (x - 2a, y, z)$ $\qquad \underline{R}_{-2} = (x + 2a, y, z)$

$\underline{R}_3 = (-x - 2a, y, z)$ $\qquad \operatorname{sgn} Q = 1$ ha $(x + 2na, y, z)$

$\qquad\qquad = -1$ ha $(-x + 2na, y, z)$

$\underline{R}_4 = (x - 4a, y, z)$

$$\phi(x,y,z) = \frac{1}{4\pi\varepsilon}\left[Q \sum_{n=-\infty}^{\infty} \left[\frac{1}{\sqrt{(x-X-2na)^2+(y-Y)^2+(z-Z)^2}} - \frac{1}{\sqrt{(x+X-2na)^2+(y-Y)^2+(z-Z)^2}}\right]\right]$$

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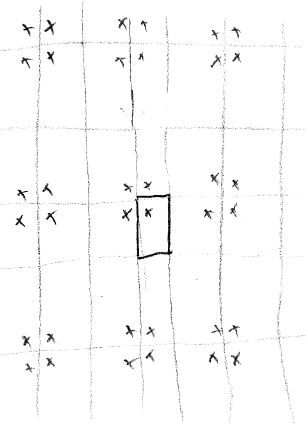
Felhő esetén $\phi(x,y,z) = \frac{1}{4\pi\varepsilon}\int_0^a dX \int_{-\infty}^{\infty} dY \int_{-\infty}^{\infty} dZ\, \rho(X,Y,Z)\left[\sum_{n=-\infty}^{\infty}\left(\frac{1}{r_{\dots}} - \frac{1}{r'_{\dots}}\right)\right]$



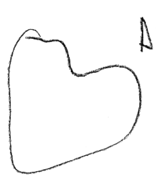
$\operatorname{sgn} Q = 1$ ha $(x+2na, -y+2mb, z)$ $n,m \in \mathbb{Z}$

$= -1$ ha $(-x+2na, +y+2mb, z)$

Felhő esetén

$$\phi(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int_0^a dX \int_0^b dY \int_{-\infty}^{\infty} dZ\, \rho(X,Y,Z) \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \cdot$$

$$\cdot\left(\frac{1}{\sqrt{(x-X-2na)^2+(y+Y-2mb)^2+(z-Z)^2}} - \frac{1}{(x+X-2na)^2+(y-Y-2mb)^2+(z-Z)^2}\right)$$



$\Delta\phi = \rho$

$\phi(\text{falon}) = 0$

helyett

$\Delta\phi = \lambda\phi$ és $\phi(\text{falon}) = 0$

$(\Delta - \lambda)\phi = 0$ $\phi = 0$ persze, hogy jó, de ez triviális

Állítás: vannak diszkrét λ megoldások, tetsz. határfeltételekre, melyek teljes, ortonormált bázist adnak.

$$\langle \phi_{nmp} | \phi_{n'm'p'} \rangle = \delta_{nn'}\delta_{mm'}\delta_{pp'}$$

Ebben tetszőleges $f(x,y,z)$ $\exists\, c_{nmp} \in \mathbb{C}$: $f(x,y,z) = \sum_{n,m,p} c_{nmp}\phi_{nmp}(x,y,z)$ $/\cdot \phi^*_{n'm'p'}$

$$\langle \phi_{n'm'p'} | f \rangle = \sum_{nmp} c_{nmp} \langle \phi^*_{n'm'p'} | \phi_{nmp} \rangle = c_{n'm'p'}$$

$$f(\underline{r}) = \sum_{n,m,p} \underbrace{\langle \phi_{nmp} | f \rangle}_{\int d^3\underline{R}\, \phi^*_{nmp}(\underline{R})\, f(\underline{R})} \phi_{nmp}(\underline{r})$$

$$f(\underline{r}) = \int d^3\underline{R} \underbrace{\left(\sum_{n,m,p} \phi_{n,m,p}(\underline{r})\, \phi^*_{n,m,p}(\underline{R}) \right)}_{\delta(\underline{r}-\underline{R})} f(\underline{R}) \qquad f(\underline{r}) = \int d^3\underline{R}\, \delta(\underline{r}-\underline{R})\, f(\underline{R})$$

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Most $\Delta\phi = -\frac{1}{\varepsilon_0}\delta(\underline{x}-\underline{y})$ -nek keresem a Green-jét $G(\underline{x},\underline{y}) = \sum_{n,m,l} c_{nml}(\underline{y})\, \phi_{nml}(\underline{x})$ alakban

$$\Delta_{\underline{x}} G(\underline{x},\underline{y}) = \sum_{nml} c_{nml}(\underline{y})\, \Delta_x \phi_{nml}(\underline{x}) = \sum_{nml} c_{nml}(\underline{y})\, \lambda_{nml}\, \phi_{nml}(\underline{x}) \stackrel{!}{=} -\frac{1}{\varepsilon_0}\delta(\underline{x}-\underline{y}) =$$

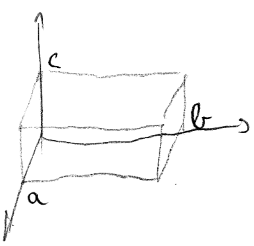
$$= -\frac{1}{\varepsilon_0}\sum_{nml} \phi^*_{nml}(y)\, \phi_{nml}(x)$$

$$c_{nml}(y)\lambda_{nml} = -\frac{1}{\varepsilon_0}\phi^*_{nml}(y)$$

$$c_{nml}(y) = -\frac{1}{\varepsilon_0}\frac{\phi^*_{nml}(y)}{\lambda_{nml}}$$

$$G(\underline{x},y) = -\frac{1}{\varepsilon_0}\sum_{nml}\frac{\phi^*_{nml}(y)\,\phi_{nml}(x)}{\lambda_{nml}}$$

Példa $\qquad \Delta\phi = \lambda\phi$



$$\phi(x,y,z) = \alpha(x)\cdot\beta(y)\cdot\gamma(z)$$

$$\Delta\phi = \alpha''\beta\gamma + \alpha\beta''\gamma + \alpha\beta\gamma'' = \lambda\alpha\beta\gamma$$

$$\underbrace{\frac{\alpha''}{\alpha}}_{-k_\alpha^2} + \underbrace{\frac{\beta''}{\beta}}_{-k_\beta^2} + \underbrace{\frac{\gamma''}{\gamma}}_{-k_\gamma^2} = \lambda \qquad \alpha'' = -k_\alpha^2\alpha$$

$$\alpha(x) = A\cos k_\alpha x + B\sin k_\alpha x \qquad n_\alpha \in \mathbb{Z}^+$$

$$\alpha(0) = 0 \Rightarrow A = 0 \qquad \Downarrow$$

$$\alpha(x) = a \Rightarrow \sin(k_\alpha a) = 0 \qquad k_\alpha a = n_\alpha\pi$$

$$k_\alpha = \frac{n_\alpha\pi}{a} \qquad k_\beta = \frac{n_\beta\pi}{b} \qquad k_\gamma = \frac{n_\gamma\pi}{c}$$

$$\lambda = -\frac{n_\alpha^2\pi^2}{a^2} - \frac{n_\beta^2\pi^2}{b^2} - \frac{n_\gamma^2\pi^2}{c^2}$$

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$$\phi_{n,m,l}(x,y,z) = \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)\sin\left(\frac{l\pi}{c}z\right)$$

$n =: n_\alpha$

$m =: n_\beta$

$l =: n_\gamma$

$$G(\underline{r}, \underline{R}) = \frac{-1}{\varepsilon_0}\sum_{n,m,l}\sin\left(\frac{n\pi}{a}X\right)\sin\left(\frac{m\pi}{b}Y\right)\sin\left(\frac{l\pi}{b}Z\right)\cdot$$

$$\cdot\sin\left(\frac{n\pi}{a}x\right)\cdot\sin\left(\frac{m\pi}{b}y\right)\sin\left(\frac{l\pi}{c}z\right)\cdot\frac{1}{-\left(\frac{n\pi}{a}\right)^2-\left(\frac{m\pi}{b}\right)^2-\left(\frac{l\pi}{c}\right)^2}$$

Normálás hiányzik

$\langle\phi_{nml}|\phi_{n'm'l'}\rangle \neq \delta_{nn'}\delta_{mm'}\delta_{ll'}$ egyelőre

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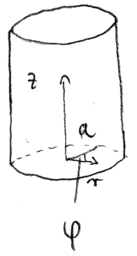
Henger



$\Delta\phi = \lambda\phi$ egyenlet hengerben

$$\Delta\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\phi = F(r)\,H(\varphi)\,S(z)$$

$$\Delta\phi = F''HS + \frac{1}{r}F'HS + \frac{1}{r^2}FH''S + FHS'' = \lambda FHS$$

$$\frac{F''}{F} + \frac{1}{r}\frac{F'}{F} + \frac{1}{r^2}\frac{H''}{H} + \frac{S''}{S} = \lambda$$

$\frac{S''}{S} := -q^2 \Rightarrow S(z) = A\cos qz + B\sin qz$

$S(0) = 0 \Rightarrow A = 0$

$S(h) = 0 \Rightarrow S(z) = B\sin\left(\frac{n\pi}{h}z\right)$

$q = \frac{n\pi}{h}$

$$r^2\frac{F''}{F} + r\frac{F'}{F} + \frac{H''}{H} = (\lambda + q^2)r^2$$

$\frac{H''}{H} := -m^2$

$$r^2\left(\frac{F''}{F} + \frac{1}{r}\frac{F'}{F}\right) - (\lambda + q^2)r^2 = m^2$$

$H(\varphi) = C\cos(m\varphi) + D\sin(m\varphi)$

$H(\varphi) = e^{im\varphi}$

$$F''(r) + \frac{1}{r}F'(r) + \left[-(\lambda+q^2) - \frac{m^2}{r^2}\right]F(r) = 0$$

Bessel, $\lambda + q^2 := -l^2$, $u := lr$

$$B(u) := F(r) \Rightarrow \frac{dF}{dr} = \frac{dB}{du}\frac{du}{dr} = l\,B'(u)$$

$$\Rightarrow \frac{d^2F}{dr^2} = l^2 B''(u)$$

$$B''(u) + \frac{1}{u}B'(u) + \left(1 - \frac{m^2}{u^2}\right)B(u) = 0$$

$\xi_{ml} :=$ m-edik J bessel l-edik zérus helye

Megoldásaik: $J \sim u^{|m|}$ kicsire

$N \sim \frac{1}{u^{|m|+1}}$ kicsire

$$\phi(a) \overset{!}{=} 0 \Rightarrow R(ka) \overset{!}{=} 0 \Rightarrow J_m(ka) \overset{!}{=} 0 \Rightarrow ka \overset{!}{=} \xi_{ml} \Rightarrow k_{ml} = \frac{\xi_{ml}}{a}$$

$$\lambda = -k_{ml}^2 - q_n^2 = -\frac{\xi_{ml}^2}{a^2} + \frac{n^2\pi^2}{h^2}$$

$$\phi_{mln}(r,\varphi,z) = J_m\left(\frac{\xi_{ml}}{a} r\right) e^{im\varphi} \sin\left(\frac{n\pi}{h} z\right) \cdot N$$

normálás

$$G(r,R) = \left[\frac{-1}{\varepsilon_0} \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_m^*\left(\frac{\xi_{ml}}{a} R\right) e^{-im\alpha} \sin\left(\frac{n\pi}{h} z\right) J_m\left(\frac{\xi_{ml}}{a} r\right) e^{im\varphi} \sin\left(\frac{n\pi}{h} z\right)}{-\left[\frac{\xi_{ml}^2}{a^2} + \frac{n^2\pi^2}{h^2}\right] J_{m+1}^2(\xi_{ml})} \right] \cdot \frac{2}{\pi a h}$$

normálás

Normálás:

$$1 = \int_V d^3r\, |\phi|^2 = N^2 \int_0^h dz \int_0^{2\pi} d\varphi \int_0^a dr\, r\, J_m^2\left(\frac{\xi_{ml}}{a} r\right) \left|e^{im\varphi}\right| \sin^2\left(\frac{n\pi}{h} z\right) =$$

$$= 2\pi N^2 \left(\int_0^h dz \sin^2\left(\frac{n\pi}{h} z\right) \right) \underbrace{\int_0^a dr\, r\, J_m^2\left(\frac{\xi_{ml}}{a} r\right)}_{\frac{a}{2} J_{m+1}^2(\xi_{ml})} = \frac{2\pi h}{2} \frac{a}{2} J_{m+1}^2(\xi_{ml})\, N^2 = 1$$

$$N = \sqrt{\frac{2}{\pi h a}} \frac{1}{J_{m+1}(\xi_{ml})}$$

$\Delta\phi = \lambda\phi$ -t akarom megoldani gömbre

$$\Delta\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\vartheta^2} + \frac{1}{r^2}\operatorname{ctg}\vartheta \frac{\partial\phi}{\partial\vartheta} + \frac{1}{r^2} \cdot \frac{1}{\sin^2\vartheta}\frac{\partial^2\phi}{\partial\varphi^2}$$

$$\Delta^{(r)} = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

$$\Delta^{(\vartheta,\varphi)} = \frac{\partial^2}{\partial\vartheta^2} + \operatorname{ctg}\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2} \qquad \phi = F(r)\cdot H(\vartheta,\varphi)$$

$$\Delta\phi = \left(\Delta^{(r)} F(r)\right) H(\vartheta,\varphi) + \frac{1}{r^2} F(r)\left(\Delta^{(\vartheta,\varphi)} H(\vartheta,\varphi)\right) = \lambda F H$$

$$r^2 \frac{\Delta^{(r)} F}{F} + \frac{\Delta^{(\vartheta,\varphi)} H}{H} = \lambda r^2$$

$$r^2\left(\frac{\Delta^{(r)} F}{F} - \lambda\right) = -\frac{\Delta^{(\vartheta,\varphi)} H}{H} = C$$

$$F''(r) + \frac{2}{r}F'(r) + \left(-\lambda - \frac{C}{r^2}\right)F = 0 \qquad F(r) := \frac{1}{r} f(r)$$ Így ez Besselre vezet

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$$\frac{\partial^2 H}{\partial \vartheta^2} + \mathrm{ctg}\,\vartheta \frac{\partial H}{\partial \vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial \varphi^2} H = -CH$$

$$H(\vartheta,\varphi) = B(\vartheta)\cdot e^{im\varphi} \rightarrow \frac{d^2}{d\vartheta^2}B(\vartheta) + \mathrm{ctg}\,\vartheta\frac{dB}{d\vartheta} - \frac{m^2}{\sin^2\vartheta}B = -C B(\vartheta)$$

$$\frac{\partial^2 H}{\partial \varphi^2} H = -m^2 H$$

Ennek megoldása $B(\vartheta) = N \sin^{|m|}\vartheta \; \underbrace{P_{lm}(\cos\vartheta)}_{l-|m| \text{ zérushely}}$

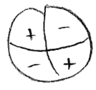
$l=0 \quad Y_{00} = \frac{1}{\sqrt{4\pi}}$

$l=1 \quad Y_{11}$ 

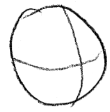
Y_{10} 1db zérus az egyenlítőn 

Y_{1-1} 

$l=2 \quad Y_{22}$ 

Y_{21} 

Y_{20} 

Y_{2-1} 

Y_{2-2} 

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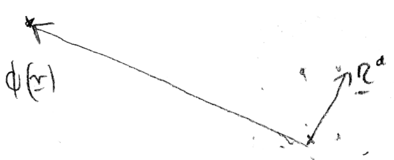
Multipól sorfejtés

Egyrészt: $4\pi\varepsilon_0 \phi(\underline{r}) = \sum_a \frac{q_a}{|\underline{r} - \underline{R}^a|} = \sum_a q_a \left[\left(\frac{1}{r} + \frac{\partial}{\partial x_k}\left(\frac{1}{|\underline{r}-\underline{R}|}\right)\right)\Big|_{R=0} (-R^a_k) + \frac{1}{2}\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_l}\frac{1}{r} + \right.$



$$= \frac{1}{r}\sum_a q_a + \left(-\partial_k \frac{1}{r}\right)\sum_a q_a R^a_k + \frac{1}{2}\partial_k \partial_l \frac{1}{r}\sum_a q_a R^a_k R^a_l +$$

$$+ \frac{1}{6}\left(\partial_k \partial_l \partial_m \frac{1}{r}\right)\left(\sum_a q_a R^a_k R^a_l R^a_m\right) + \dots$$

$$-\partial_a \frac{1}{r} = \cdot\frac{1}{r^2}\partial_a r = \frac{e_a}{r^2}$$

$$\partial_a \partial_\ell \frac{1}{r} = -\partial_\ell \frac{e_a}{r^2} = -\frac{\partial_\ell e_a}{r^2} - e_a \partial_\ell \frac{1}{r^2} = \frac{-\delta_{\ell a} - e_\ell e_a}{r^3} - e_a\left(-\frac{2}{r^3}\right)\partial_\ell r = -\frac{\delta_{\ell a}}{r^3} + \frac{e_\ell e_a}{r^3} + \frac{2 e_a e_\ell}{r^3} =$$

$$= \frac{3 e_a e_\ell - \delta_{a\ell}}{r^3}$$

$$4\pi\varepsilon_0\, \phi^{\text{kvadr.}}(r) = \frac{1}{3\cdot 2}\,\frac{3 e_a e_\ell - \delta_{a\ell}}{r^3}\,\overbrace{\sum_a q_a\left(3 R_a^a R_\ell^a - \delta_{a\ell}\left(R^a\right)^2\right)}^{Q_{a\ell}}$$

$$Q_{a\ell} = Q_{\ell a} \qquad \frac{3 e_a e_\ell - \delta_{a\ell}}{r^3}\,\delta_{a\ell} = \frac{3 e_a e_a - \delta_{aa}}{r^3} = 0$$

$$Sp\,Q = 0$$

Ez volt 2 indexre. Állítás: a többi indexre is úgy kell csinálni, hogy Spúrtalan legyen.

Ha $f(cx_1, cx_2 \dots cx_a) = c^h f(x_1, x_2 \dots x_a)$ akkor f egy h-adfokú homogén függvény

Némileg részletesen a kvadrupólt!

$$Q_{11} = \sum_a q_a\left(3XX - R^2\right) = \sum_a q_a\left(2X^2 - Y^2 - Z^2\right)$$

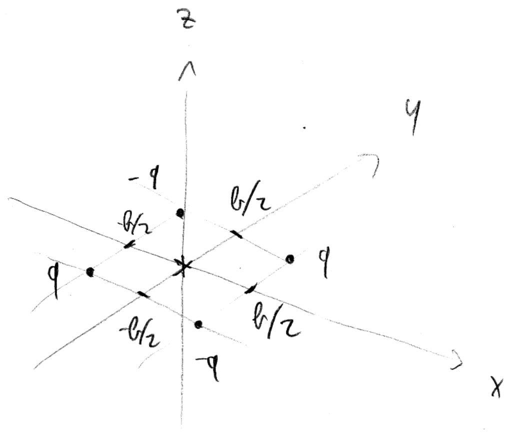
$$Q_{22} = \sum_a q_a\left(2Y^2 - X^2 - Z^2\right)$$

$$Q_{33} = \sum_a q_a\left(2Z^2 - Y^2 - X^2\right)$$

$$Q_{12} = Q_{21} = 3\sum_a q_a XY$$

$$Q_{13} = Q_{31} = 3\sum_a q_a XZ$$

$$Q_{23} = Q_{32} = 3\sum_a q_a ZY$$



$$Q_{11} = q\left(2\left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 - 0^2\right) - q\left(2\left(-\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 - 0^2\right) + q\left(2\left(-\frac{b}{2}\right) - \left(-\frac{b}{2}\right)^2 - 0\right) - q\left(2\frac{b}{2}\right)^2 - \left(-\frac{b}{2}\right)^2 - 0^2\Big)$$

$$Q_{11} = 0 = Q_{22} = Q_{33} \qquad Q_{13} = 0 \quad Q_{23} = 0$$

$$Q_{12} = 3\left(q\frac{b}{2}\frac{b}{2} - q\left(\frac{-b}{2}\right)\left(\frac{b}{2}\right) + q\left(\frac{-b}{2}\right)\left(\frac{-b}{2}\right) - q\frac{b}{2}\left(-\frac{b}{2}\right)\right) = 3qb^2$$

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z

$+q \quad (0,0,b)$

$-2q \quad (0,0,0)$

$+q \quad (0,0,-b)$

$$Q_{11} = q(-b^2) - 2q\,0 + q(-(-b)^2) = -2q b^2$$

$$Q_{22} = -2q b^2$$

$$Q_{33} = 4 q b^2$$

$$\underline{\underline{Q}} = 2qb^2 \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Vizsgáljuk meg az elektromos teret!

$$4\pi\varepsilon_0 \phi(r) = \frac{1}{6} \frac{3 e_k e_l - \delta_{kl}}{r^3} Q_{kl} = 2qb^2 \frac{1}{6} \frac{1}{r^3} \Big[-1(3e_1e_1 - \delta_{11}) + (-1)(3e_2e_2 - \delta_{22}) + $$
$$+ 2(3e_{33} - \delta_{33}) \Big] =$$

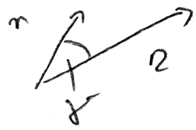
$$= \frac{qb^2}{3r^3}\Big(-3e_1e_1 + 1 - 3e_2e_2 + 1 + 6e_3e_3 - 2\Big] = \frac{qb^2}{r^3}\Big(-e_1e_1 - e_2e_2 + 2e_3e_3\Big) = \frac{qb^2}{r^3}\Big(3e_3^2 - 1\Big) =$$

$$= \frac{qb^2}{r^3}\Big(3\cos^2\vartheta - 1\Big) = qb^2\left(\frac{3z^2}{r^5} - \frac{1}{r^3}\right)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \underline{\underline{Q}}' \quad \text{milyen? HF}$$

Ekkor $4\pi\varepsilon_0 \phi(\underline{r}) = \frac{1}{6} \frac{3e_k e_l - \delta_{kl}}{r^3} Q_{kl} = \ldots = 3qb^2 \frac{\sin^2\vartheta \cos\varphi \sin\varphi}{r^3}$

Szögfüggvények sorfejtés



$$\frac{1}{|\underline{r} - \underline{R}|} = \frac{1}{\sqrt{(\underline{r} - \underline{R})^2}} = \frac{1}{\sqrt{r^2 + R^2 - 2\underline{r}\underline{R}}} = \frac{1}{\sqrt{r^2 + R^2 - 2rR\cos\gamma}} =$$

$$= \frac{1}{r} \frac{1}{\sqrt{1 + \frac{R^2}{r^2} - 2\frac{R}{r}\cos\gamma}} = \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^l P_l(\cos\gamma) = \sum_{l=0}^{\infty} \frac{R^l}{r^{l+1}} P_l(\cos\gamma)$$

$$P_l(\cos\gamma) = \sum_m^{\text{véges}} f(\vartheta,\varphi)\, g(\Theta,\phi) = \frac{4\pi}{2l+1}\sum_{m=-l}^{l} Y_{lm}(\vartheta,\varphi)\, Y_{lm}^*(\Theta,\phi)$$

$$\underline{r} = r\begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix} \qquad \underline{R} = R\begin{pmatrix} \sin\Theta\cos\phi \\ \sin\Theta\sin\phi \\ \cos\Theta \end{pmatrix}$$

$$\frac{1}{|\underline{r}-\underline{R}|} = \sum_{\ell=0}^{\infty} \frac{R^\ell}{r^{\ell+1}} \frac{4\pi}{2\ell+1} \sum_m Y_{\ell m}(\vartheta,\varphi)\, Y^*_{\ell m}(\Theta,\phi)$$

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$$4\pi\varepsilon_0 \phi(\underline{r}) = \sum_a q_a \sum_{\ell=0}^{\infty} \frac{R_a^\ell}{r^{\ell+1}} \frac{4\pi}{2\ell+1} \sum_m Y_{\ell m}(\vartheta,\varphi)\, Y_{\ell m}^*(\Theta,\phi) =$$

$$= \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(\frac{Y_{\ell m}(\vartheta,\varphi)}{r^{\ell+1}} \sqrt{\frac{4\pi}{2\ell+1}} \right) \left(\sum_a q_a R_a^\ell Y^*_{\ell m}(\vartheta_a, \varphi_a) \sqrt{\frac{4\pi}{2\ell+1}} \right)$$

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Elektrontakiba 2D-ben

vezető drót, töltött tere

$$\operatorname{div} \underline{E} = \frac{1}{\varepsilon} \varrho$$

$$\oint \underline{E}\, d\underline{F} = \int_V \frac{1}{\varepsilon} \varrho \, dV$$

$$\int_{\text{palást}} \underline{E}\, d\underline{F} = \int_{\text{palást}} E\, dA = \int_{\text{palást}} \overset{\text{áll}}{E(r)}\, dA = E(r)\, 2\pi R L = \frac{1}{\varepsilon} q L$$

$$E(r) = \frac{\varrho L}{2\pi r L \varepsilon_0} = \frac{\varrho}{2\pi r \varepsilon_0}$$

$$\phi(r) = \frac{-Q}{4\pi\varepsilon_0} \ln r$$

ϱ vonalmenti töltéssűrűség

$w = f(z) \qquad z = x + iy$

$$w = \psi + i\phi = \psi(x,y) + i\phi(x,y)$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\phi}{\partial y} \qquad \frac{\partial\psi}{\partial y} = -\frac{\partial\phi}{\partial x} \qquad \text{C.R.}$$

$$\Delta\psi = 0$$

$$\Delta\phi = 0$$

a ψ = áll és ϕ = áll vonalak merőlegesek,

$$\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right) \begin{pmatrix} \frac{\partial\psi}{\partial x} \\ \frac{\partial\psi}{\partial y} \end{pmatrix} = \frac{\partial\phi}{\partial x} \underbrace{\frac{\partial\psi}{\partial x}}_{=\frac{\partial\phi}{\partial y}} + \frac{\partial\phi}{\partial y} \underbrace{\frac{\partial\psi}{\partial y}}_{=-\frac{\partial\phi}{\partial x}} = 0$$

elveszett hurok

ezt tudjuk az erővonalakat

$$\Delta Q = \sigma \Delta r L = \varepsilon_0 |\underline{E}| |\Delta \underline{r}| L = \varepsilon_0 |\underline{E} \times \overset{\text{mert merőlegesek}}{\Delta \underline{r}}| L = \varepsilon_0 L |E_x \Delta y - E_y \Delta x| = \varepsilon_0 L \left| -\frac{\partial\phi}{\partial x}\Delta y + \frac{\partial\phi}{\partial y}\Delta x \right| =$$

$$= \varepsilon_0 L\, \underline{\nabla}\psi\, \Delta\underline{r} = \varepsilon_0 L\, \Delta\psi$$

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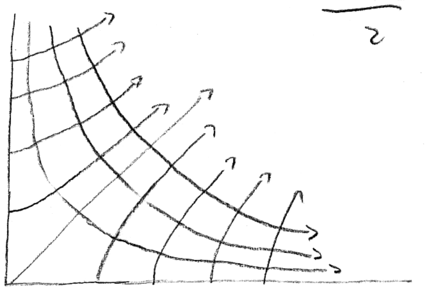
pl. $w = z^2 = (x + y\cdot i)^2 = \underbrace{(x^2 - y^2)}_{\psi} + i\underbrace{(2xy)}_{\phi}$

$\phi = \text{del} = 2xy$

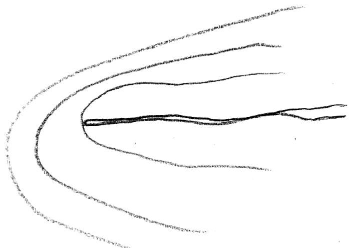
$\psi = \text{del} = x^2 - y^2$



$w = \sqrt{z} = \sqrt{r e^{i\varphi}} = \sqrt{r}\, e^{i\frac{\varphi}{2}} = \sqrt{r}\Big(\underbrace{\cos\frac{\varphi}{2}}_{\psi} + i \underbrace{\sin\frac{\varphi}{2}}_{\phi}\Big)$ $\qquad \psi = \sqrt{r}\cos\frac{\varphi}{2}$

$\phi = \sqrt{r}\sin\frac{\varphi}{2} = \sqrt{r}\sqrt{\frac{1-\cos\varphi}{2}} = \sqrt{r}\sqrt{\frac{1 - x/r}{2}} = \sqrt{\frac{r-x}{2}} = \sqrt{\frac{\sqrt{x^2+y^2} - x}{2}}$

$\phi =$ const vanalak: $\quad c^2 = \frac{\sqrt{x^2+y^2} - x}{2}$

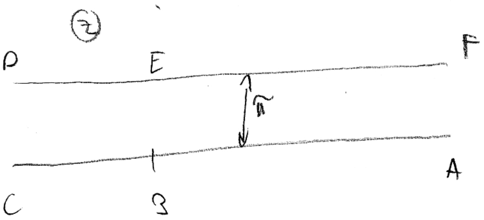


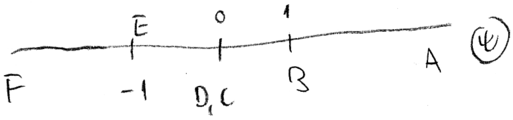
$x^2 + y^2 = 4c^4 + x^2 + 4c^2 x$

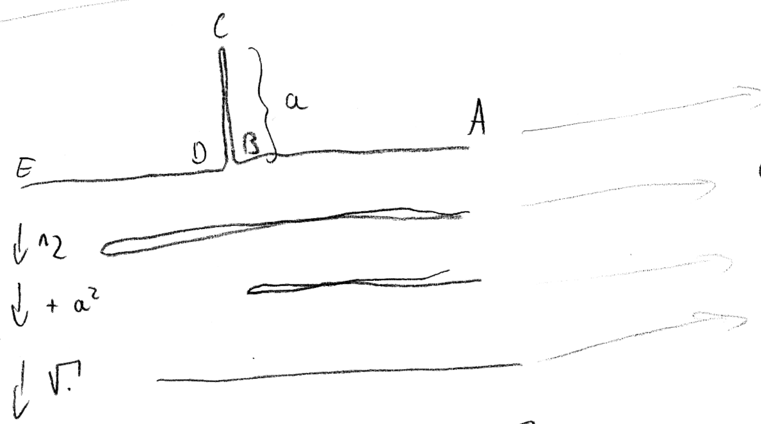
$y^2 = 4c^2(x + c^2)$

$w = e^z = e^{x+iy} = e^x e^{iy} = e^x(\cos y + i \sin y)$ $\qquad \psi = e^x \cos y$

$\phi = e^x \sin y$







A	B	C	D	E
∞	$0+\varepsilon$	ia	$0-\varepsilon$	$-\infty$
$\infty + i\varepsilon$	$0 + i\varepsilon$	$-a^2$	$0 - i\varepsilon$	$\infty - i\varepsilon$
$\infty + i\varepsilon$	$a^2 + i\varepsilon$	0	$a^2 - i\varepsilon$	$\infty - i\varepsilon$
∞	a	0	$-a$	$-\infty$

$w = \sqrt{z^2 + a^2} = \sqrt{(x+iy)^2 + a^2} = \sqrt{x^2 - y^2 + 2ixy + a^2} = \psi + i\phi$

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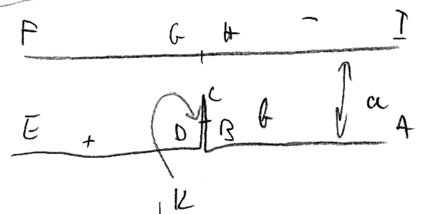
$$\psi^2 - \phi^2 + 2i\phi\psi = x^2 - y^2 + a^2 + 2ixy$$

$$\left.\begin{aligned} \psi^2 - \phi^2 &= x^2 - y^2 + a^2 \\ 2\phi\psi &= 2xy \end{aligned}\right\} \quad \psi = \frac{xy}{\phi}$$

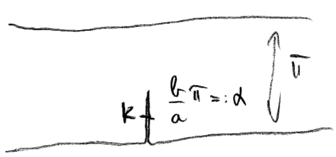
$$\frac{x^2y^2}{\phi^2} - \phi^2 = x^2 - y^2 + a^2$$



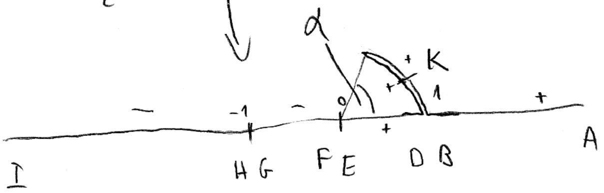
Lineáris tört függvény

$$\psi = \frac{az+b}{cz+d}$$

$$w_1 = z\frac{\pi}{a}$$



$$w_2 = e^{w_1}$$

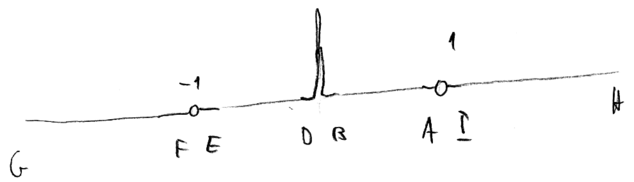


nevezhetem: $-1 \to \infty$

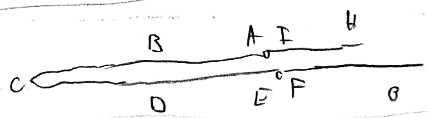
$1 \to 0$

$$w_3 = \frac{w_2 - 1}{w_2 + 1}$$

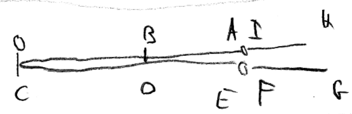
K hova kerül $\frac{e^{i\beta}-1}{e^{i\beta}+1} = \frac{\cos\beta + i\sin\beta - 1}{\cos\beta + i\sin\beta + 1} = \frac{-2\sin^2\frac{\beta}{2} + i\sin\beta}{2\cos^2\frac{\beta}{2} + i\sin\beta} = \dots = i\,\mathrm{tg}\frac{\beta}{2}$



$\downarrow\ {}^2$



$+\mathrm{tg}^2\frac{\alpha}{2}$ $\downarrow$



$\downarrow \sqrt{\ }$

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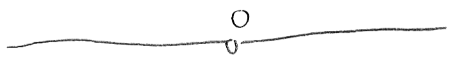
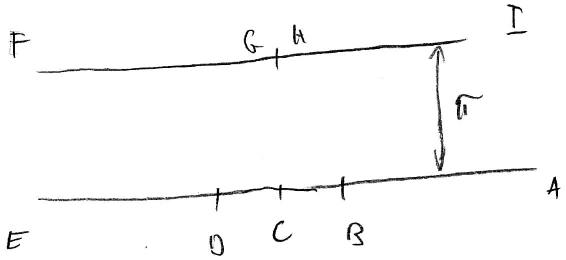
$\cdot \cos\frac{\alpha}{2}$

$\frac{1+w_7}{1-w_7} = w_8$



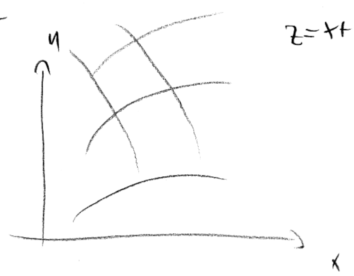
ápr 17 ZH

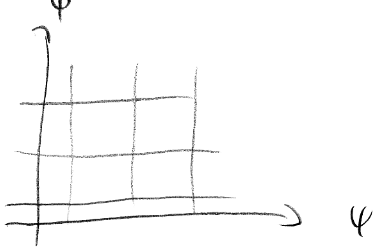
$\ln w_8$



$$\ln \left[\frac{\sqrt{\left(\frac{e^{2\frac{\pi}{a}}-1}{e^{2\frac{\pi}{a}}+1}\right)^2 + \mathrm{tg}^2\frac{\alpha}{2}} \cdot \cos\frac{\alpha}{2} + 1}{\sqrt{\left(\frac{e^{2\frac{\pi}{a}}-1}{e^{2\frac{\pi}{a}}+1}\right)^2 + \mathrm{tg}^2\frac{\alpha}{2}} \cdot \cos\frac{\alpha}{2} - 1} \right]$$

$z = x + iy \xrightarrow{f(z) = w = \psi + i\phi}$





03.19.

$Q \sim \psi_2 - \psi_1$

$w'(z) = f'(z) = -E_y - iE_x$ nem biztos hogy a ϕ-nek jobb a gradiensét venni

$E_n = \sqrt{E_x^2 + E_y^2}$

$\sigma = \varepsilon_0 \cdot E_n$

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$\cdot\, Q \qquad \phi = \frac{-Q}{2\pi\varepsilon_0} \ln r + C$

C-t megválaszthatom úgy, hogy

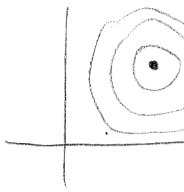
$$\phi = \frac{-Q}{2\pi\varepsilon_0} \ln \frac{r}{R}$$

↑ ekkor r=R-ben lesz a pot 0

komplexesen

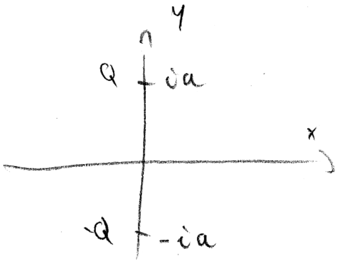
$$\ln z = \ln r e^{i\varphi} = \ln r + i\varphi$$

$w = -i \frac{Q}{2\pi\varepsilon_0} \ln z$ és ha nem origóban van?



$$w = -i \frac{Q}{2\pi\varepsilon_0} \ln(z - z_0)$$

ha több töltés van $\quad w = \frac{-i}{2\pi\varepsilon_0} \sum_n Q_n \ln(z - z_n)$



ekkor

$$w = \frac{-i}{2\pi\varepsilon_0}\left[Q \ln(z - ia) + (-Q) \ln(z + ia)\right] = \frac{-iQ}{2\pi\varepsilon_0} \ln \frac{z - ia}{z + ia}$$

És a térerősség? $\quad w' = \frac{-iQ}{2\pi\varepsilon_0}\left(\frac{1}{z - ia} - \frac{1}{z + ia}\right) = \frac{-iQ}{2\pi\varepsilon_0} \frac{2ia}{z^2 + a^2} =$

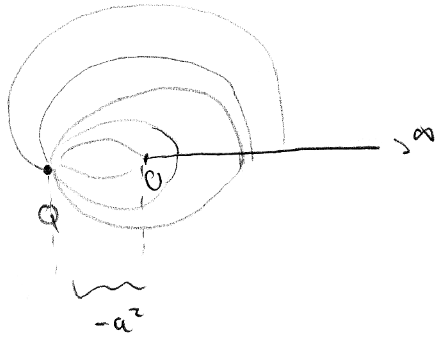
$$= \frac{Qa}{\pi\varepsilon_0} \frac{1}{(x + iy)^2 + a^2} = \frac{Qa}{\pi\varepsilon_0} \frac{1}{(x^2 - y^2 + a^2) + i\,2xy} =$$

$$= \frac{Qa}{\pi\varepsilon_0} \frac{(x^2 - y^2 + a^2) - i\,2xy}{(x^2 - y^2 + a^2)^2 + 4x^2y^2} = -E_y - iE_x \quad \text{ha } y = 0$$

$E_x = 0$ ✓

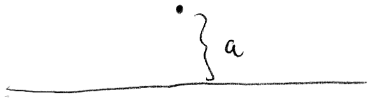
$$E_y = -\frac{Qa}{\pi\varepsilon_0} \frac{1}{x^2 + a^2}$$

$$\sigma = \varepsilon_0 \cdot E_y = -\frac{Q \cdot a}{\pi} \frac{1}{x^2 + a^2}$$



$\sqrt{z}$

$\longrightarrow$

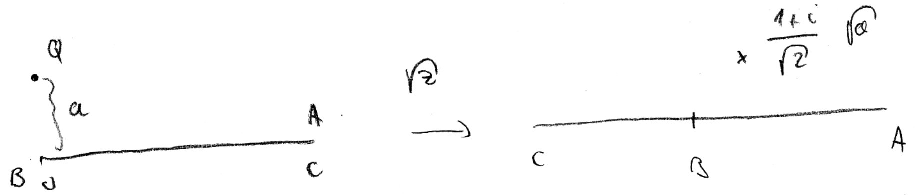


$$w = -\frac{Qi}{2\pi\varepsilon_0} \ln \frac{w_1 - ia}{w_1 + ia} =$$

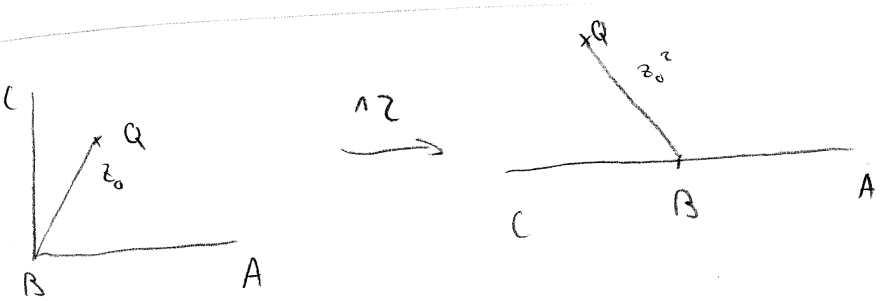
$$= -\frac{Qi}{2\pi\varepsilon_0} \ln \frac{\sqrt{z} - ia}{\sqrt{z} + ia}$$

$$w' = -\frac{Qi}{2\pi\varepsilon_0}\left(\frac{1}{\sqrt{z} - ia} \frac{1}{2\sqrt{z}} - \frac{1}{\sqrt{z} + a} \frac{1}{2\sqrt{z}}\right) = -\frac{Qi}{2\pi\varepsilon_0} \frac{1}{2\sqrt{z}} \frac{2ia}{z + a^2}$$

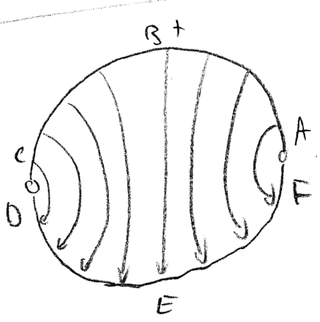
megoldom ua



$$\psi = \frac{-iQ}{2\pi\varepsilon_0} \ln \frac{\sqrt{z} - \sqrt{a}\,\frac{1+i}{\sqrt{2}}}{\sqrt{z} + \sqrt{a}\,\frac{1-i}{\sqrt{2}}}$$



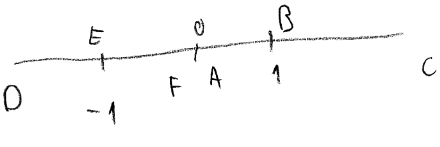
$$\psi(z) = \frac{-Q}{2\pi\varepsilon_0} \ln \frac{z^2 - z_0^2}{z^2 - z_0^{2*}}$$

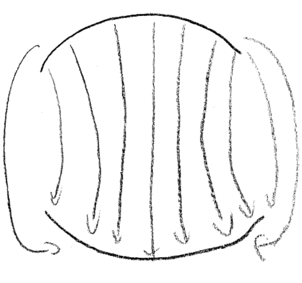


	A	B	C	D	E	F
z	$R+i\varepsilon$	iR	$-R+i\varepsilon$	$-R-i\varepsilon$	$-iR$	$R-i\varepsilon$
ψ_1	0	1	∞	∞	-1	0

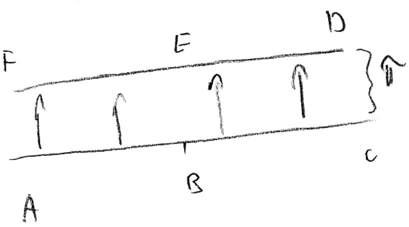
$$\psi_1 = i\,\frac{R-z}{R+z}$$

$$\psi_1(B) = i\,\frac{R-iR}{R+iR} = i\,\frac{1-i}{1+i} = i\,\frac{(1-i)^2}{(1+i)(1-i)} = i\,\frac{1-2i+i^2}{1^2+1} = 1$$





$$\psi_2 = \ln \psi_1$$



$\psi_{\text{végső}} = \psi_3 = \frac{\mu_0}{\pi}\psi_2$ így a két lemez között μ_0 fen van $\psi = \frac{\mu_0}{\pi} \ln\left(i\,\frac{R-z}{R+z}\right)$

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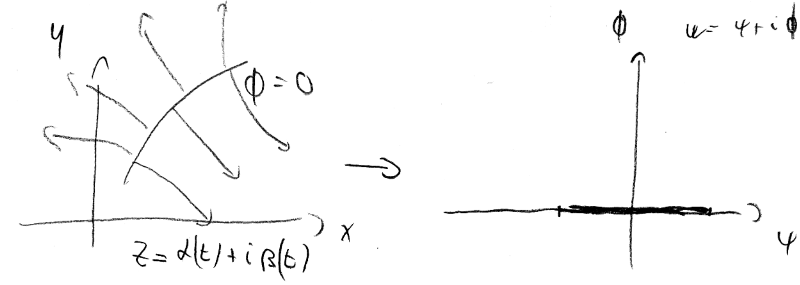
Paraméteres elektróda

$x = \alpha(t)$ $\quad t \in [t_1, t_2]$

$y = \beta(t)$



$z = \alpha(t) + i\beta(t)$

$\downarrow$

$z = \alpha(hw) + i\beta(hw) = \alpha(h\psi + ih\phi) + i\beta(h\psi + ih\phi)$

$\phi = 0 \quad z = \alpha(\underbrace{h\psi}_{t = h\psi}) + i\beta(h\psi)$

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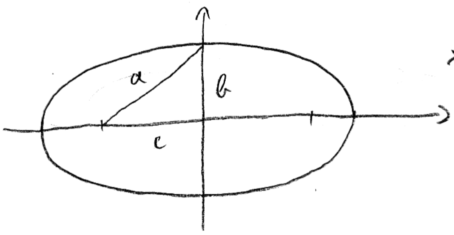
pl: 

$x(t) = a\cos(t)$

$y(t) = b\sin(t)$

$c^2 = a^2 - b^2$

$\varepsilon = \frac{c}{a}$

$a := c \cdot \mathrm{ch}\,\gamma$

$b := c \cdot \mathrm{sh}\,\gamma$

$\varepsilon = \frac{1}{\mathrm{ch}\,\gamma} < 1$

$z = a\cos t + i b \sin t$

$z(w) = a\cos(hw) + ib\sin(hw) = c(\mathrm{ch}\gamma \cos hw) + ic(\mathrm{sh}\gamma \sin hw) =$

$\cos ix = \mathrm{ch}\,x$

$\sin ix = i\,\mathrm{sh}\,x$

$\mathrm{ch}\,ix = \cos x$

$\mathrm{sh}\,ix = i\sin x$

$= c\left[\mathrm{ch}\gamma\, \mathrm{ch}(ihw) + \mathrm{sh}\gamma\, \mathrm{sh}(ihw)\right] =$

$= c\,\mathrm{ch}(\gamma + ihw) = z(w)$

$\gamma + ihw = \mathrm{arch}\,\frac{z}{c}$

$w(z) = \frac{\mathrm{arch}\,\frac{z}{c} - \gamma}{ih}$

$z = x + iy = c\,\mathrm{ch}(\gamma + ihw) = c\,\mathrm{ch}(\gamma + ih(\psi + i\phi)) = c\,\mathrm{ch}(\gamma + ih\psi - h\phi) = c\,\mathrm{ch}[(\gamma - h\phi) + ih\psi] =$

$= c\left[\mathrm{ch}(\gamma - h\phi)\,\mathrm{ch}(ih\psi) + \mathrm{sh}(\gamma - h\phi)\,\mathrm{sh}(ih\psi)\right] =$

$= c\left[\mathrm{ch}(\gamma - h\phi)\cos h\psi + i\,\mathrm{sh}(\gamma - h\phi)\sin(h\psi)\right] =)$

$x = c\,\mathrm{ch}(\gamma - h\phi)\cos h\psi$

$y = c\,\mathrm{sh}(\gamma - h\phi)\sin h\psi$

$A := c\,\mathrm{ch}(\gamma - h\phi_1)$

$B := c\,\mathrm{sh}(\gamma - h\phi_1)$

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5

$x = A\cos h\psi$ $\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 = 1$ $A^2 - B^2 = c^2$

$y = B\sin h\psi$

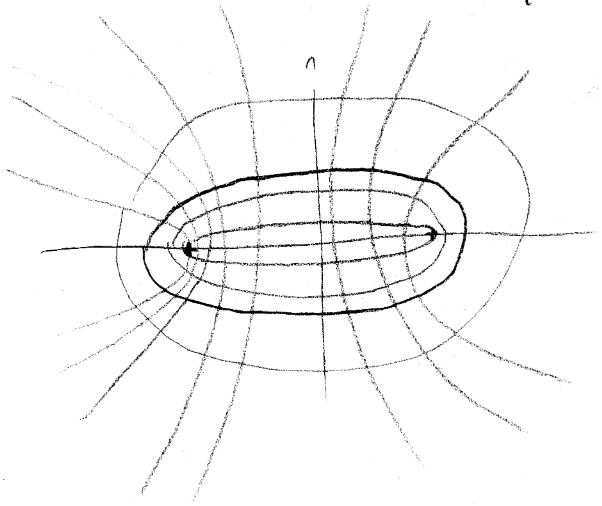
$\psi = \text{const}$ $A' := c\cos h\psi$ $x = A'\,\mathrm{ch}(\gamma - k\phi)$ $\left(\frac{x}{A'}\right)^2 - \left(\frac{y}{B'}\right)^2 = 1$

$B' := c\sin h\psi$ $y = B'\,\mathrm{sh}(\gamma - k\phi)$

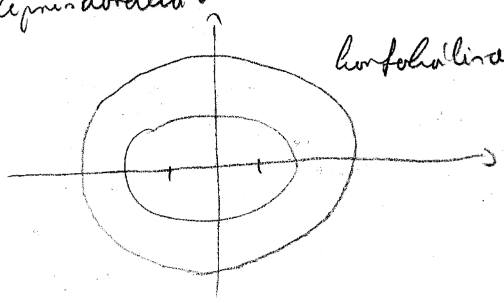
$A'^2 - B'^2 = c^2$

$Q = \varepsilon_0 \Delta\psi L$

$C = \frac{Q}{u} = \frac{\varepsilon_0 \Delta\psi L}{\Delta\phi}$



Elliptikus koordináták



konfokális

$\gamma_2: \ \phi = \phi_2$

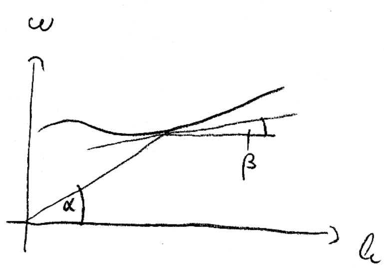
$\gamma_1: \ \phi = \phi_1$

03.25.

Hullámok

$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u(x,t) = 0$ 1D → probléma

$u = e^{ikx} e^{-i\omega t} = e^{ik\left(x - \frac{\omega}{k}t\right)}$



$\frac{\partial}{\partial t} u = (-i\omega) u$

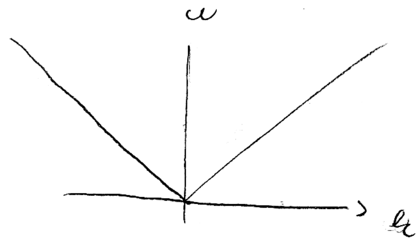
$\operatorname{tg}\alpha = \frac{\omega}{k} = v_f$

$\frac{\partial}{\partial x} u = (ik) u$

$\operatorname{tg}\beta = \frac{d\omega}{dk} = v_{cs}$

$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u(x,t) = L(-i\omega, ik)\, u = 0 \Rightarrow L(-i\omega, ik) = 0 \Rightarrow \omega(k)$

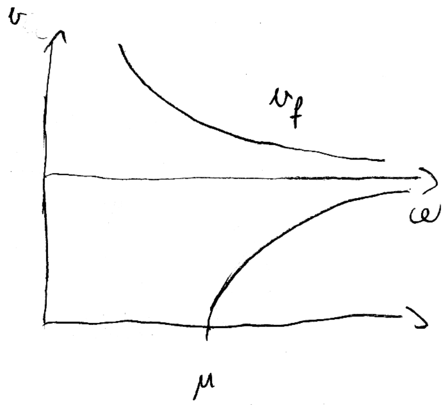
„A" – hullámegyenlet $\frac{\partial^2}{\partial t^2} u = c^2 \frac{\partial^2 u}{\partial x^2}$



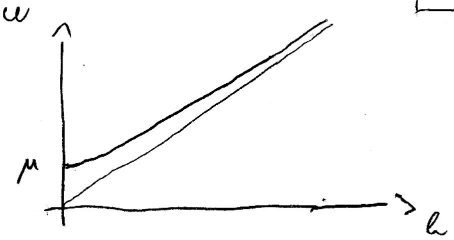
$$(-i\omega)^2 = c^2 (ik)^2$$

$$-\omega^2 = c^2(-k^2) \Rightarrow \omega = \pm ck \quad \text{ekkor} \quad v_f = v_g = c$$

Klein-gordon $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - \mu^2 u$



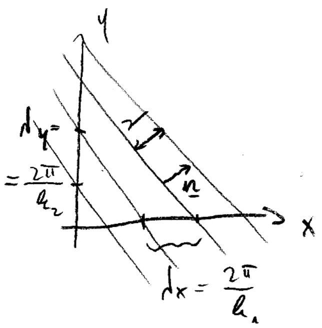
$$(-i\omega)^2 = c^2 (ik)^2 - \mu^2 \Rightarrow \omega^2 = c^2 k^2 + \mu^2$$



Több dimenzióban

$$L\left(\frac{\partial}{\partial t}, \nabla\right) u(\underline{r}, t) = 0 = L(-i\omega, i\underline{k}) \Rightarrow \omega(\underline{k})$$

megoldáskeresés $u(\underline{r}, t) = u_0 \cdot e^{i(\underline{k}\underline{r} - \omega t)}$



$$\underline{v}_g = \frac{\partial \omega(\underline{k})}{\partial \underline{k}} \qquad (\underline{v}_g)_\ell = \frac{\partial \omega(\underline{k})}{\partial k_\ell}$$

$$(\underline{v}_\omega)_\ell = \frac{\omega(\underline{k})}{k_\ell}$$

A rezgő mennyiség is lehet vektor $\qquad$ $\underline{u}_0$ = a polarizációvektor

$\underline{u}(\underline{r}, t) \qquad \underline{\underline{L}}\left(\frac{\partial}{\partial t}, \nabla\right) \underline{u}(\underline{r}, t) = 0 \qquad$ ekkor a megoldást így keressük: $\underline{u} = \underline{u}_0 \cdot e^{i(\underline{k}\underline{r} - \omega t)}$

$\underline{a}_\alpha(\underline{k})$

$$0 = \underline{\underline{L}}\, \underline{u} = \underline{\underline{L}}(-i\omega, i\underline{k})\, \underline{a}$$

$\det \underline{\underline{L}}(-i\omega, i\underline{k}) = f(\omega, \underline{k}) \overset{!}{=} 0$ Ekkor van nem triviális megoldás $\Rightarrow \omega_\alpha(\underline{k}) \quad \alpha \in (0, 1, \dots 3)$

Egy általános megoldás $\underline{a}_\alpha(\underline{k})\, e^{i\underline{k}\underline{r} - i\omega_\alpha(\underline{k})t}$ ezek szuperpozíciója

$$\underline{u}(\underline{r},t) = \int \sum_\alpha \underline{a}_\alpha(\underline{k})\, e^{i\underline{k}\underline{r} - i\omega_\alpha(\underline{k})t}\, d^3k$$

E és B együtt

$$\left.\begin{array}{ll} \operatorname{div} \underline{D} = 0 & \text{mert legyen } \rho = 0 \\ \operatorname{div} \underline{B} = 0 & \\ \operatorname{rot} \underline{H} = \dfrac{\partial D}{\partial t} & \text{legyen } \underline{j} = 0 \\ \operatorname{rot} \underline{E} = -\dfrac{\partial B}{\partial t} & \end{array}\right\}$$

$$\operatorname{rot} \underline{E} + \frac{\partial B}{\partial t} = 0$$

$$\operatorname{rot} B - \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = 0$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_z}{\partial t} = 0$$

$$\underline{F} := \begin{pmatrix} E_x \\ E_y \\ E_z \\ B_x \\ B_y \\ B_z \end{pmatrix}$$

$$\underline{\underline{L}} = \begin{array}{|c|c|c|c|c|c|} \hline 0 & -\partial_z & \partial_y & \partial_t & 0 & 0 \\ \hline \partial_z & 0 & -\partial_x & 0 & \partial_t & 0 \\ \hline -\partial_y & \partial_x & 0 & 0 & 0 & \partial_t \\ \hline -\frac{1}{c^2}\partial_t & 0 & 0 & 0 & -\partial_z & \partial_y \\ \hline 0 & -\frac{1}{c^2}\partial_t & 0 & \partial_z & 0 & -\partial_x \\ \hline 0 & 0 & -\frac{1}{c^2}\partial_t & -\partial_y & \partial_x & 0 \\ \hline \end{array}$$

$$\partial_t := \partial_0 \qquad \partial_x := \partial_1 \qquad \partial_y := \partial_2 \qquad \partial_z := \partial_3$$

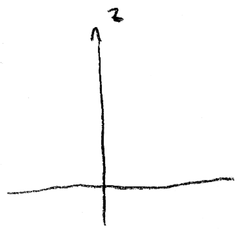
$c = 1$ mértékegység

$$\left(\gamma^0 \cdot \partial_0 + \gamma^1 \partial_1 + \gamma^2 \partial_2 + \gamma^3 \partial_3\right) \underline{F} = 0 \qquad \left(\sum_{k=0}^{3} \gamma^k \partial_k\right) \underline{F} = 0 \qquad \left(\underline{\underline{\gamma}}\, \partial\right) \underline{F} = 0$$

$$\gamma^0 = \left[\begin{array}{ccc|ccc} & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \\ \hline -1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \end{array}\right] \qquad \gamma^1 = \left[\begin{array}{ccc|ccc} & & & & & \\ & & -1 & & & \\ & 1 & & & & \\ \hline & & & & & \\ & & & & & -1 \\ & & & & 1 & \end{array}\right]$$

$$\gamma^2 = \left[\begin{array}{ccc|ccc} & & 1 & & & \\ & & & & & \\ -1 & & & & & \\ \hline & & & & & 1 \\ & & & & & \\ & & & -1 & & \end{array}\right] \qquad \gamma^3 = \left[\begin{array}{ccc|ccc} & -1 & & & & \\ 1 & & & & & \\ & & & & & \\ \hline & & & & -1 & \\ & & & 1 & & \\ & & & & & \end{array}\right]$$

hullám plazmában



$\underline{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot k$

$E_x = E_0 \cdot e^{i \overbrace{\underline{k}\underline{r}}^{k \cdot z}} e^{-i\omega t}$

$E_y = 0 \qquad \underline{E}_0 = \begin{pmatrix} E_0 \\ 0 \\ 0 \end{pmatrix}$

$E_z = 0$

$\operatorname{div} \underline{E} = 0$ ✓

$B_x = 0$

$B_y = B_0 \cdot e^{ikz} e^{-i\omega t}$

$B_z = 0$

$\operatorname{div} \underline{B} = 0$ ✓

$(\operatorname{rot} \underline{B})_x = -ik B_0 e^{ikz} e^{-i\omega t}$

$(\operatorname{rot} \underline{E})_z = (\operatorname{rot} \underline{E})_x = 0 \quad (\operatorname{rot} \underline{E})_y = ik E_0 e^{ikz} e^{-i\omega t}$

$\operatorname{rot} \underline{E} = -\frac{\partial \underline{B}}{\partial t}$

$\operatorname{rot} \underline{H} = \frac{\partial \underline{D}}{\partial t} + \underline{j} \qquad \underline{j} = -en \cdot \underline{v}$

$\underline{j} = -e \cdot n \cdot \underline{\dot{u}}$

elektron: $m \underline{\ddot{u}} = -e \cdot (\underline{E} + \underbrace{\underline{v} \times \underline{B}}_{\approx 0})$

$(\operatorname{rot} \underline{E})_y = -\left(\frac{\partial \underline{B}}{\partial t}\right)_y$

$ik E_0 = +i\omega B_0$

$(\operatorname{rot} \underline{H})_x = \left(\frac{\partial \underline{D}}{\partial t}\right)_x + (\underline{j})_x$

$\underline{u}(t) = \underline{R}\, e^{i\underline{k}\underline{r}} e^{-i\omega t}$

$\underline{\dot{u}}(t) = -i\omega \underline{R} e^{i\underline{k}\underline{r}} e^{-i\omega t}$

$\underline{\ddot{u}}(t) = (-i\omega)^2 \underline{R}\, e^{i\underline{k}\underline{r}} e^{-i\omega t} \qquad m\underline{\ddot{u}} = -e\underline{E}$

$-m\omega^2 \underline{R} e^{i\underline{k}\underline{r}} e^{-i\omega t} = -e \underline{E}_0 e^{i\omega t} e^{ikz}$

$m\omega^2 \underline{R} = e\underline{E}_0$

$\underline{R} = \frac{e}{m\omega^2} \underline{E}_0$

$\frac{1}{\mu_0}(-ik B_0) = -i\omega \varepsilon_0 E_0 + i \frac{e^2 n}{m\omega} E_0$

$\omega^2 = c^2 k^2 - \frac{c^2 e^2 n}{m} \mu_0$

$\omega^2 = c^2 k^2 - \frac{e^2 n}{m \varepsilon_0}$

$\underline{\dot{u}}(t) = -i\omega \frac{e}{m\omega^2} \underline{E}_0 \, e^{i\underline{k}\underline{r}} e^{-i\omega t} \qquad \underline{j} = -en\underline{\dot{u}}$

$\underline{j} = i \frac{e^2 n}{m\omega} \underline{E}_0 e^{i\underline{k}\underline{r}} e^{-i\omega t}$

$/ \cdot k\mu_0 \quad \frac{-\cancel{i}kB_0}{\mu_0} = -\cancel{i}\omega \varepsilon_0 \frac{\cancel{i}\omega B_0}{\cancel{i}k} + \cancel{i} \frac{e^2 n}{m\omega} \frac{\omega B_0}{k}$

$\operatorname{div} \underline{D} = \rho \qquad \varepsilon \underline{E} = \underline{D}$

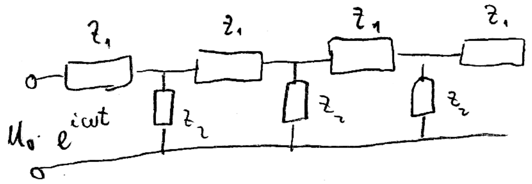
$\operatorname{div} \underline{B} = 0 \qquad \underline{B} = \mu \cdot \underline{H}$

$\operatorname{div} \underline{E} = -\frac{\partial \underline{B}}{\partial t}$

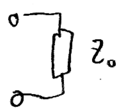
$\operatorname{rot} \underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t}$

$-k^2 = -\omega^2 \varepsilon_0 \mu_0 + \frac{e^2 n}{m} \mu_0$

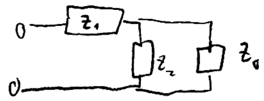
$\boxed{\omega^2 = k^2 c^2 - \frac{e^2 n}{m} \varepsilon_0}$



$\equiv$



$\overset{!}{=}$



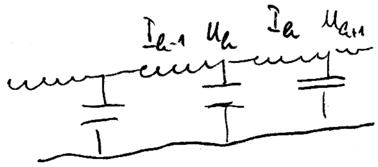
$$Z_1 + \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_0}} = Z_0$$

$$Z_1(Z_2+Z_0) + Z_2 Z_0 = Z_0(Z_2+Z_0)$$

$$Z_0^2 - Z_1 Z_0 - Z_1 Z_2 = 0$$

$$Z_0 = \frac{Z_1 \pm \sqrt{Z_1^2 + 4 Z_1 Z_2}}{2}$$

pl



$Z_1 = i\omega L$

$Z_2 = \frac{1}{i\omega C}$

$$Z_0 = \frac{i\omega L + \sqrt{(i\omega L)^2 + 4 i\omega L \frac{1}{i\omega C}}}{2} = \frac{i\omega L + L\sqrt{\frac{4}{LC} - \omega^2}}{2} \qquad \frac{4}{LC} := \Omega^2$$

$$Z_0 = \frac{L}{2}\left(i\omega + \sqrt{\Omega^2 - \omega^2}\right)$$

$$U_l = Z_2 (I_{l-1} - I_l)$$

$$U_{l+1} - U_l = -Z_1 \cdot I_l$$

$$\left.\begin{aligned} -I_l &= (U_{l+1} - U_l)/Z_1 \\ -I_{l-1} &= (U_l - U_{l-1})/Z_1 \end{aligned}\right\} \quad I_l - I_{l-1} = -\frac{2U_l - U_{l-1} - U_{l+1}}{Z_1}$$

$$-U_l = Z_2 \frac{2U_l - U_{l-1} - U_{l+1}}{Z_1} \qquad U_{l+1} + U_{l-1} = \frac{(2Z_2 + Z_1)}{Z_2} U_l$$

$$U_l = U_0 \cdot \alpha^l$$

$$U_0 \alpha^{l+1} + U_0 \alpha^{l-1} = \left(2 + \frac{Z_1}{Z_2}\right) U_0 \cdot \alpha^l$$

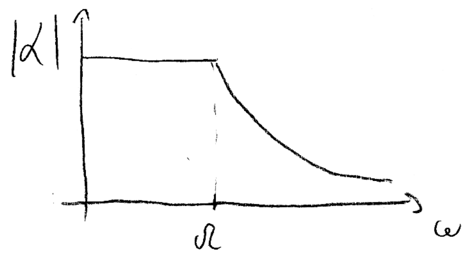
$$\alpha + \frac{1}{\alpha} = \left(2 + \frac{Z_1}{Z_2}\right) \qquad \alpha^2 - \left(2 + \frac{Z_1}{Z_2}\right)\alpha + 1 = 0 \qquad \alpha_{1,2} = \frac{2 + \frac{Z_1}{Z_2} \pm \sqrt{4 + 4\frac{Z_1}{Z_2} + \frac{Z_1^2}{Z_2^2} - 4}}{2}$$

$Z_1 =: i\omega L \qquad Z_2 =: \frac{1}{iLC\omega} \qquad LC =: \frac{4}{\Omega^2}$

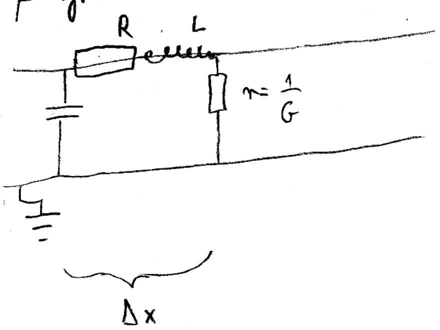
$$\frac{Z_1}{Z_2} = -\omega^2 LC$$

$$\alpha^2 - (2-\omega^2 LC)\alpha + 1 = 0$$

$$\alpha = 1 - \frac{2\omega^2}{\Omega^2} \pm \sqrt{\left(1 - \frac{2\omega^2}{\Omega^2}\right)^2 - 1} = 1 - \frac{2\omega^2}{\Omega^2} \pm \sqrt{\frac{4\omega^2}{\Omega^2} + \frac{4\omega^4}{\Omega^4}}$$



folytonos hálózat



$$u(x+\Delta x) - u(x) = -(R\,\Delta x)\cdot I(x) - L\,\Delta x\,\frac{dI}{dt}$$

$$\frac{\partial u}{\partial x} = -R\,I(x) - L\cdot\frac{\partial I}{\partial t}$$

$$I(x+\Delta x) - I(x) = -\frac{u(x)}{r}\,\Delta x - C\,\frac{du}{dt}\,\Delta x$$

$$\frac{\partial I}{\partial x} = -G\cdot u - C\cdot\frac{du}{dt}$$

$$\frac{dI}{dt} = -\frac{R}{L}I - \frac{1}{L}\frac{du}{dx}$$

$$\frac{d^2 I}{dt^2} = -\frac{R}{L}\frac{dI}{dt} - \frac{1}{L}\frac{\partial^2 u}{\partial t\,\partial x} = -\frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}\frac{\partial^2 I}{\partial x^2} - \frac{G}{L\cdot C}\left(R\cdot I + L\frac{\partial I}{\partial t}\right) =$$

$$= -\left(\frac{G}{C} + \frac{R}{L}\right)\frac{\partial I}{\partial t} + \frac{1}{LC}\frac{\partial^2 I}{\partial x^2} - \frac{GR}{LC}I \qquad \frac{1}{LC} := v^2 \qquad \mu^2 := \frac{GR}{LC}$$

$$\frac{\partial^2 I}{\partial t} - \beta\frac{\partial I}{\partial t} = v^2\frac{\partial^2 I}{\partial x^2} - \mu^2 I$$ Telegráf

Klasszikus megoldás

$$\frac{\partial^2 I}{\partial t^2} + \beta \frac{\partial I}{\partial t} = \frac{\partial^2 I}{\partial x^2} - \mu^2 I \qquad I(x,t) = e^{i(kx-\omega t)}$$

$$-\omega^2 + \beta(i\omega) = -V^2 k^2 - \mu^2 \qquad \text{ha } \omega \in \mathbb{R},\ k \in \mathbb{C}$$

$$V^2 k^2 = \omega^2 - \mu^2 + i\beta\omega$$

$$k^2 = \frac{\omega^2 - \mu^2}{V^2} + i \frac{\beta}{V^2}\omega \qquad I = e^{iqx} e^{-i\omega t} e^{-\kappa x}$$

$$k = q + i\kappa$$

$$\underline{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \cdot e^{i(kz-\omega t)} \qquad E_z = 0$$

$\operatorname{div} \underline{D} = 0 \qquad \underline{D} = \varepsilon \underline{E}$

$\operatorname{div} \underline{B} = 0$

$\operatorname{rot} \underline{E} = -\frac{\partial B}{\partial t} \qquad \underline{B} = \mu \underline{H}$

$\underline{k} = (0,0,k)$

$$\underline{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \cdot e^{i(kz-\omega t)} \qquad B_z \neq 0$$

$\operatorname{rot} H = \frac{\partial D}{\partial t}$

$$B_z \cdot e^{i(kz-\omega t)} = B_0$$

$$\nabla \underline{D} = 0 \Rightarrow \partial_x E_x + \partial_y E_y = 0$$

$$\nabla \underline{B} = 0 \Rightarrow \partial_x B_x + \partial_y B_y + ik B_0 = 0$$

nem jó ?

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = 0 \Rightarrow \begin{cases} -ik E_y - i\omega \cdot B_x = 0 \\ ik E_x - i\omega \cdot B_y = 0 \\ \partial_x E_y - \partial_y E_x - i\omega \cdot B_0 = 0 \end{cases}$$

$$\frac{\partial B_0}{\partial y} = ik B_y - i\omega \varepsilon \mu E_x = i B_y \left(k - \frac{\varepsilon \mu \omega^2}{k}\right)$$

$$\frac{1}{\mu} \nabla \times B - \varepsilon \frac{\partial E}{\partial t} = 0 \qquad \begin{aligned} &\frac{1}{\mu}\left(\partial_y B_0 - ik B_y\right) + \varepsilon i\omega E_x = 0 \\ &\frac{1}{\mu}\left(ik B_x - \partial_x B_0\right) + \varepsilon i\omega E_y = 0 \\ &\frac{1}{\mu}\left(\partial_x B_y - \partial_y B_x\right) = 0 \end{aligned}$$

$$\frac{\partial B_0}{\partial x} = ik B_x + \varepsilon \mu i\omega E_y = i B_x \left(k - \frac{\varepsilon \mu \omega^2}{k}\right)$$

$$\frac{\partial B_0}{\partial x} = ih B_x\left(1-\frac{\varepsilon\mu\omega^2}{h^2}\right) \qquad \frac{\partial B_0}{\partial y} = ih B_y\left(1-\frac{\varepsilon\mu\omega^2}{h^2}\right)$$

$$B_x = -\frac{i}{h}\frac{\partial B_0}{\partial x}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}}$$

$$B_y = -\frac{i}{h}\frac{\partial B_0}{\partial y}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}}$$

$$E_x = -\frac{i\omega}{h^2}\frac{\partial B_0}{\partial y}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}}$$

$$E_y = \frac{i\omega}{h^2}\frac{\partial B_0}{\partial x}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}}$$

$E_z = 0$ ✓

$\operatorname{div} \underline{E} = 0$ automatikusan teljesül

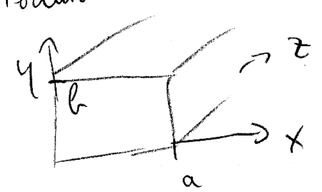
$\operatorname{div} \underline{B} = 0 \qquad \partial_x B_x + \partial_y B_y + ihB_0 = 0$

$$-\frac{i}{h}\frac{\partial^2 B_0}{\partial x^2}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}} - \frac{i}{h}\frac{\partial^2 B_0}{\partial y^2}\frac{1}{1-\frac{\varepsilon\mu\omega^2}{h^2}} + ihB_0 = 0$$

$$\frac{\partial^2 B_0}{\partial x^2} + \frac{\partial^2 B_0}{\partial y^2} - h^2\left(1-\frac{\varepsilon\mu\omega^2}{h^2}\right)B_0 = 0 \qquad \Delta_{xy} B_0 = \left(h^2 - \varepsilon\mu\omega^2\right)B_0$$

04.17.

Módusok



$\underline{E}$ fallal párh. komponense 0

$E_y = 0 \qquad x=0,\ x=a$

$E_x = 0$

$y=0 \quad y=b$

$E_z = 0$

$$E_x(x,y) = C_1 \cdot \cos\frac{\pi n x}{a}\sin\frac{m\pi y}{b}$$

$E_y = C_2 \sin \ldots \cos \ldots$

$E_z = 0$

Ha $f \sim e^{i(kz-\omega t)}$

$$\operatorname{rot} E = -\frac{\partial B}{\partial t} \sim i\omega B$$

$$B = \frac{1}{i\omega} \operatorname{rot} E$$

$\operatorname{div} \underline{B} = 0 \ldots$

---

külön teljesülnek a hullámegyenletek

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \Delta E$$ tiszta hullámegyenlet

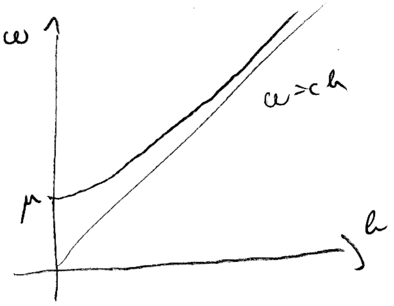
$$-\frac{\omega^2}{c^2} E = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E = -\left(\frac{n\pi}{a}\right)^2 E - \left(\frac{m\pi}{b}\right)^2 E - k^2 E$$

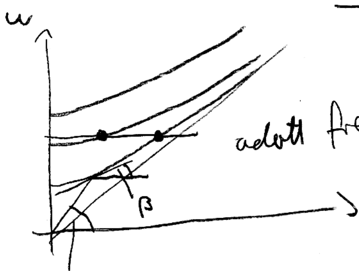
$$\frac{\omega^2}{c^2} = \underbrace{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}_{\frac{\mu^2_{n,m}}{c^2}} + k^2$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u - \mu^2 u$$

$$\Downarrow$$

$$\omega^2 = c^2 k^2 + \mu^2$$





k

adott frekiben véges sok megoldás, de össz. végtelen sok

α $\quad \operatorname{tg} \alpha = \frac{\omega}{k} = v_f > c$

$\operatorname{tg} \beta = \frac{d\omega}{dk} = v_g < c$. $\quad v_f \cdot v_g = c^2$

4 irányba haladó hullám $\uparrow\downarrow\rightleftarrows$

$e^{i\left(\frac{n\pi}{a}x + \frac{m\pi}{b}y + kz - \omega t\right)}$ $\quad \underline{k}^{(1)} = \left(\frac{n\pi}{a}, \frac{m\pi}{b}, k\right)$

$e^{i\left(-\frac{n\pi}{a}x + \frac{m\pi}{b}y + \ldots\right)}$ $\quad \underline{k}^{(2)} = \left(-\frac{n\pi}{a}, \frac{m\pi}{b}, k\right)$

$e^{i\left(-\frac{n\pi}{a}x - \frac{m\pi}{b}y + \ldots\right)}$ $\quad \underline{k}^{(4)} = \left(-\frac{n\pi}{a}, -\frac{m\pi}{b}, k\right)$

$e^{i\left(\frac{n\pi}{a}x - \frac{m\pi}{b}y + \ldots\right)}$ $\quad \underline{k}^{(3)} = \left(+\frac{n\pi}{a}, -\frac{m\pi}{b}, k\right)$

$$\underline{k}^2 = \frac{\omega^2}{c^2}$$

$\uparrow$ mindegyikre

$\mathrm{div}\,\underline{E} = \frac{1}{\varepsilon_0}\varrho$

$\mathrm{div}\,\underline{B} = 0$

$\mathrm{rot}\,\underline{E} = -\frac{\partial \underline{B}}{\partial t}$

$\mathrm{rot}\,\underline{B} = \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + \mu_0 \underline{j}$ $\qquad \frac{1}{c^2} = \frac{1}{\varepsilon_0\mu_0}$

$\underline{E}, \underline{B}$ 6 mennyiség, de lehet 4-gyel is

$\underline{B} = \mathrm{rot}\,\underline{A}$

$\underline{A} \to \underline{A}' = \underline{A} + \mathrm{grad}\,\lambda(\underline{r}, t)$

$\underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t}$

$\frac{1}{c^2}\frac{\partial\phi}{\partial t} + \mathrm{div}\,\underline{A} = 0$ Lorenz-mérték

$\Box \underline{A} = \mu_0 \underline{j}$ $\qquad$ Ez csak 4, de Hertz 3-mat tud csinálni.

$\Box \phi = \frac{1}{\varepsilon_0}\varrho$

Hertz: $\quad \varepsilon_0\mu_0\frac{\partial\phi}{\partial t} + \mathrm{div}\,\underline{A} = 0$ Lorenz-feltétel

$$\left.\begin{array}{l} \phi = -\frac{1}{\varepsilon_0}\,\mathrm{div}\,\underline{Z}(\underline{r}, t) \\ \underline{A} = \mu_0 \frac{\partial \underline{Z}}{\partial t} \end{array}\right\}$$ létezik ilyen $\underline{Z}$? Ha igen, a Lorenz-feltétel kiléegül.

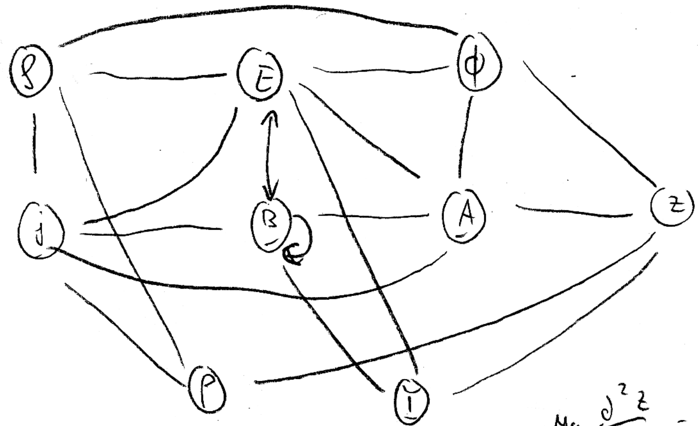
$$\left.\begin{array}{l} \mathrm{div}\,\Box\underline{A} = \mu_0\,\mathrm{div}\,\underline{j} \\ \frac{1}{c^2}\frac{\partial}{\partial t}\Box\phi = \frac{1}{\varepsilon_0}\frac{1}{c^2}\frac{\partial}{\partial t}\varrho \end{array}\right\} \Box\overbrace{\left(\mathrm{div}\,\underline{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right)}^{0} = \mu_0\overbrace{\left(\mathrm{div}\,\underline{j} + \frac{\partial\varrho}{\partial t}\right)}^{0}$$

$\frac{\partial\varrho}{\partial t} + \mathrm{div}\,\underline{j} = 0$

ha $\exists\,\underline{P}$: $\varrho = -\mathrm{div}\,\underline{P}(\underline{r}, t)$ $\quad\Rightarrow$ akkor a kontinuitási egyenlet teljesül

$\underline{j} = \frac{\partial}{\partial t}\underline{P}(\underline{r}, t)$

$\Box \underline{Z} = \underline{P}$ $\qquad$ Menet: $(\varrho, \underline{j}) \to \underline{P} \xrightarrow{\Box^{-1}} \underline{Z} \xrightarrow{\partial} (\underline{A}, \phi) \xrightarrow{\partial} (\underline{E}, \underline{B})$



$$\underline{B} = \text{rot}\,\underline{A} = \text{rot}\,\mu_0 \frac{\partial \underline{Z}}{\partial t} = \mu_0 \frac{\partial}{\partial t}\,\text{rot}\,\underline{Z}$$

$$\underline{E} = -\nabla\phi - \frac{\partial \underline{A}}{\partial t} = -\nabla\left(-\frac{1}{\varepsilon_0}\,\text{div}\,\underline{Z}\right) - \frac{\partial}{\partial t}\,\mu_0 \frac{\partial \underline{Z}}{\partial t} =$$

$$= \frac{1}{\varepsilon_0}\,\text{grad}\,\text{div}\,\underline{Z} - \mu_0 \frac{\partial^2 \underline{Z}}{\partial t^2} = \frac{1}{\varepsilon_0}\,\text{rot}\,\text{rot}\,\underline{Z} + \frac{1}{\varepsilon_0}\,\Delta \underline{Z} -$$

$$\mu_0 \frac{\partial^2 \underline{Z}}{\partial t^2} = \frac{1}{\varepsilon_0}\,\text{rot}\,\text{rot}\,\underline{Z} + \frac{1}{\varepsilon_0}\left(\Delta \underline{Z} - \mu_0\varepsilon_0 \frac{\partial^2 \underline{Z}}{\partial t^2}\right) =$$

$$= \frac{1}{\varepsilon_0}\left(\text{rot}\,\text{rot}\,\underline{Z} - \underline{P}\right)$$

$$\boxed{\begin{aligned} \underline{B} &= \mu_0 \frac{\partial}{\partial t}\underline{Y} \\ \underline{E} &= \frac{1}{\varepsilon_0}\,\text{rot}\,\underline{Y} - \underline{P} \end{aligned}}$$

← tök Tuti

Vákuumban $\Box \underline{Y} = 0 \Rightarrow \underline{B} = \mu_0 \frac{\partial}{\partial t}\underline{Y}$

$\underline{E} = \frac{1}{\varepsilon_0}\,\text{rot}\,\underline{Y}$

$\underline{Y} = \text{rot}\,\underline{Z}$

$$\boxed{\begin{aligned} \Box \underline{X} &= 0 \\ \underline{E} &= -\frac{\partial \underline{X}}{\partial t} \\ \underline{B} &= \text{rot}\,\underline{X} \end{aligned}}$$

Ha $\underline{A}$-t a $\underline{E}$-hez

ϕ-t a $\underline{B}$-hez

veszem be

Hengerben $(\text{grad}\,\phi)_\rho = \frac{\partial \phi}{\partial \rho}$

$$\text{div}\,\underline{v} = \frac{\partial v_\rho}{\partial \rho} + \frac{1}{\rho} v_\rho + \frac{1}{\rho}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z}$$

$$(\text{grad}\,\phi)_\varphi = \frac{1}{\rho}\frac{\partial \phi}{\partial \varphi}$$

$$(\text{rot}\,\underline{v})_\rho = \frac{1}{\rho}\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z}$$

$$(\text{grad}\,\phi)_z = \frac{\partial \phi}{\partial z}$$

$$(\text{rot}\,\underline{v})_\varphi = \frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho}$$

$$(\text{rot}\,\underline{v})_z = \frac{\partial v_\varphi}{\partial \rho} + \frac{v_\varphi}{\rho} - \frac{1}{\rho}\frac{\partial v_\rho}{\partial \varphi}$$

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

$\underline{Z} = \left(0, 0, Z_z(\varrho, \varphi, z)\right)$ $\underline{W} = \left(W_\varrho = 0,\ W_\varphi = 0,\ W_z = W(\varrho, \varphi, z)\right)$

04.17
5.

$\underline{Y} = \mathrm{rot}\,\underline{Z}$

$Y_\varrho = \frac{1}{\varrho}\frac{\partial Z}{\partial \varphi}$ $Y_\varphi = -\frac{\partial Z}{\partial \varrho}$ $Y_z = 0$

TM

$B_\varrho = \mu_0 \frac{1}{\varrho}\frac{\partial^2 Z}{\partial \varphi \partial t}$ $B_\varphi = -\mu_0 \frac{\partial^2 Z}{\partial \varrho \partial t}$ $B_z = 0$

$\boxed{E_\varrho = \frac{1}{\varepsilon_0}\frac{\partial^2 Z}{\partial \varrho \partial z}}$ $\boxed{E_\varphi = \frac{1}{\varepsilon_0}\frac{1}{\varrho}\frac{\partial^2 Z}{\partial \varphi \partial z}}$ $E_z = ?$

$\varepsilon_0 E_z = (\mathrm{rot}\,\underline{Y})_z = \frac{\partial Y_\varphi}{\partial \varrho} + \frac{Y_\varphi}{\varrho} - \frac{1}{\varrho}\frac{\partial Y_\varrho}{\partial \varphi} =$

$= -\frac{\partial^2 Z}{\partial \varrho^2} - \frac{1}{\varrho}\frac{\partial Z}{\partial \varrho} - \frac{1}{\varrho^2}\frac{\partial^2 Z}{\partial \varphi^2} = \left(-\Delta + \frac{\partial^2}{\partial z^2}\right) Z =$

$\Box Z = 0 \Downarrow \Delta - \frac{\partial^2}{\partial t^2} = \Box$

$= \left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right) Z$

$$\boxed{E_z = -\mu_0 \frac{\partial^2 Z}{\partial t^2} + \frac{1}{\varepsilon_0}\frac{\partial^2 Z}{\partial z^2}}$$

$X_\varrho = \frac{1}{\varrho}\frac{\partial W}{\partial \varphi}$ $X_\varphi = -\frac{\partial W}{\partial \varrho}$ $X_z = 0$

TE

$E_\varrho = -\frac{1}{\varrho}\frac{\partial^2 W}{\partial \varphi \partial t}$ $E_\varphi = \frac{\partial^2 W}{\partial \varrho \partial t}$ $E_z = 0$

$$\boxed{B_\varrho = \frac{\partial^2 W}{\partial z \partial \varrho} \quad B_\varphi = \frac{1}{\varrho}\frac{\partial^2 W}{\partial z \partial \varphi} \quad B_z = \frac{\partial^2 W}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 W}{\partial t^2}}$$

Ilyen tuti lesz a végén

$\Box W = 0$ ezt akarom megoldani $W(t, \varrho, \varphi, z)$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} W = \frac{\partial^2 W}{\partial \varrho^2} + \frac{1}{\varrho}\frac{\partial W}{\partial \varrho} + \frac{1}{\varrho^2}\frac{\partial^2 W}{\partial \varphi^2} + \frac{\partial^2 W}{\partial z^2}$$

megoldás keresés $W = e^{i(kz - \omega t)} \cdot u(\varrho, \varphi)$

$$-\frac{\omega^2}{c^2} u(\varrho, \varphi) = \frac{\partial^2 u}{\partial \varrho^2} + \frac{1}{\varrho}\frac{\partial u}{\partial \varrho} + \frac{1}{\varrho^2}\frac{\partial^2 u}{\partial \varphi^2} - k^2 u$$

$$\frac{\partial^2 u}{\partial \varrho^2} + \frac{1}{\varrho}\frac{\partial u}{\partial \varrho} + \frac{1}{\varrho^2}\frac{\partial^2 u}{\partial \varphi^2} + \left(\frac{\omega^2}{c^2} - k^2\right) u = 0$$

$$u(\varrho, \varphi) = F(\varrho)\, e^{im\varphi} \quad m \in \mathbb{Z}$$

$$F''(\varrho) + \frac{1}{\varrho} F'(\varrho) - \frac{m^2}{\varrho^2} F(\varrho) + q^2 F(\varrho) = 0$$

$$F''(\varrho) + \frac{1}{\varrho} F'(\varrho) + \left(q^2 - \frac{m^2}{\varrho^2}\right) F(\varrho) = 0$$

$$u := q \cdot \varrho \qquad F(\varrho) = B(u)$$

$$B''(u) = \frac{1}{u} B'(u) + \left(1 - \frac{m^2}{u^2}\right) B(u) = 0$$

$$J_m(u) \sim u^m$$

$$N_m(u) \sim \frac{1}{u^{m+1}}$$

$$F(\varrho) = J_m(q\varrho)$$

$$u(\varrho, \varphi) = J_m(q, \varrho) e^{im\varphi}$$

$$W(t, \varrho, \varphi, z) = J_m(q\varrho) e^{im\varphi} e^{i(kz - \omega t)}$$

Sugárzások

04.23

Nem hoztam füzetet

Hertz-vektorokkal fogunk dolgozni

$\underline{Z} \to (\underline{A}, \phi) \to \underline{E}, \underline{B}$ $\qquad \Box \underline{Z} = \frac{1}{\varepsilon_0}\underline{P}$ ↘ megoldása

$$\underline{Z}(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3R \, \frac{\underline{P}\left(\underline{R}, t - \frac{|\underline{r}-\underline{R}|}{c}\right)}{|\underline{r}-\underline{R}|}$$

$-\rho = \mathrm{div}\,\underline{P}$

$\underline{j} = \frac{\partial \underline{P}}{\partial t}$

Legyen pontszerű forrás

$$\underline{P}(\underline{r},t) = \delta(\underline{r})\,\underline{p}(t)$$

$$\underline{Z}(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3R \, \frac{\delta(\underline{R})\,\underline{p}\left(t-\frac{|\underline{r}-\underline{R}|}{c}\right)}{|\underline{r}-\underline{R}|} = \frac{1}{4\pi\varepsilon_0}\frac{\underline{p}\left(t-\frac{|\underline{r}|}{c}\right)}{|\underline{r}|}$$

$$4\pi\varepsilon_0^2\phi = -\mathrm{div}\, 4\pi\varepsilon_0 \underline{Z} = -\mathrm{div}\frac{\underline{p}\left(t-\frac{r}{c}\right)}{r} = -\partial_l \frac{p_l\left(t-\frac{|r|}{c}\right)}{r} =$$

$$= \frac{1}{r^2} e_l p_l\left(t-\frac{|r|}{c}\right) - \frac{1}{r}\dot p_l\, \partial_l\left(t-\frac{r}{c}\right) = \frac{1}{r^2}p_l\left(t-\frac{|r|}{c}\right)e_l +$$

$$+\frac{1}{rc}e_l\dot p_l\left(t-\frac{r}{c}\right) = \frac{1}{r^2}\underline{e}\underline{p} + \frac{1}{rc}\underline{e}\dot{\underline{p}}$$

$$4\pi\varepsilon_0^2 A_l = \frac{1}{c^2}\frac{\partial Z_l}{\partial t}4\pi\varepsilon_0 = \frac{1}{c^2}\frac{\partial}{\partial t}\frac{p_l\left(t-\frac{r}{c}\right)}{r} = \frac{\dot p_l}{rc^2}$$

$$4\pi\varepsilon_0^2 B_k = 4\pi\varepsilon_0^2(\mathrm{rot}\,\underline{A})_k = \cancel{4\pi\varepsilon_0^2}\,\varepsilon_{klm}\partial_l\left(\frac{\dot p_m}{rc^2}\right) =$$

$$= \frac{1}{c^2}\varepsilon_{klm}\partial_l\left(\frac{1}{r}\dot p_m\right) = \frac{1}{c^2}\varepsilon_{klm}\left(-\frac{1}{r^2}e_l\dot p_m + \frac{1}{r}\ddot p_m\left(\frac{-e_l}{c}\right)\right) =$$

$$= \frac{1}{r^2c^2}\left(-\varepsilon_{klm}e_l\dot p_m\right) - \frac{1}{rc^3}\varepsilon_{klm}e_l\ddot p_m$$

$$4\pi\varepsilon_0^2\underline{B} = -\frac{\underline{e}\times\dot{\underline{p}}}{r^2c^2} - \frac{\underline{e}\times\ddot{\underline{p}}}{rc^3}$$

Hogy is voltak a képletek?

$\mathrm{div}\,\underline{E} = \frac{\rho}{\varepsilon_0}$

$\mathrm{div}\,\underline{B} = 0$

$\mathrm{rot}\,\underline{E} = -\frac{\partial B}{\partial t}$

$\mathrm{rot}\,\underline{B} = \mu_0\underline{j} + \mu_0\frac{\partial E}{\partial t}\varepsilon_0$

$\underline{B} = \mathrm{rot}\,\underline{A} \qquad \underline{E} = -\mathrm{grad}\,\phi - \frac{\partial A}{\partial t}$

$$\mathrm{rot}\,\mathrm{rot}\,\underline{A} = (\mathrm{grad}\,\mathrm{div} - \Delta)\underline{A} = \mu_0\underline{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\mathrm{grad}\,\phi - \frac{\partial A}{\partial t}\right)$$

$$\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \Delta A = \mu_0\underline{j} - \mathrm{grad}\left(\frac{1}{c^2}\frac{\partial\phi}{\partial t} + \mathrm{div}\,\underline{A}\right)$$

$\mathrm{div}\,\underline{A} + \frac{\partial\phi}{\partial t}\cdot\frac{1}{c^2} = 0$ Lorenz mérték

$\Box\underline{A} = \mu_0\underline{j}$

$$\mathrm{div}\,\underline{E} = \mathrm{div}\left(-\mathrm{grad}\,\phi - \frac{\partial A}{\partial t}\right) = -\Delta\phi - \frac{\partial}{\partial t}\mathrm{div}\,\underline{A} = -\Delta\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = \frac{1}{\varepsilon_0}\rho$$

$\Box\phi = \frac{\rho}{\varepsilon_0}$

$\phi = -\mathrm{div}\,\underline{Z} \qquad \underline{E} = -\mathrm{grad}\,\phi - \frac{\partial A}{\partial t}$

$\underline{A} = \frac{1}{c^2}\frac{\partial \underline{Z}}{\partial t} \qquad \underline{B} = \mathrm{rot}\,\underline{A}$

$$4\pi\varepsilon_0 E_e = -\mathrm{grad}_a\left(4\pi\varepsilon_0\phi\right) - \frac{\partial}{\partial t}\left(4\pi\varepsilon_0 A_a\right) = -\partial_a\left(\frac{e p}{r^2} + \frac{e\dot{p}}{rc}\right) - \frac{\partial}{\partial t}\left(\frac{\dot{p}_a}{rc^2}\right) =$$

$$= -\partial_a\left(\frac{x_e p_e}{r^3} + \frac{x_e \dot{p}_e}{r^2 c}\right) - \frac{\ddot{p}_a}{rc^2} = -\frac{\delta_{ae} p_e}{r^3} + \frac{3}{r^4} e_a x_e p_e - \frac{x_e}{r^3}\dot{p}_e\left(-\frac{e_a}{c}\right) -$$

$$-\frac{\delta_{ae}\dot{p}_e}{r^2 c} + \frac{2}{r^3 c} e_a x_e \dot{p}_e - \frac{x_e}{r^2 c}\ddot{p}_e\left(-\frac{1}{c}e_a\right) - \frac{1}{rc^2}\ddot{p}_a =$$

$$= \frac{3 e_a e_e - \delta_{ae}}{r^3} p_e + \frac{3 e_a e_e - \delta_{ae}}{r^2 c}\dot{p}_e + \frac{e_a e_e - \delta_{ae}}{rc^2}\ddot{p}_e$$

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\underline{e}\circ\underline{e} - \underline{I}}{r^3}\underline{p} + \frac{3\underline{e}\circ\underline{e} - \underline{I}}{r^2 c}\dot{\underline{p}} + \frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}}\right]$$

$$\underline{B} = \frac{1}{4\pi\varepsilon_0}\left[\qquad -\frac{\underline{e}\times\dot{\underline{p}}}{r^2c^2} - \frac{\underline{e}\times\ddot{\underline{p}}}{rc^3}\right]$$

távol $\quad \underline{E} = \frac{1}{4\pi\varepsilon_0}\frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}} = \frac{1}{4\pi\varepsilon_0}\frac{\underline{e}(\underline{e}\ddot{\underline{p}}) - (\underline{e}\underline{e})\ddot{\underline{p}}}{rc^2} = \frac{1}{4\pi\varepsilon_0}\frac{\underline{e}\times(\underline{e}\times\ddot{\underline{p}})}{rc^2}$

$$\underline{E} = -\frac{1}{c}\underline{e}\times\underline{B}$$

$$\underline{B} = \frac{-1}{4\pi\varepsilon_0}\frac{\underline{e}\times\ddot{\underline{p}}}{rc^3} \qquad \underline{B} = \frac{1}{c}\underline{e}\times\underline{E}$$

$$\boxed{\underline{S} = \frac{1}{\mu_0}\underline{E}\times\underline{B}} \qquad \underline{\sigma} = \varepsilon_0\,\underline{E}\circ\underline{E} + \frac{1}{\mu_0}\underline{B}\circ\underline{B} - \underline{I}\,\frac{\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2}{2}$$

$$(4\pi\varepsilon_0)^2\underline{\sigma} = \varepsilon_0\left(\frac{3\underline{e}\circ\underline{e} - \underline{I}}{r^3}\dot{\underline{p}}\right)\circ\left(\frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}}\right) + \varepsilon_0\left(\frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}}\right)\circ\left(\frac{3\underline{e}\circ\underline{e} - \underline{I}}{r^2 c}\dot{\underline{p}}\right) +$$

$$+\varepsilon_0\left(\frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}}\right)\circ\left(\frac{\underline{e}\circ\underline{e} - \underline{I}}{rc^2}\ddot{\underline{p}}\right) +$$

$$+\frac{1}{\mu_0}\frac{\underline{e}\times\dot{\underline{p}}}{r^2c^2}\circ\frac{\underline{e}\times\ddot{\underline{p}}}{rc^3} + \frac{1}{\mu_0}\frac{\underline{e}\times\ddot{\underline{p}}}{rc^2}\circ\frac{\underline{e}\times\dot{\underline{p}}}{r^2c^2} + \frac{\underline{e}\times\ddot{\underline{p}}}{rc^3}\circ\frac{\underline{e}\times\ddot{\underline{p}}}{rc^3} - \underline{I}\cdot\frac{\varepsilon_0 E^2 + \frac{1}{\mu}B^2}{2}$$

$$\underline{\underline{T}}\,\underline{e} = \varepsilon_0 \underline{E}(\underline{E}\underline{e}) + \frac{1}{\mu_0}\underline{B}(\underline{B}\underline{e}) - \underline{e}\,\frac{\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2}{2}$$

$$4\pi\varepsilon_0 \underline{E}\,\underline{e} = \frac{3\underline{e}(\underline{e}\underline{p}) - \underline{p}}{r^3}\underline{e} + \frac{3\underline{e}(\underline{e}\dot{\underline{p}}) - \dot{p}}{r^2 c}\underline{e} + \frac{\underline{e}(\underline{e}\ddot{\underline{p}}) - \ddot{p}\underline{e}}{r c^2} = \frac{2\underline{e}\underline{p}}{r^3} + \frac{2\underline{e}\dot{\underline{p}}}{r^2 c}$$

$$4\pi\varepsilon_0 \underline{B}\,\underline{e} = 0$$

$$(4\pi\varepsilon_0)^2 \underline{\underline{T}}\underline{e} = \varepsilon_0 (4\pi\varepsilon_0 \underline{E})\left(\frac{2\underline{e}\underline{p}}{r^3} + \frac{2\underline{e}\dot{p}}{r^2 c}\right) - \underline{e}\,\frac{\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2}{2}(4\pi\varepsilon_0)^2 =$$

$$= \varepsilon_0 \frac{\underline{e}\ \ \underline{\underline{0}}\underline{e} - \underline{\underline{I}}}{r c^2}\ddot{p}\ \frac{2\underline{e}\dot{p}}{r^2 c} - \underline{e}(\ldots)$$

$$M = -\oint (\underline{r} \times \underline{\underline{T}}\,\underline{e})\,dA$$

$$\underline{e} \times \underline{\underline{T}}\,\underline{e} = \frac{\varepsilon_0\, \underline{e} \times (\underline{e} \times (\underline{e} \times \ddot{\underline{p}}))}{r c^2}\ \frac{2\underline{e}\dot{\underline{p}}}{r^2 c} = \varepsilon_0 \frac{-\underline{e} \times \ddot{p}}{r c^2}\ \frac{2\underline{e}\dot{p}}{r^2 c}$$

$$\underline{e} \times \underline{\underline{T}}\,\underline{e} = \frac{-1}{(4\pi\varepsilon_0)^2}\, 2\varepsilon_0\, \frac{\underline{e}\dot{p}(\underline{e} \times \ddot{p})}{r^3 c^3}$$

$$M = -\int \underline{e} \times \underline{\underline{T}}\,\underline{e}\ r\,dA = \frac{1}{(4\pi\varepsilon_0)^2}\,\frac{2\varepsilon_0}{c^3} \oint \underline{e}\dot{p}(\underline{e} \times \ddot{p}) \sin\vartheta\, d\vartheta\, d\varphi$$

$$\underline{p}(t) = p_0 \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix} \qquad \dot{\underline{p}} = p_0\omega \begin{pmatrix} -\sin\omega t \\ \cos\omega t \\ 0 \end{pmatrix} \qquad \ddot{\underline{p}} = -p_0\omega^2 \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix} \qquad \underline{e} = \begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix}$$

forgó dipól

$$\underline{e}\dot{\underline{p}} = p_0\omega \sin\vartheta(-\cos\varphi\sin\omega t + \sin\varphi\cos\omega t) = p_0\omega\sin(\varphi - \omega t)\sin\vartheta$$

$$(\underline{e} \times \ddot{\underline{p}})_x = e_y \cancel{\ddot{p}_z} - e_z \ddot{p}_y = -\cos\vartheta(-p_0\omega^2)\sin\omega t$$

$$(\underline{e} \times \ddot{\underline{p}})_y = e_z \ddot{p}_x - e_x \cancel{\ddot{p}_z} = -p_0\omega^2\cos\vartheta\cos\omega t$$

$$(\underline{e} \times \ddot{\underline{p}})_z = e_x \ddot{p}_y - e_y \ddot{p}_x = -p_0\omega^2\sin\vartheta(\cos\varphi\sin\omega t - \sin\varphi\cos\omega t) = p_0\omega^2\sin\vartheta\sin(\varphi - \omega t)$$

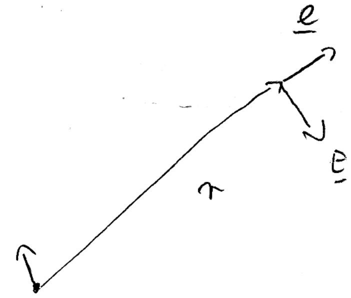
$$\underline{M} = \frac{1\cdot 2}{16\pi^2\varepsilon_0 c^3}\int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} p_0\omega \sin\vartheta \sin(\varphi-\omega t)\, p_0\omega^2 \begin{pmatrix}\cos\vartheta \sin\omega t\\ -\cos\vartheta\cos\omega t\\ \sin\vartheta\sin(\varphi-\omega t)\end{pmatrix}\sin\vartheta \, d\vartheta \, d\varphi$$

04.23

$M_x = M_y = 0$

$$M_z = \frac{1}{8\pi\varepsilon_0 c^3} p_0^2\omega^3 \overbrace{\int_0^{\pi}\sin^3\vartheta \, d\vartheta}^{4/3} = \frac{p_0^2\omega^3}{6\pi\varepsilon_0 c^3}$$

04.30



ha $\rho(\underline{r},t) = \delta(\underline{r})\,\underline{p}(t)$ $\quad \underline{p}(t) = \underline{p}_0 e^{i\omega t}$

$$\underline{Z}(\underline{r},t) = \frac{\underline{p}(t-\frac{r}{c})}{4\pi\varepsilon_0 r}$$

$$\underline{E}(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\,\underline{e}\circ\underline{e}-\underline{\underline{I}}}{r^3}\underline{p} + \frac{3\,\underline{e}\circ\underline{e}-\underline{\underline{I}}}{r^2 c}\dot{\underline{p}} + \frac{\underline{e}\circ\underline{e}-\underline{\underline{I}}}{rc^2}\ddot{\underline{p}}\right)_{t-\frac{r}{c}}$$

$$\underline{B}(\underline{r},t) = \frac{\mu_0}{4\pi}\left(-\frac{\underline{e}\times\dot{\underline{p}}}{r^2} - \frac{\underline{e}\times\ddot{\underline{p}}}{rc}\right)_{t-\frac{r}{c}}$$

$$\underline{S} = \frac{1}{\mu_0}\underline{E}\times\underline{B}$$

$$\underline{\underline{T}} = \varepsilon_0 \underline{E}\circ\underline{E} + \frac{1}{\mu_0}\underline{B}\circ\underline{B} + \frac{1}{2}\,\frac{\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2}{2}\,\underline{\underline{I}}$$

$$I_{\text{intenzitás}} = \oint \underline{S}\,d\underline{F} = \oint \underline{S}\,\underline{e}\, r^2 \sin\vartheta \, d\vartheta \, d\varphi$$

$d\underline{F} = \underline{e}\,dA$

$dA = r^2 \sin\vartheta \, d\vartheta \, d\varphi$

$$\underset{\substack{\uparrow\\ t}}{}\;\underline{E}_{\text{távol}} \approx\sim \frac{\underline{e}\circ\underline{e}-\underline{\underline{I}}}{rc^2}\ddot{\underline{p}} = \frac{\underline{e}(\underline{e}\ddot{\underline{p}})-\ddot{\underline{p}}}{rc^2} = \frac{\underline{e}\times\underline{e}\times\ddot{\underline{p}}}{rc^2} \sim -\frac{1}{c}\underline{e}\times\underline{B}_t$$

$$\underline{B}_t \sim \underline{e}\times\underline{E}_t$$

$$\underline{S} \sim \underline{E}\times\underline{B} = -\frac{1}{c}(\underline{e}\times\underline{B})\times\underline{B} = \frac{1}{c}\,\underline{e}\,B^2$$

$$\underline{S}\,\underline{e} = \frac{1}{c}B^2 \sim \frac{1}{r^2c^3}(\underline{e}\times\ddot{\underline{p}})^2$$

$$I \sim \frac{1}{c^3}\int(\underline{e}\times\ddot{\underline{p}})^2 \sin\vartheta\, d\vartheta\, d\varphi$$

vigyázz a komplex négyzetére!

$$I \sim \frac{1}{c^3}\int(\underline{e}\times p_0)^2\,\omega^4\cos^2\omega t$$

ha $p = p_0 e^{i\omega t}$.

$$\ddot{p} = p_0(-\omega^2)\,e^{i\omega t}$$

$$\underline{e} = \begin{pmatrix}\sin\vartheta\cos\varphi\\ \sin\vartheta\sin\varphi\\ \cos\vartheta\end{pmatrix}$$

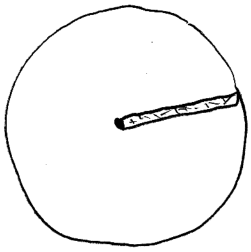
ha $p_0 = \begin{pmatrix}0\\0\\1\end{pmatrix}p_0$ $\quad (e\times p_0)^2 = p_0^2\sin^2\vartheta$

$$I = \frac{\mu_0}{4\pi}\frac{\omega^4}{c^3}\frac{1}{2}\int p_0^2\sin^2\vartheta\,\sin\vartheta\, d\vartheta\, d\varphi$$

$$\cos\vartheta = du$$

$$I = \frac{\mu_0}{2}\frac{\omega^4}{c^3}\frac{p_0^2}{2}\int_1^{-1}(1-u^2)(-du) = (\dots)\frac{p_0^2\omega^4}{c^3}$$

Példa



$$\frac{dE}{dt} = -I \qquad \frac{d\omega}{dt} = -\frac{k}{\Theta}\omega^3$$

$$\frac{dJ}{dt} = -M \qquad \frac{d\omega}{\omega^3} = -\frac{k}{\Theta}dt$$

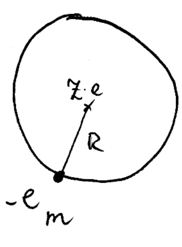
$$J = \Theta\omega$$

$$\frac{dJ}{dt} = \Theta\frac{d\omega}{dt} = -M = -k\omega^3 \qquad -\frac{1}{2\omega^2} + \frac{1}{2\omega_0^2} = -\frac{k}{\Theta}(t-t_0)$$

$$E = \frac{\Theta}{2}\omega^2$$

$$\frac{dE}{dt} = \frac{\Theta}{2}\,2\omega\frac{d\omega}{dt} = -I = -k\omega^4 \qquad \omega^2 = \frac{\omega_0^2}{1+\frac{2k}{\Theta}\omega_0^2(t-t_0)}$$

Példa



$$\frac{1}{4\pi\varepsilon_0}\frac{Ze\cdot e}{R^2} = \frac{mv^2}{R} = mR\omega^2$$

$$\frac{Ze^2}{4\pi\varepsilon_0 m} = R^3\omega^2$$

$$J = mRv = mR^2\omega$$

$$E = \frac{mv^2}{2} - \frac{Ze^2}{4\pi\varepsilon_0 r} = \frac{mR^2\omega^2}{2} - \frac{Ze\cdot e}{4\pi\varepsilon_0 R} = \frac{m}{2}\frac{Ze^2}{4\pi\varepsilon_0 m}\frac{1}{R} - \frac{Ze^2}{4\pi\varepsilon_0 R} =$$

$$= -\frac{Ze^2}{8\pi\varepsilon_0 R} := -\frac{\alpha}{2R}$$

$$R^3\omega^2 = \frac{\alpha}{m} \qquad E = -\frac{\alpha}{2R}$$

$$J = mR^2\omega \qquad \alpha = \frac{Ze^2}{4\pi\varepsilon_0}$$

$$\frac{dJ}{dt} = m\left(R^2\dot\omega + 2R\dot R\omega\right) = -k\omega^3$$

$$\frac{dE}{dt} = \dots\dot R + \dots\dot\omega = -k\omega^4$$

$$3R^2\dot R\omega^2 + 2R^3\omega\dot\omega = 0 = \left(R^3\omega^2\right)^{\cdot}$$

$$\left(\dot E = \frac{\alpha}{2R^2}\dot R\right)$$

$$J = mR^2\omega \leftarrow R^3\omega^2 = \alpha/m$$

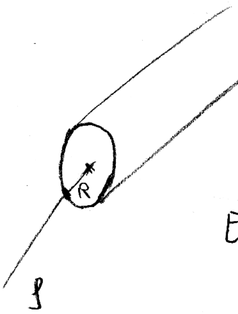
$$J = m\left(\frac{\alpha}{m\omega^2}\right)^{2/3}\omega = \alpha^{2/3} m^{1/3}\omega^{1/3}$$

$$p_0 = e\cdot R$$

$$\dot J = \left(\alpha^2 m\right)^{1/3}\left(-\frac{1}{3}\right)\omega^{-4/3}\dot\omega = M = \beta(eR)^2\omega^3 = \beta e^2\left(\frac{\alpha}{m\omega^2}\right)^{2/3}\omega^3 \sim \omega^{5/3}$$

$$\frac{\dot\omega}{\omega^3} = \text{áll} \qquad \frac{d\omega}{dt} = -\omega^3$$

Vissza a [illegible]



$$w_z(\varrho,\varphi,z,t) = J_m(q\varrho)\, e^{im\varphi}\, e^{i(kz-\omega t)} \qquad q^2 = \frac{\omega^2}{c^2} - k^2$$

$$J_m'(qR) = 0$$

m-edik Bessel deriv. zérushely.

$$E_\varrho = \frac{m\omega}{\varrho} J_m\left(\frac{\eta_{mn}\varrho}{R}\right) e^{im\varphi}\, e^{i[kz - \omega_{mn}(k)t]}$$

η_{mn} a hely

$$J_m'(\eta_{m,n}) = 0$$

$$E_\varphi = -i\frac{\omega\eta_{mn}}{R} J_m'\left(\frac{\eta_{mn}\varrho}{R}\right) e^{im\varphi}\, e^{i(kz-\omega t)}$$

TE

$$E_z = 0$$

$$q = \frac{\eta_{mn}}{R}$$

$$\omega^2 = c^2k^2 + c^2\frac{\eta_{mn}^2}{R^2}$$

$$B_\varrho = ik\frac{\eta_{mn}}{R} J_m'\left(\frac{\eta_{mn}\varrho}{R}\right) e^{im\varphi}\, e^{i(kz-\omega t)}$$

$$B_\varphi = -mk J_m(\quad) \text{ —\|—}$$

$$B_z = \left(\frac{\eta_{mn}}{R}\right)^2 J_m(\quad) \text{ —\|—}$$

TM

$$E_\rho = ik \frac{\xi_{mn}}{R} J_m' \left(\xi_{mn} \frac{\rho}{R}\right) e^{im\varphi} e^{i(kz-\omega t)}$$

$$E_\varphi = -\frac{mk}{\rho} J_m(\ldots) -''-$$

$$E_z = -\left(\frac{\xi_{mn}}{R}\right)^2 \underbrace{J_m(..) -''-}_{z}$$

$$B_\rho = \frac{m\omega}{c^2} \frac{1}{\rho} J_m(\ldots) -''-$$

$$B_\varphi = \frac{i\omega}{c^2} \frac{\xi_{mn}}{R} J_m'(\ldots) -''-$$

$$B_z = 0$$

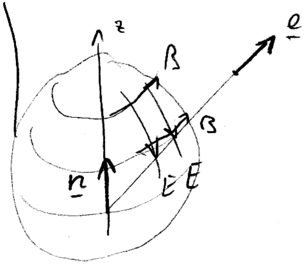
Eldin graf

$$\underline{E}(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\underline{e}\circ\underline{e}-\underline{\underline{I}}}{r^3}\,\underline{p}\left(t-\frac{r}{c}\right) + \frac{3\underline{e}\circ\underline{e}-\underline{\underline{I}}}{r^2 c}\,\dot{\underline{p}}\left(t-\frac{r}{c}\right) + \frac{\underline{e}\circ\underline{e}-\underline{\underline{I}}}{c^2 r}\,\ddot{\underline{p}}\right)$$

$$\underline{B}(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\left(-\frac{\underline{e}\times\dot{\underline{p}}}{r} - \frac{\underline{e}\times\ddot{\underline{p}}}{cr}\right)$$

$$\underline{S} \sim \underline{E}\times\underline{B} \sim \underline{e}\,|\underline{e}\times\ddot{\underline{p}}|^2 \sim \frac{\underline{e}\,\omega^4 p_0^2 \sin^2\vartheta}{r^2}$$

$$\underline{e} := \frac{1}{r}\underline{r} = \begin{pmatrix}\sin\vartheta\cos\varphi\\ \sin\vartheta\sin\varphi\\ \cos\vartheta\end{pmatrix}$$



$$E \sim \frac{\underline{e}\times(\underline{e}\times\ddot{\underline{p}})}{c^2 r} \sim \underline{e}\,\omega^4$$

$$B \sim \frac{\underline{e}\times\ddot{\underline{p}}}{cr}$$

$$\underline{p} = \begin{pmatrix}0\\0\\1\end{pmatrix} p\, e^{i\omega t}$$

$p \sim e\cdot l$

$I \sim \frac{ev}{l} \sim e\omega \sim \frac{p\omega}{l}$

$v \sim l\omega$

$p \sim \frac{Il}{\omega}$

Losová antennára

$$E_\vartheta(r,\vartheta) = \int I(z)\frac{\omega\sin\vartheta}{r}\, e^{i\omega\left(t-\frac{r}{c}\right)}\,dz$$ 1 dlb naappra

$$E_\vartheta = \frac{1}{\sin\vartheta}\,\frac{\cos\frac{\pi}{2}\cos\vartheta}{r}\, e^{i\omega\left(t-\frac{r}{c}\right)}$$

$$E_{\vartheta,\text{rolna}} = \sum E_\vartheta = \ldots = \frac{\sin N(\ldots)}{\sin(\ldots)}$$