MODERN COSMOLOGY

astronomical, physical and logical approaches

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Introduction: what is cosmology and what is it not?

Brief history of scientific cosmology

Keystones of cosmology
- astrophysical observations
- physical theories
- "aesthetical" or simplifying assumption

Construction of Einstein-Friedman family of cosmological models

Specific models of matter

Different solutions of EF equation
- cosmological model family for the present Universe

Modern observational projects and calculations

Interpretation of results

Consequences of measurements:
identity card and history of the Universe

Theoretical status of modern (precision) cosmology

Possibility of logical and axiomatic foundation of cosmology

INTRODUCTION

① WHY cosmology ?

What is the reason of my invitation? A possible answer: A logician (I.N.) wants to develop the logical foundation of space-time. It is interesting for him – and hopefully for You – to know what other branches of science say about the REAL SPACE-TIME about us.

② WHAT is cosmology and what is it not ?

COSMOLOGY – physics of the Universe as a whole

Cosmology is NOT philosophy

the term "Universe" does not mean the "totality" of philosophers

- it is a specific physical system around us in a sphere about 14 billion light year
- it is observable, measureble
- it can be characterized by physical quantities

Cosmology is NOT MATHEMATICS

- not a pure collection of mathematical models of interesting space-times

Cosmology is NOT LOGIC

- we do not know whether the properties of the Universe are logical necessities or not

COSMOLOGY is a branch of NATURAL SCIENCES

- it is based on observations and measurements
- uses mathematical models to describe the reality
- gives numerical data for describing the phenomena
- gives predictions which are falsificable

MODERN COSMOLOGY is very conservative (contrary ~1970)

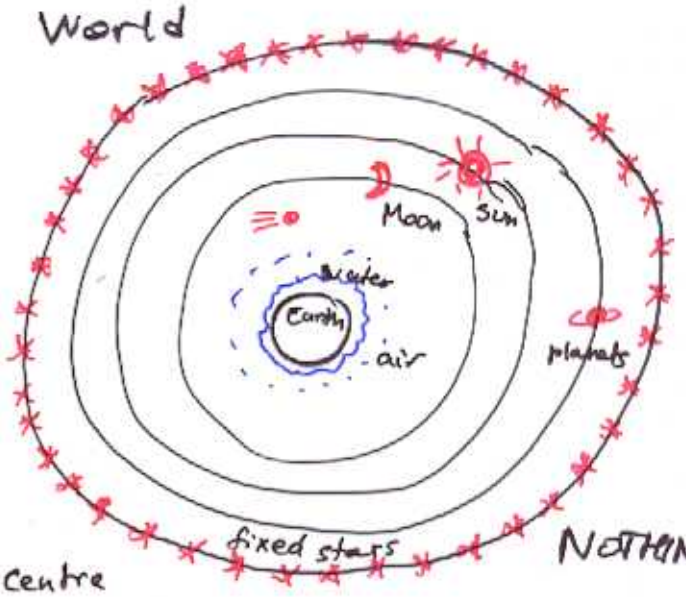
- it uses Occam's razor
- avoids exotic and ad hoc hypotheses
- uses well-proved physical theories

BRIEF HISTORY OF SCIENTIFIC COSMOLOGY

⓪ Religious speculations about the creation and structure of the World

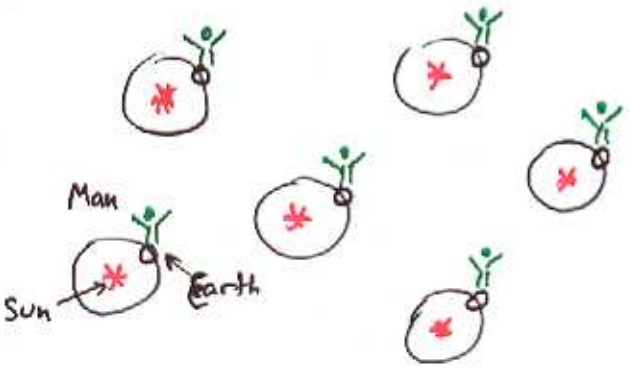
① **Ptolemay** and followers

- finite in space
- finite/infinite in time
- the Earth is the centre
- mathematically elaborated model → predicts ecliples



② **Copernic** ~~Earth~~ SUN is the centre

③ **Giordano Bruno** invents INFINITY (~1600, fired)



- infinite in space /no boundary/
- infinite in time /no creation/
- "cosmic democracy"
  every observers (LGM)
  look SIMILAR world around them
  ↓
  homogeneous and isotropic
- NO CENTER

! Conflict to (new) newtonian physics

general gravity → attracts matter to big clusters
NO FIXED STARS ! (Newton knew...)

↓ Paradoxes (19th century) Olbers (luminosity)
Seliger (gravity)
Boltzmann (thermodynamics)

New telescopes, photography:

- greater world, same structure

④ Revolution(s) in 20th century

A/ Einstein 1915 : GENERAL RELATIVITY

- new geometrical theory of gravity & spacetime

B/ Einstein 1917

- correction to new equations:
  COSMOLOGICAL TERM (Λ)
  - to get the modern form of Bruno's STATIC world

↓

Einstein's Static Cosmological Model

- finite in space
- infinite in time
- NO BOUNDARY
- STATIC

(BUT UNSTABLE...)

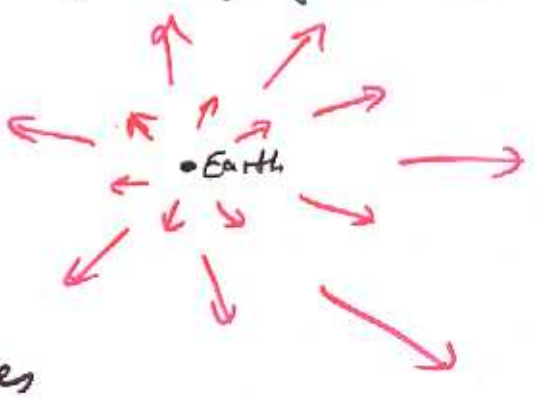
C) Friedman (1921) solves Einstein's original equations (without Λ)

→ **EXPANDING MODELS**

D) Hubble (1929) observes the radially diverging motion of galaxies

$$v = Hr$$

- v: radial velocity
- H: Hubble constant
- r: radial distance



REMARKS

- this is not a MOTION of galaxies but the EXPANSION of space according to Friedman's solution
- there is no centre:
  each observer sees similar world and expansion
  (Bruno's cosmic democracy)

~1930: GEOMETRICAL COSMOLOGY

⑤ Physical cosmology

A) GAMOW (1948) predicts the HOT EARLY UNIVERSE
(Hoyle: BIG BANG)

- in earlyer epochs the Universe was dense and hot
- relicts of this state: radiation of hot plasma
  → now it is transferred to microwave region

B) Penzias & Wilson (1964) observe this radiation: CMBW
Cosmic Microwave Background Radiation

C) Detailed models of early epoch
using physics of hot and dense matter
- plasma physics
- nuclear physics
- physics of elementery particles
- quantum field theory

Reconstruction of the history of the Universe

- history of physical parameters $T(t)$, $p(t)$...
- history of material components (particles, fields)
- emerging STRUCTURES from uniform early state

nuclei
atoms
galaxies
stars
planets
mathematicians
⋮

D) Proliferation of competitor and exotic models
- steady state
- great numbers
- multidimensional
- multiverse
⋮

⑥ Precision cosmology 1990 - 2010 - ....

- New observation instruments
  (eg. space telescopes)
- New observation and data collecting projects
  (SLOAN Sky Survey ~10 Terabyte)
- High capacity computers
  - data acquisition
  - statistical processing
  - numerical simulations
- Perturbative cosmology
  - theoretical calculation of fluctuations around classical models
- Fitting theoretical model parameters to measured data

↓

~2000-2006: ACCURATE (1%) VALUES of cosmological parameters

- coherent results from independent observational methods

↓

Selection of the correct model with proper parameters

↓

Description of main properties of past and present Universe

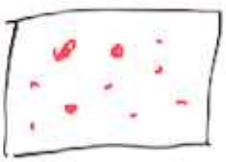
KEYSTONES OF COSMOLOGY

① Astronomical observations

a) Classical observations

α) DISTRIBUTION AND MOTION OF MATTER

basic experience: ISOTROPY (no preferred direction around us)



large scale "galaxy gas"

100 million light years

distribution is isotropic

Hubble constant does not depend on direction

mass density $\sim 10^{-28}$ kg/m^3

(1 proton / 10 m^3)

β) COSMIC MICROWAVE BACKGROUND RADIATION

- isotropic
- thermal $T = 2.725 \pm 0.002$ K

glow of early hot plasma

γ) FREQUENCY OF ISOTOPES OF LIGHT ELEMENTS

created in hot fireball (few minutes after Big Bang)

very sensitive test of cosmological models

b) Modern observations

α) Fluctuations of CMBW

β) Supernovae Ia luminosity

γ) Large scale distribution of galaxies

Detailed discussion: see later

② Physical theories

OCCAM'S RAZOR:

- use ONLY well-proved theories of "local physics"
- do not introduce exotic "cosmic" physical laws and ideas (except if unavoidable)

a) Quantum theory and its daughters:
models of different kinds of matter
(nuclei, plasma, particles, fields ...)

b) Classical & quantum thermodynamics

c) **GENERAL RELATIVITY (GR)**

M: "world" or space-time : (3+1)D pseudo-Riemann manifold

line element: $ds^2 = g_{\mu\nu}(p)\, dx^\mu dx^\nu \quad p \in M$

$g_{\mu\nu} = g_{\nu\mu}$ METRIC TENSOR

tangent space : (3+1)D Minkowski space

In GR there is NO gravitational force

bodies are moving on GEODESIC lines : $x^\mu(\tau)$

$$\frac{d^2x^\mu(\tau)}{d\tau^2} + \Gamma^\mu_{\ell m}\frac{dx^\ell}{d\tau}\frac{dx^m}{d\tau} = 0$$

$\Gamma^\mu_{\ell m}$ ← Levi-Civita connexion coefficients

τ ← affine parameter

What determines the metrics?

Distribution and motion of MATTER – represented by $T_{\mu\nu}(x)$
STRESS-ENERGY TENSOR
contains: energy density, energy flux, stress tensor

$$g_{\mu\nu}(x) \to \frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\sigma} \to R_{\mu\nu} \to R$$

$R_{\mu\nu}$: tensor of curvature; R: Ricci scalar of curvature

EINSTEIN's gravitational field equations:

$$\boxed{R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = \alpha T_{\mu\nu}}$$

← stress-energy tensor

$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$: Einstein tensor; Λ: cosmological constant

$\alpha = \text{const} = \frac{8\pi G}{c^4}$ ← G: Cavendish constant; c: velocity of light

Auxilary equation $p = p(\varepsilon)$
equation of state : it depends on model of matter

③ "Aesthetical" or simplifying assumptions

— make possible the solution of equations
— are based on astronomical observations

a) COSMOLOGICAL PRINCIPLE

(originated from Giordano Bruno ~1600)

principle of "cosmic democracy"

We are NOT the centre of the world
→ the world has NO CENTRE

Combining it with the OBSERVED fact of isotropy
→ **HOMOGENEITY of the world** (in a given instant!)

It's only an assumption → it is to be CHECKED (in models / by observation)

b) Weyl's postulate

World lines of mass points in the cosmic "substrate" (i.e. the galaxies) DO NOT INTERSECT each other (except the past and/or future singularity)
→ the substrate is PERFECT FLUID

Relativistic hydrodynamics:

$$T_{ik} = (\varepsilon + p) u_i u_k - p g_{ik}$$

ε: energy density
p: hydrostatic pressure } of fluid
u_i: 4-velocity vector field

Let CHOOSE a CO-MOVING frame
in which galaxies are IN REST: $u^i = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ← time varies; ← spatial coordinates are constant
these lines are GEODESICS!

→ $T_{ik} = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$ other components are zero

Equation of state: $p = p(\varepsilon)$

CONSTRUCTION OF EINSTEIN-FRIEDMAN (-ROBERTSON-WALKER) FAMILY OF COSMOLOGICAL MODELS

(1) GLOBAL TIME

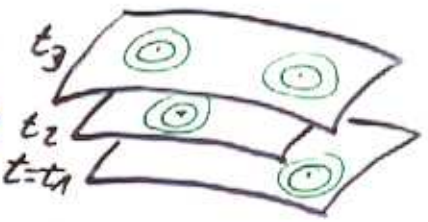
cosmological principle: in a given instant space is isotropic & homogeneous

↓

existence of "a given instant"

In GR it means:

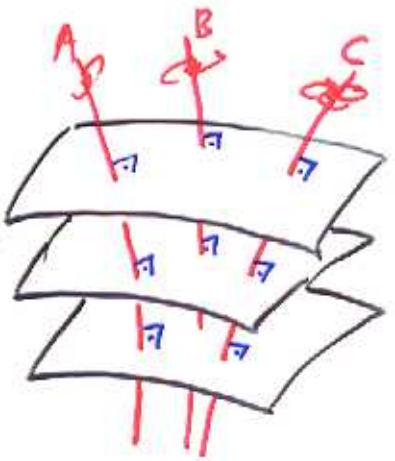
(3+1)D space-time can be FOLIATED into space-like hypersurfaces which are SPHERICALLY SYMMETRIC about ANY point of them



Weyl-postulate: galaxies are IN REST

→ world-lines of galaxies are ORTHOGONAL to the space-like surfaces

+ these are GEODESICS in this metrics (world lines of FREE particles)



↓

$$ds^2 = g_{ik}(x)\,dx^i dx^k = dt^2 - dl^2$$

ds^2: space-time line element

dt: universal time

dl: spatial line element

② ISOTROPIC SPACES

Space-like slices are spherically symmetric about ANY point of them (e.g.: EARTH)

use 3D spherical coordinates:

$$d\ell^2 = a^2\left(dr^2 + S^2(r)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right)$$

- a: scale factor
- r: radial coordinate
- ar: radial distance
- $d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$: line element on a 2D sphere (S^2), $\vartheta \in [0,\pi]$, $\varphi \in [0, 2\pi[$
- $S(r)$: perimeter of a circle (of radius r) : $2\pi a\, S(r)$

$g_{kl} \to R_{kl} \to R \to$ let it be constant!

differential eqs for $S(r)$:

$$S''(r) = -k\,S(r)$$
$$S'(r)^2 + k\,S(r)^2 = 1$$

where $k \in \{1, 0, -1\}$

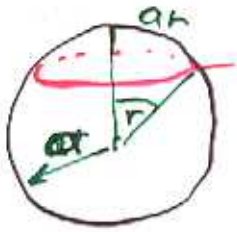
Solutions:

$k = 1$	$S(r) = \sin r$
$k = 0$	$S(r) = r$
$k = -1$	$S(r) = \sinh r$

Case a/ $k=+1$ CLOSED topology of space

$$dl^2 = a^2\left[dr^2 + \sin^2 r\,(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right]$$

line element of S^3: 3D "surface" of a sphere of radius a



$2\pi a \sin r < 2\pi a r$

The "surface" ie the total VOLUME is finite:

$$V = \int_0^{\pi} dr \int_0^{\pi} d\vartheta \int_0^{2\pi} d\varphi \underbrace{a^3 \sin^2 r \sin\vartheta}_{\sqrt{\det g}} = 2\pi^2 a^3$$

Curvature: $\frac{1}{a^2}$

There is NO BOUNDARY, "edge" of a finite spherical space!

Case b) $k=1$ OPEN topology, FLAT geometry

$$dl^2 = a^2\left[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right] = dx^2 + dy^2 + dz^2$$

$x = ar\sin\vartheta\cos\varphi \quad y = ar\sin\vartheta\sin\varphi \quad z = ar\cos\vartheta$

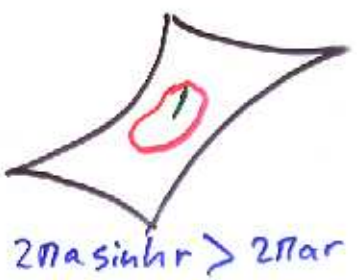
Euclidean, flat space Volume $= \infty$

Curvature $= 0$

Case c) $k=-1$ OPEN topology, HYPERBOLIC geometry

$$dl^2 = a^2\left[dr^2 + \sinh^2 r\,(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right]$$

Line element of 3D Bolyai-Lobatschewski space



$2\pi a \sinh r > 2\pi a r$

Volume $= \infty$

Curvature $= -\frac{1}{a^2}$

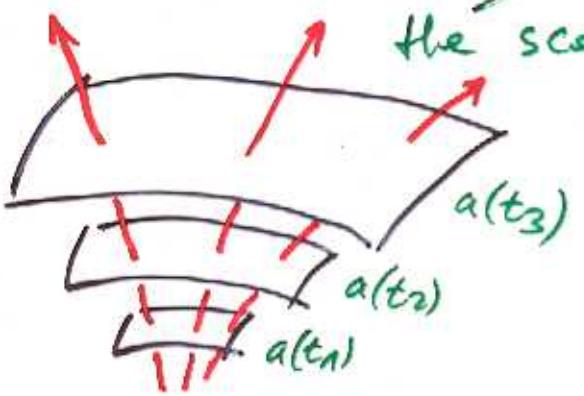
Parameter k = sign of the CURVATURE

③ EXPANDING UNIVERSE

Line element of a sliced (3+1)D Einstein Space-time:

⊛ $ds^2 = dt^2 - dl^2 = dt^2 - a^2(t)\left[dr^2 + S^2(r)(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right]$

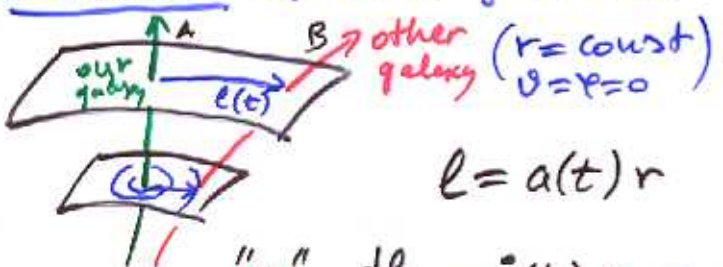
the scale parameter may VARY with time!



⊛ : Einstein-Friedman-Robertson-Walker line-element

REMARK: → rescaling of units → if $t_{NOW} = t_0$ let $a(t_0) = 1$

Distance of extrapolaxies



$l = a(t)\, r$

GALAXY B is IN REST in Weyl's comoving frame

$$"v" = \frac{dl}{dt} = \dot{a}(t)\, r = \frac{\dot{a}(t)}{a(t)}\, a(t)\, r = \frac{\dot{a}(t)}{a(t)}\, l = H(t)\, l$$

$v = Hl$ Hubble's law of expansion

$H = H(t) = \frac{\dot{a}(t)}{a(t)}$ (present value: $H_0 = \frac{\dot{a}(t_0)}{a(t_0)} = \dot{a}(t_0)$)

Hubbles "constant" varies with time!

Deceleration parameter: $q = -\frac{\ddot{a}(t)\, a(t)}{\dot{a}(t)^2}$ dimensionless

e.g: $a \sim t^n \rightarrow q = \frac{1}{n} - 1$

$a \sim e^{t/\tau} \rightarrow q = -1$

⑤ Einstein equations for expanding Universe

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + S^2(r)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right]$$

where $S(r) = \begin{cases} \sin r & k=1 \\ r & k=0 \\ \sinh r & k=-1 \end{cases}$

$\downarrow$

$g_{ik} \longrightarrow R_{ik} \longrightarrow$ put in Einstein equations

$$R_{ik} - \frac{R}{2} g_{ik} - \Lambda g_{ik} = \alpha T_{ik}$$

Stress-energy tensor has Weyl format $T_{ik} = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$

$\rightarrow$

$\overset{0}{0}$ component of Einstein equations:

$$\boxed{3\frac{\dot a^2}{a^2} + 3\frac{k}{a^2} - \Lambda = \alpha\varepsilon} \qquad (1)$$

$\overset{1}{1}, \overset{2}{2}$ and $\overset{3}{3}$ components:

$$\boxed{2\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{k}{a^2} - \Lambda = -\alpha p} \qquad (2)$$

Additional equation: eq. of state:

$$\boxed{p = p(\varepsilon)} \qquad (3)$$

(1–3): EFRW set of cosmological equations

⑤ Entropy et al

Combining (1) and (2) →

$$3\,\dot{a}(\varepsilon+p) = -a\,\dot{\varepsilon} \qquad (4)$$

Using (3): $p(\varepsilon)$ → (4) is a SEPARABLE differential eq:

$$3\frac{da}{a} = -\frac{d\varepsilon}{\varepsilon + p(\varepsilon)}$$

Integrating: $\int 3\frac{da}{a} = \ln a^3 = -\int \frac{d\varepsilon}{\varepsilon + p(\varepsilon)} = -\ln f(\varepsilon) + \text{const}$

$$\boxed{f(\varepsilon)\,a^3 = C} \qquad (5)$$

C ← constant of integration

~(5) is a CONSERVATION LAW

conservation of WHAT ?

Consider a 3D sphere of radius r = const

Volume $V \sim (ar)^3 \sim a^3 \rightarrow 3\frac{da}{a} = \frac{dV}{V}$

(4) → $\frac{3\dot{a}}{a}\varepsilon + \dot{\varepsilon} = -\frac{3\dot{a}}{a}p$

$\varepsilon\frac{dV}{V} + d\varepsilon = -\frac{dV}{V}p$

Let $E = \varepsilon V$ $\qquad \underline{dE = -p\,dV}$ equivalent to (4) or (5)

First Law of Thermodynamics:

$dE = -p\,dV + T\,dS$

(4) or (5) says: $\boxed{dS = 0}$ conservation of ENTROPY

Where does entropy conservation come from?

Hidden inputs

a) — Weyl's postulate: perfect fluid

b) — equation of state $p(\varepsilon)$

There are "revolutionary" epochs in the history of the Universe, when a) & b) are not true: then Universe products entropy via irreversible processes

→ (4) cannot be integrated → numerical simulation

and "evolutionary", "consolidated" epochs:

a) and b) is true

→ (4) can be integrated to (5)

Prof. George Marx described the irreversible epochs.

Solving (5) ⟶ We get $\varepsilon = \varepsilon(a)$

putting into (4)

$$\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)} - \frac{\Lambda}{3} = \frac{\varkappa}{3}\varepsilon(a) \qquad (6)$$

Einstein-Friedman equation

first order separable differential equation

SOLUTION

$$\int dt = t - t_1 = \int_{a_1}^{a} \frac{da}{\sqrt{-k + \frac{\Lambda}{3}a^2 + \frac{\varkappa}{3}a^2\varepsilon(a)}} \qquad (7)$$

EXACT SOLUTION of (1-3) eqs of cosmology!

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{\alpha}{3}\varepsilon(a)$$ EF equation

Parameters:

$k \in \{1, 0, -1\}$ characterizes the geometry (curvature) of space-like slices

$\Lambda \in]-\infty, \infty[$ cosmological constant, its value is given by { God, Einstein, Hipps, ... }

$C > 0$ a constant of integration in (5) → hidden in the function $\varepsilon(a)$

t_0 additive constant of integration, origin of time axis

REMARKS

- A lot of solutions of EFE have one (~~one~~ or two) SINGULAR point(s) on time axis, where $a(t^*) = 0$ This is the BIG BANG (and Big Crunch, if exists). It is convenient to choose ~~then~~ in (7) $t_1 = 0$
- The EF equation includes the eq. of state (3) thus an assomption on the DOMINANT FORM of matter in a given epoch. This may vary against epochs → the calculated solutions must be fitted to each other. The "interregnums", the changes of dominant matter coincide with the "revolutionary" epochs of entropy production.

SPECIAL MODELS OF MATTER

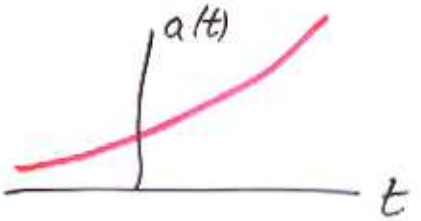
① PURE VACUUM

$$\varepsilon = p = 0$$

(1) gives: $\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{\Lambda}{3}$

especially $k=0$, $\Lambda > 0$ $T = \sqrt{3/\Lambda}$:

the solution: $a(t) = a_0 e^{t/T}$



DE SITTER model

expanding space without matter

Einstein was sad: de Sitter model ⟷ Mach's principle

Mach: matter CAUSES the properties of space-time

→ without matter NO space-time,
NO solution of EFEs ...

② DUST-LIKE MATERIAL

disjoint mass points, rarely in space

No collision → no interaction → no pressure

Equation of state (3): $p = p(\varepsilon) = 0$

Integrating to (5) $\varepsilon a^3 = C \quad (= \varepsilon(t_0) = \varepsilon_0)$

→ $E = \varepsilon V = \text{const}$: energy conservation

It is a good model for **PRESENT UNIVERSE**:
collisionless gas of galaxies

($E = mc^2 \rightarrow \varepsilon \sim \rho c^2$)

↑ total matter in a volume

(3) RADIATION

Classical thermodynamics, Stefan-Boltzmann law:

(3) $p = \varepsilon/3$ eq. of state of radiation

→ (5) $\varepsilon a^4 = C$

$\varepsilon \sim 1/a^4 = \frac{1}{a^3} \cdot \frac{1}{a}$

dilution of photon gas · redshift of a photon

$E_{ph} \sim \nu \sim \frac{1}{\lambda}$ $\lambda \sim a$

$E_{ph} \sim \frac{1}{a}$

Thermal history of Universe

~"interregnum": fit 2 solutions!

④ SCALAR FIELD (Higgs field, inflaton, quintessence ...)

$\phi(x) : M \to \mathbb{R}$

Equation of field: $\nabla_k \nabla^k \phi(x) = -V'(\phi(x))$

nonlinear Klein-Gordon equation

where $V(\phi) \geq 0$ potential energy of self-interaction of ϕ

Field theory → stress-energy tensor:

$$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu} \cdot \frac{1}{2}(\partial_\rho \phi)(\partial^\rho \phi) + V(\phi) g_{\mu\nu}$$

Cosmological principle: spatial derivative of $\phi = 0$
(ϕ is the same in a spacelike slice)

ASSUMING: $\frac{\partial \phi}{\partial t}$ is small (negligible) : $\phi \approx \phi_0 = \text{const}$

$$\to \quad T_{\mu\nu} = V(\phi_0) \cdot g_{\mu\nu}$$

Einstein equation:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = \alpha T_{\mu\nu} = \alpha V(\phi_0) g_{\mu\nu}$$

ordering

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = [\Lambda + \alpha V(\phi_0)] g_{\mu\nu}$$

$V(\phi_0)$ **simulates** the cosmological constant Λ
in epochs when $\phi \approx \phi_0 = \text{const}$

This is the **INFLATON field** theory of Λ:
a dynamical model of Λ

Scalar field (inflaton) is a MATTER which causes the effects of cosmological constant

VERY SPECIFIC material!

(1) $3\frac{\dot{a}^2}{a^2} + \frac{3k}{a^2} = \alpha(\varepsilon + V(\phi_0))$

(2) $2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -\alpha(p - V(\phi_0))$

a matter of positive energy density (ε) and negative pressure → antigravity...

DIFFERENT SOLUTIONS OF EF EQUATION

→ COSMOLOGICAL MODEL FAMILY FOR THE PRESENT UNIVERSE

In PRESENT EPOCH material is assumed to be DUST-LIKE (collisionless → pressureless gas of galaxies)

(3) $p=0 \longrightarrow$ (5) $\varepsilon = \frac{C}{a^3}$ where $C = \varepsilon(a=1) = \varepsilon(t_0) = \varepsilon_0$

t_0 = now

EINSTEIN-FRIEDMAN EQUATION:

$$\boxed{\frac{\dot a^2(t)}{a^2} = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\varkappa}{3}\frac{C}{a^3}} \quad (7)$$

Parameters: k, Λ, C — cannot be measured directly

We use the combinations of PRESENT values of data

Let $t = t_0 =$ today

→ $a = a(t_0) = 1$ (scaled)

$\frac{\dot a^2}{a^2} = \dot a^2(t_0) = H_0^2$: present value of Hubble-constt

$C = \varepsilon(t_0) = \varepsilon_0 > 0$ present value of energy density

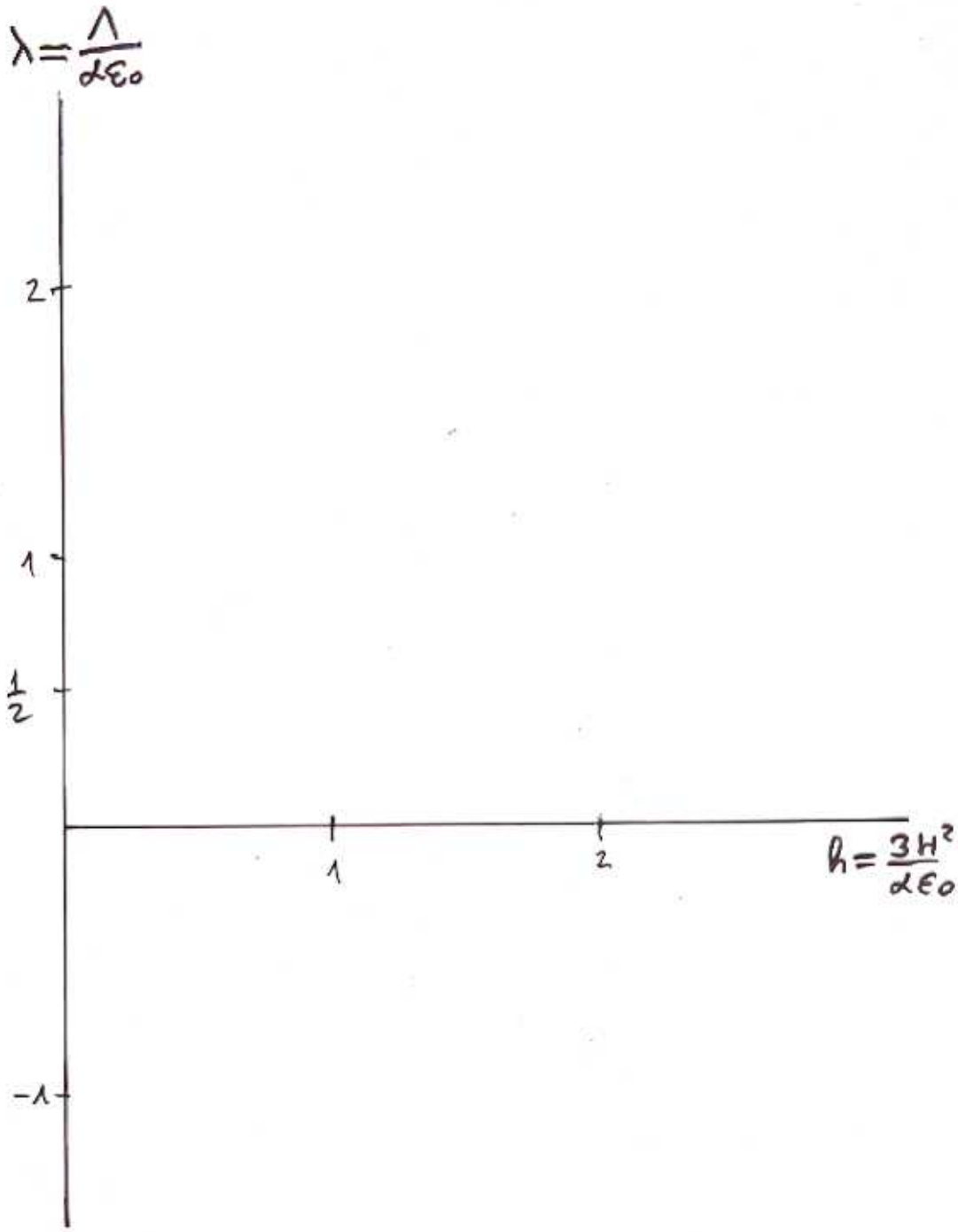
(7) → $\boxed{H_0^2 = -k + \frac{\Lambda}{3} + \frac{\varkappa \varepsilon_0}{3}}$ / $\cdot \frac{3}{\varkappa \varepsilon_0}$

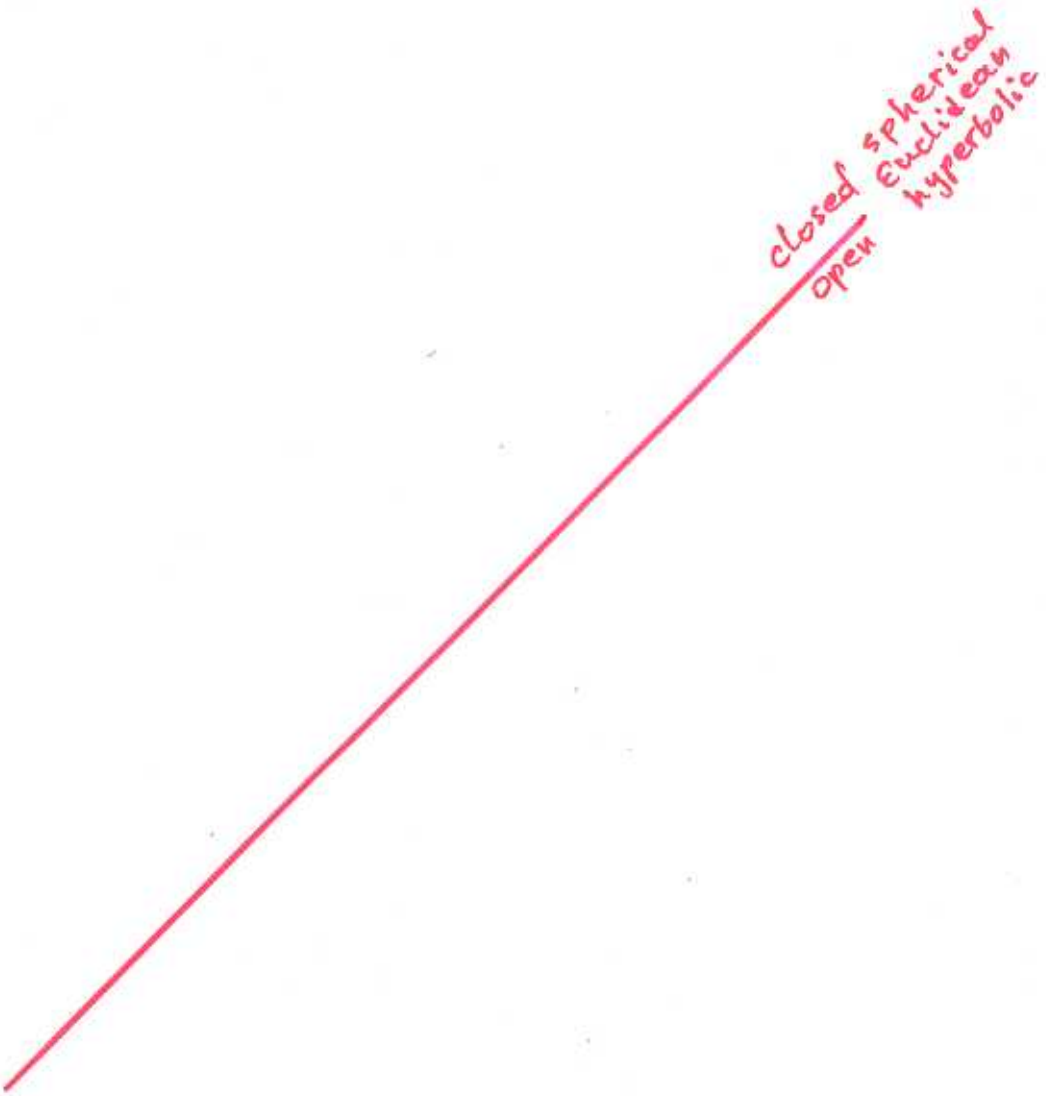
$$\frac{3H_0^2}{\varkappa\varepsilon_0} = -\frac{3k}{\varkappa\varepsilon_0} + \frac{\Lambda}{\varkappa\varepsilon_0} + 1$$

Defs: $h = \frac{3H_0^2}{\varkappa\varepsilon_0} > 0$, $\varkappa = \frac{3k}{\varkappa\varepsilon_0}$, $\lambda = \frac{\Lambda}{\varkappa\varepsilon_0}$

$$\boxed{\varkappa = \lambda + 1 - h}$$

PLOT models on λ-h plane!


$\lambda = \frac{\Lambda}{\alpha \varepsilon_0}$
2
1
$\frac{1}{2}$
-1
1
2
$h = \frac{3H^2}{\alpha \varepsilon_0}$



$k \sim \mathcal{X} = \lambda + 1 - h \longrightarrow$

$\lambda > h-1$	$k=+1$	closed	spherical
$\lambda = h-1$	$k=0$	Euclidean	
$\lambda < h-1$	$k=-1$	hyperbolic	

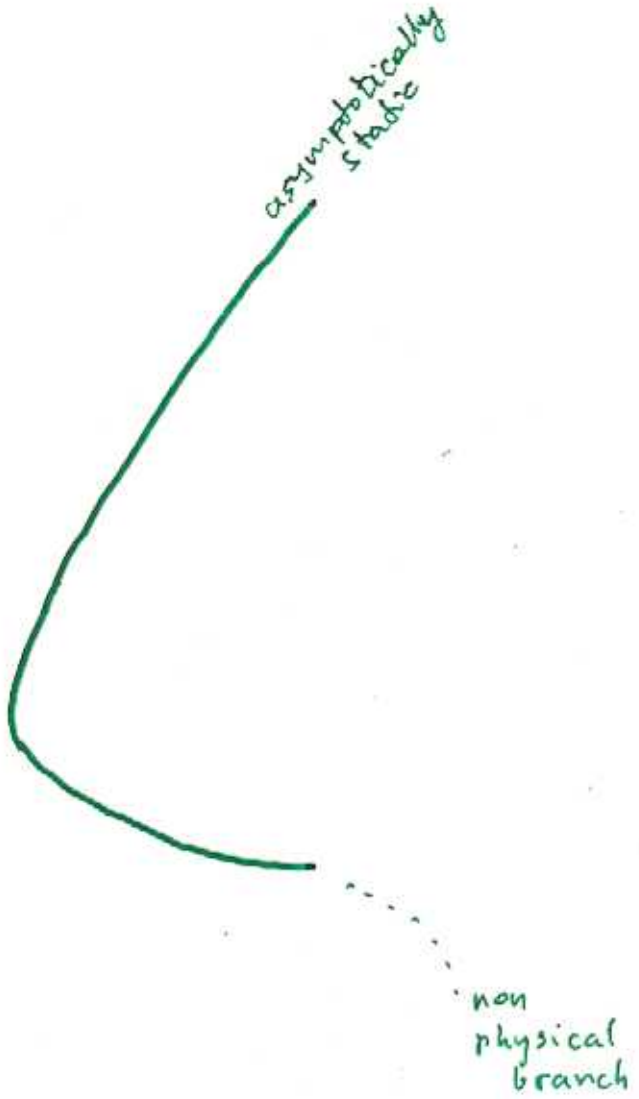
now accelerates
now decelerates

Deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1/2 - \lambda}{h}$$

differentiating (7)

$\lambda < \frac{1}{2}$	$q > 0$	now decelerates
$\lambda = \frac{1}{2}$	$q = 0$	There is NO acceleration now
$\lambda > \frac{1}{2}$	$q < 0$	now accelerates



Asymptotic behaviour of solutions

There are solutions, in which $t \to \infty$ or $t \to -\infty$

$$a(t) \longrightarrow A = \text{const}$$

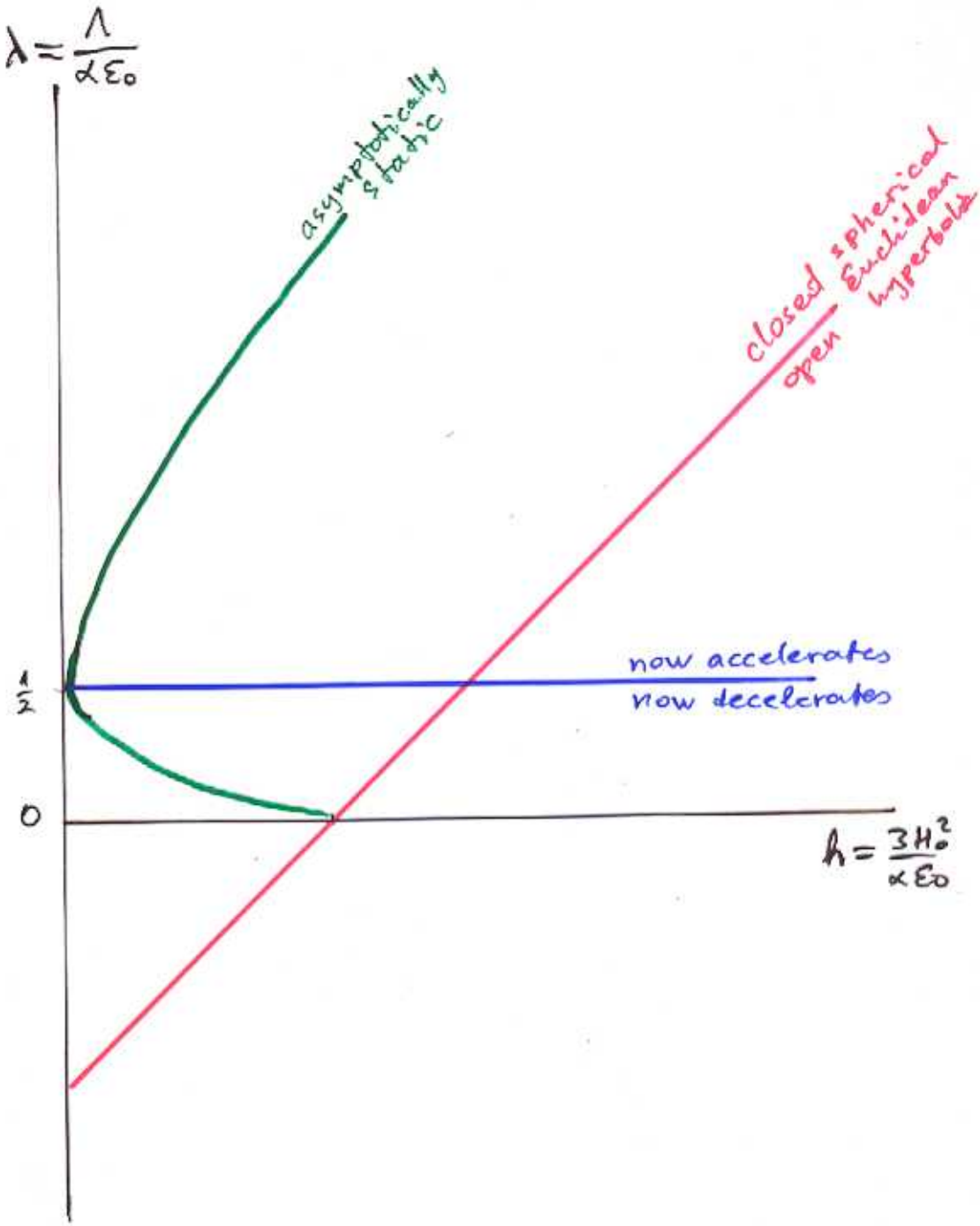
In such situation $\dot{a} = 0$ and $\ddot{a} = 0$

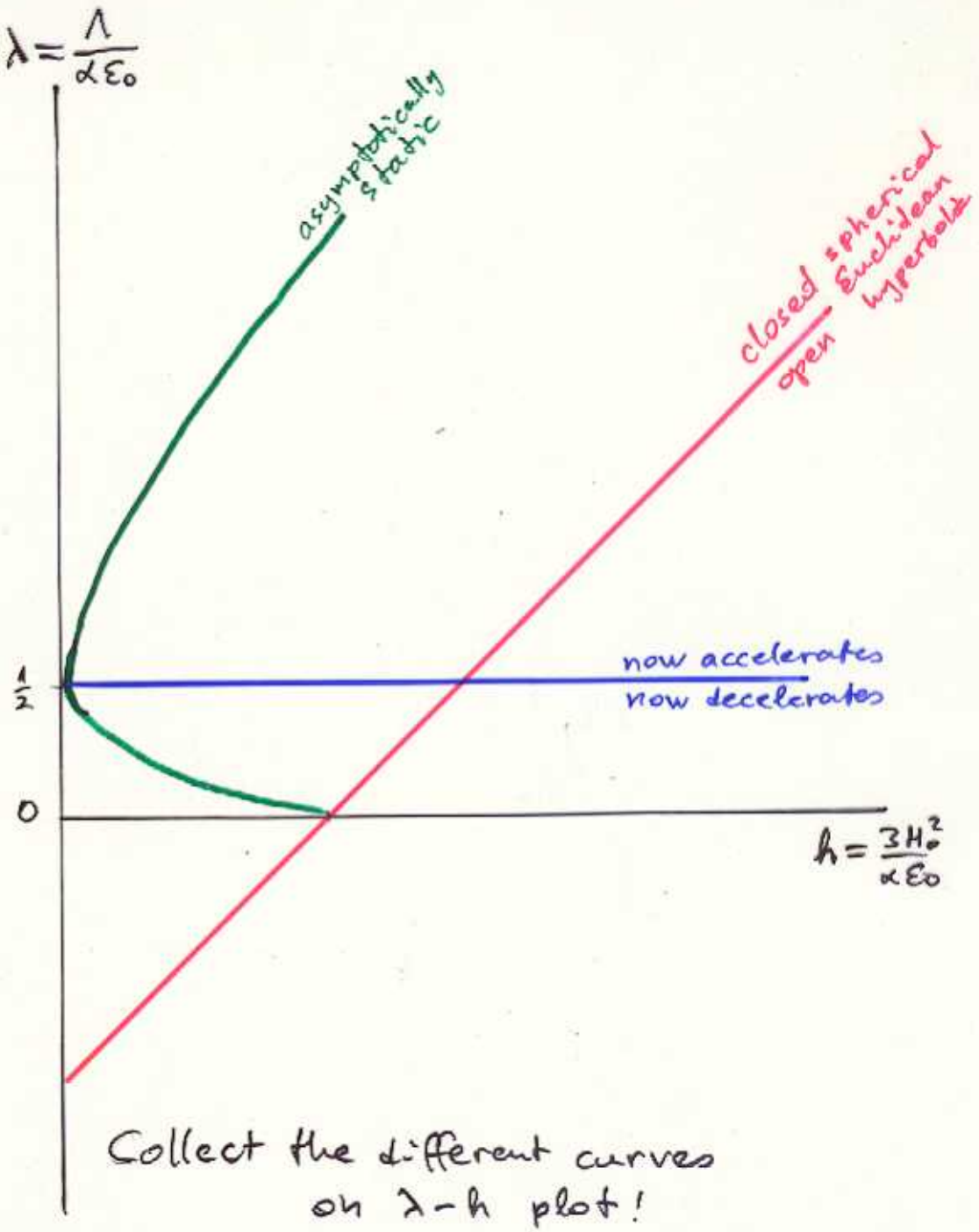
$$(1) \rightarrow \frac{3k}{A^2} - \Lambda = \alpha\, \varepsilon(A) = \frac{\alpha \varepsilon_0}{A^3}$$

$$(2) \rightarrow \frac{k}{A^2} - \Lambda = -\alpha p = 0$$

Eliminating k and A: $\lambda = \frac{4}{27}(\lambda + 1 - h)^3$

plot ↑


$\lambda = \frac{\Lambda}{\alpha \varepsilon_0}$
asymptotically static
closed spherical
Euclidean
open hyperbolic
now accelerates
now decelerates
$\frac{1}{2}$
0
$h = \frac{3H_0^2}{\alpha \varepsilon_0}$



Collect the different curves on $\lambda - h$ plot!

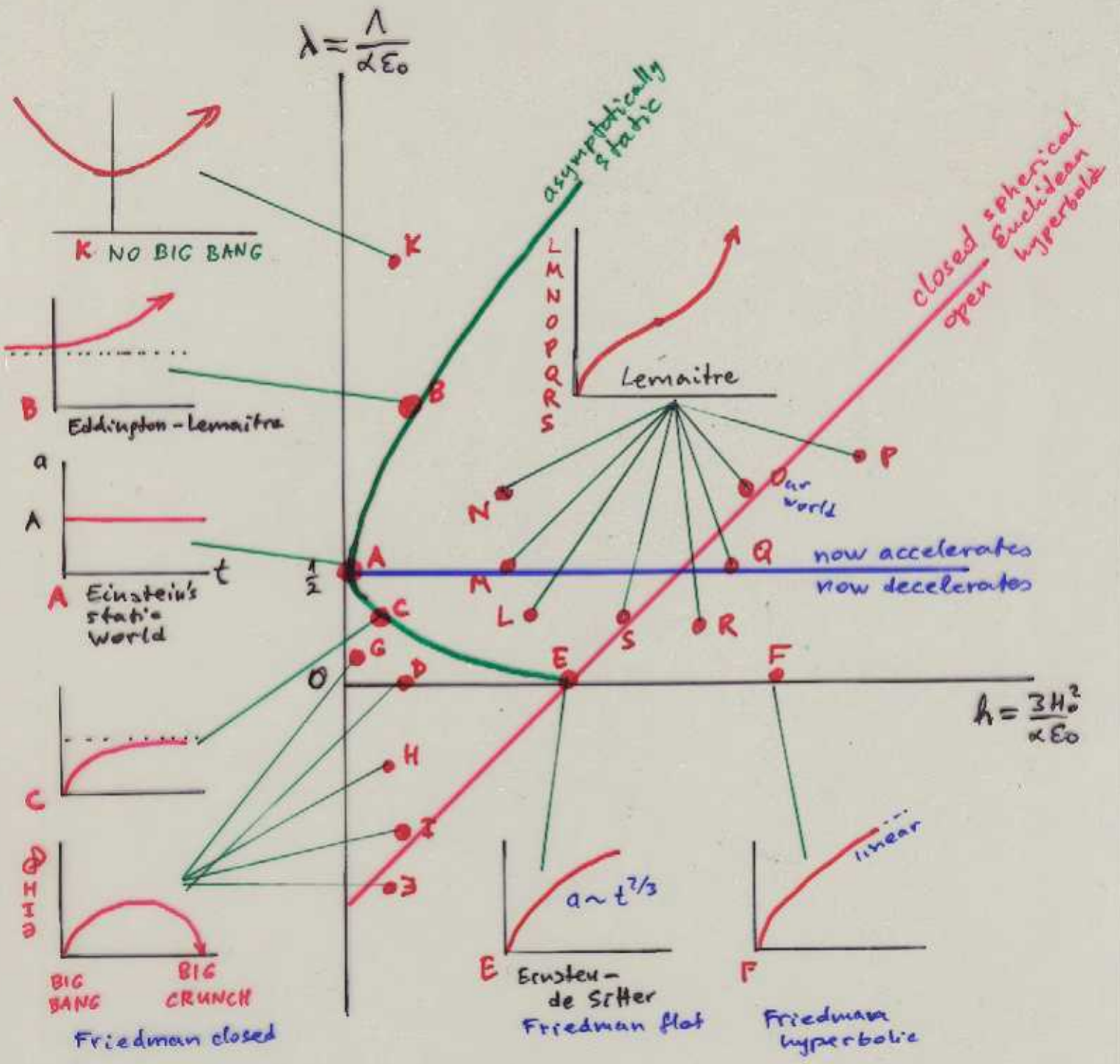
Look the qualitative solutions of EF equations for different values of λ and h:

$$\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{1}{3} + \frac{\alpha C}{3a^3} = \frac{\alpha C}{3}\left(-\frac{\varkappa}{a^2} + \lambda + \frac{1}{a^3}\right)$$

But $\varkappa = \lambda + 1 - h$

$$\frac{d\,a(H_0 t)}{d(H_0 t)} = \sqrt{\frac{\lambda}{h}(a^2 - 1) + \frac{1}{h}\left(\frac{1}{a} - 1\right) + 1}$$

$\frac{1}{H_0} \sim$ time scale $\sim$ 20 billion years


$\lambda = \frac{\Lambda}{\alpha \varepsilon_0}$
asymptotically static
closed spherical
open Euclidean
hyperbolic
K NO BIG BANG
K
B
Eddington - Lemaitre
L M N O P Q R S
Lemaitre
P
N
Our world
Q
now accelerates
now decelerates
a
Λ
t
A
Einstein's static world
½
A
M
L
S
R
C
G
O
D
E
F
$h = \frac{3H_0^2}{\alpha \varepsilon_0}$
H
I
J
C
BIG BANG
BIG CRUNCH
Friedman closed
$a \sim t^{2/3}$
E
Einstein - de Sitter
Friedman flat
linear
F
Friedman hyperbolic

REMARK

$a(t)$: curves were calculated in the frame of EFRW models, using

- DUST-LIKE equation of state ($p=0$)
- constant Λ

They are not valid:

- around the Big Bang ($p \neq 0$)
- $t \to \infty$ (if Λ is not constant)
- in other models

They are valid (probable)
between 10^{-20} s do 10^{20} s after Big Bang
(1000 billion years)

MODERN OBSERVATIONAL PROJECTS AND CALCULATIONS

(1) Fluctuations of Cosmic Microwave Background Radiation

CMBR: radiation of the ancient fireball
- ~ half million years after Big Bang
- → Doppler-shift to microwave region

$T = 2.725 \pm 0.002$ K

measured by probes:
- COBE ~1990
- WMAP ~2000
- Planck ~2010

→ detect and measure fluctuations

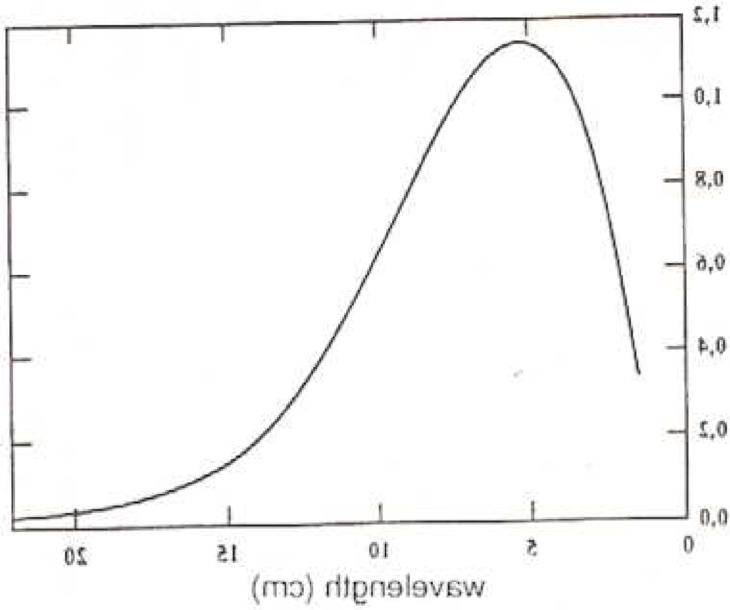
Theoretical models:
- quantum fluctuations before 10^{-34} s
  - → fluctuations of energy density / curvature
- inflation: fluctuations blow up to cosmic size
- temperature fluctuations in thermal epochs
- fluctuations in the temperature of CMBR

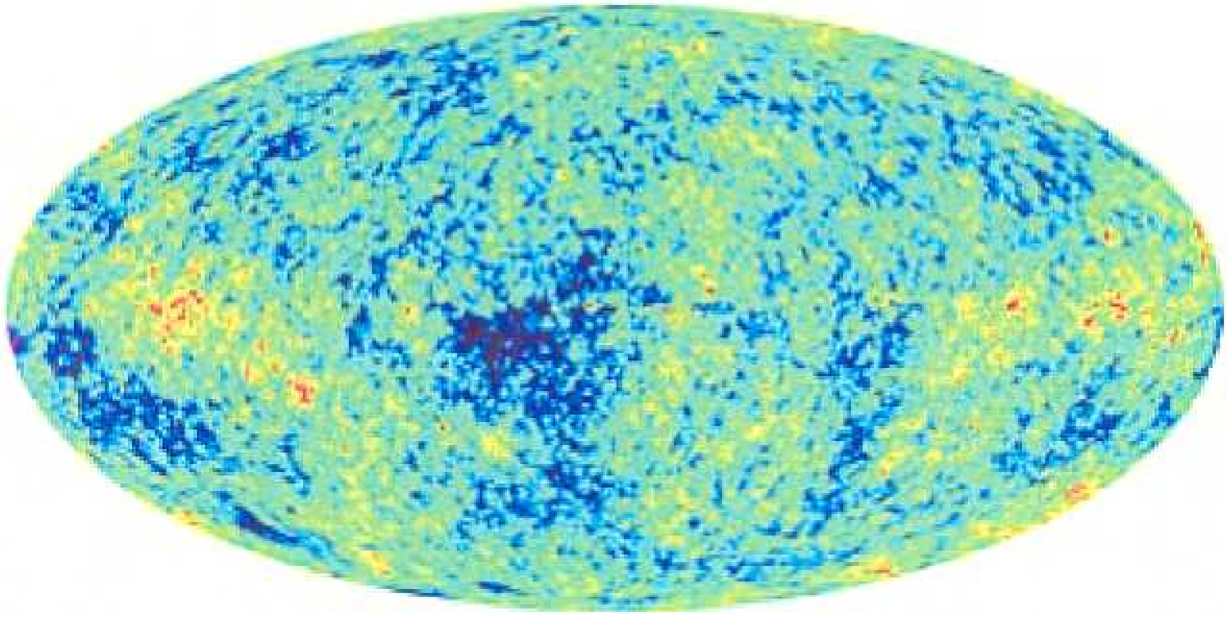
→ wavelength, amplitude, correlations

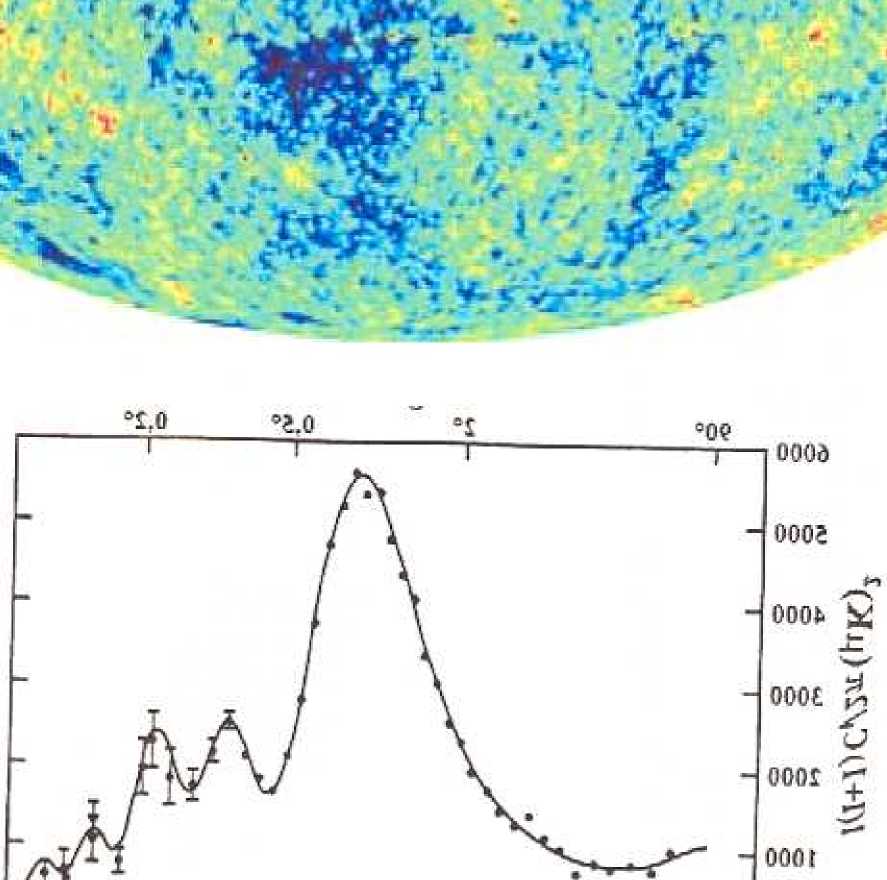
depend on parameters of expansion scenario

FITTING THE MODELS TO OBSERVED CURVES

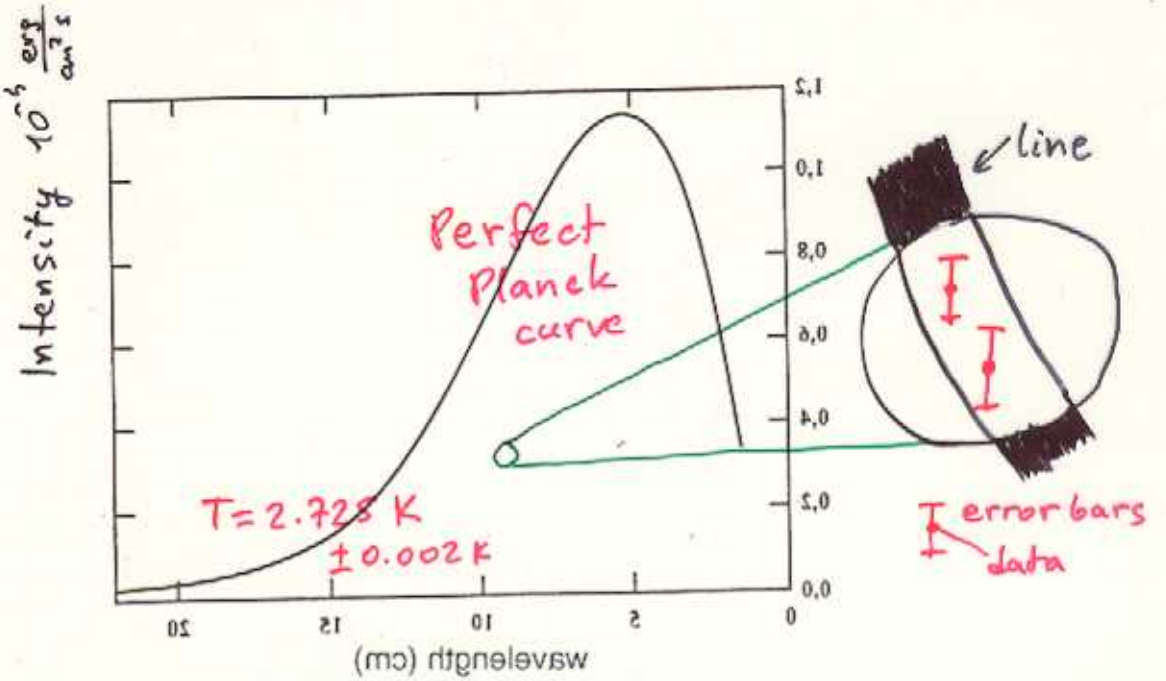
↓ SUBSET OF PARAMETER SPACE


wavelength (cm)

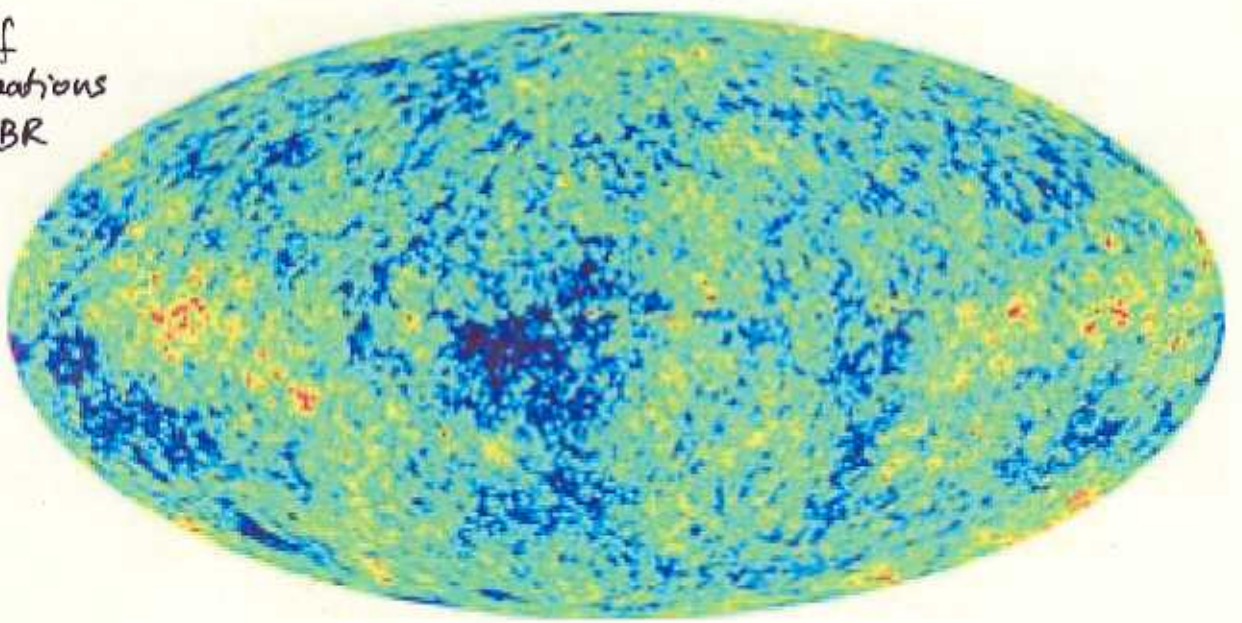


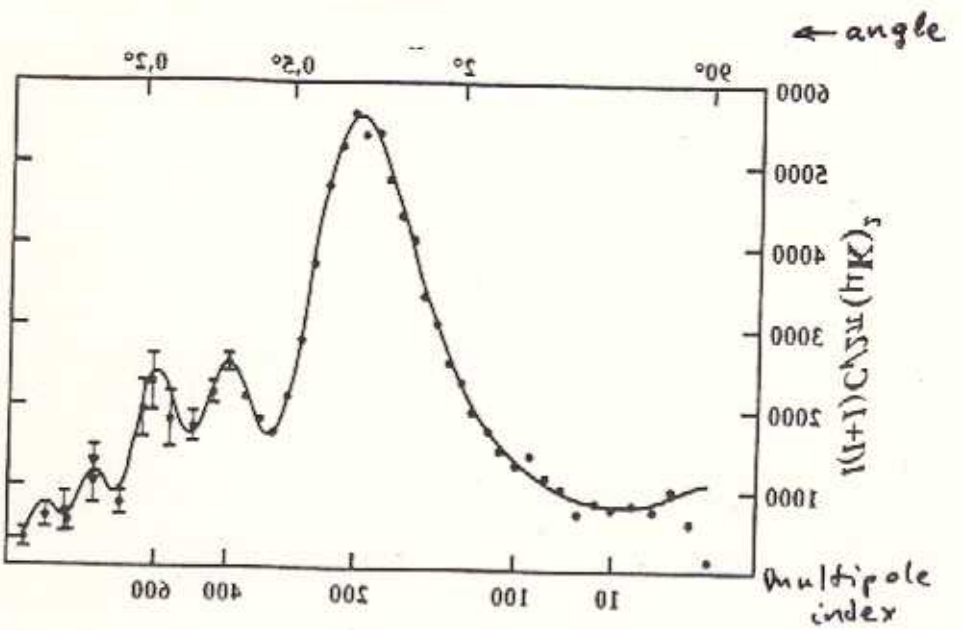


COBE and WMAP data of CMBR



Map of fluctuations in CMBR (WMAP data)

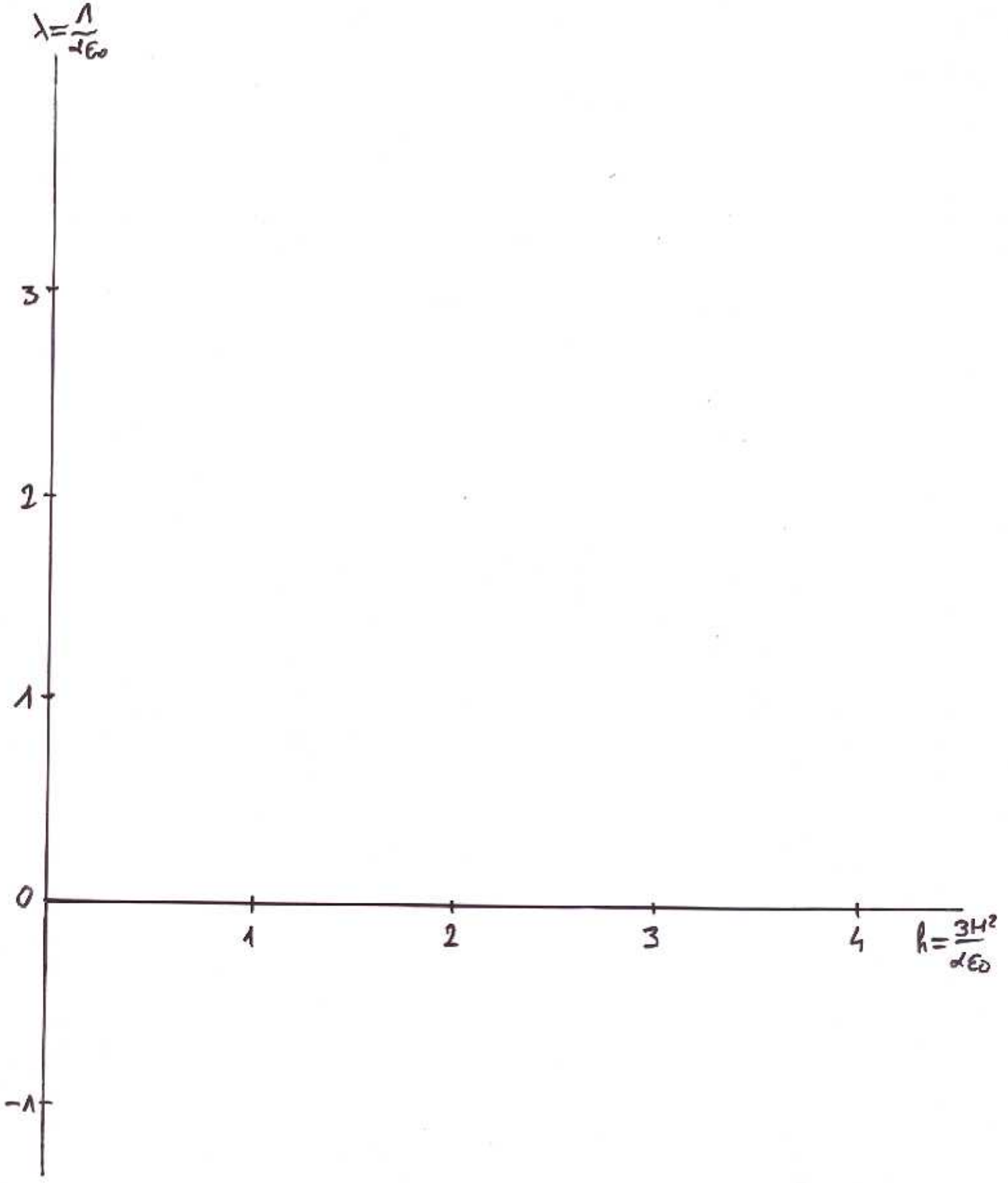


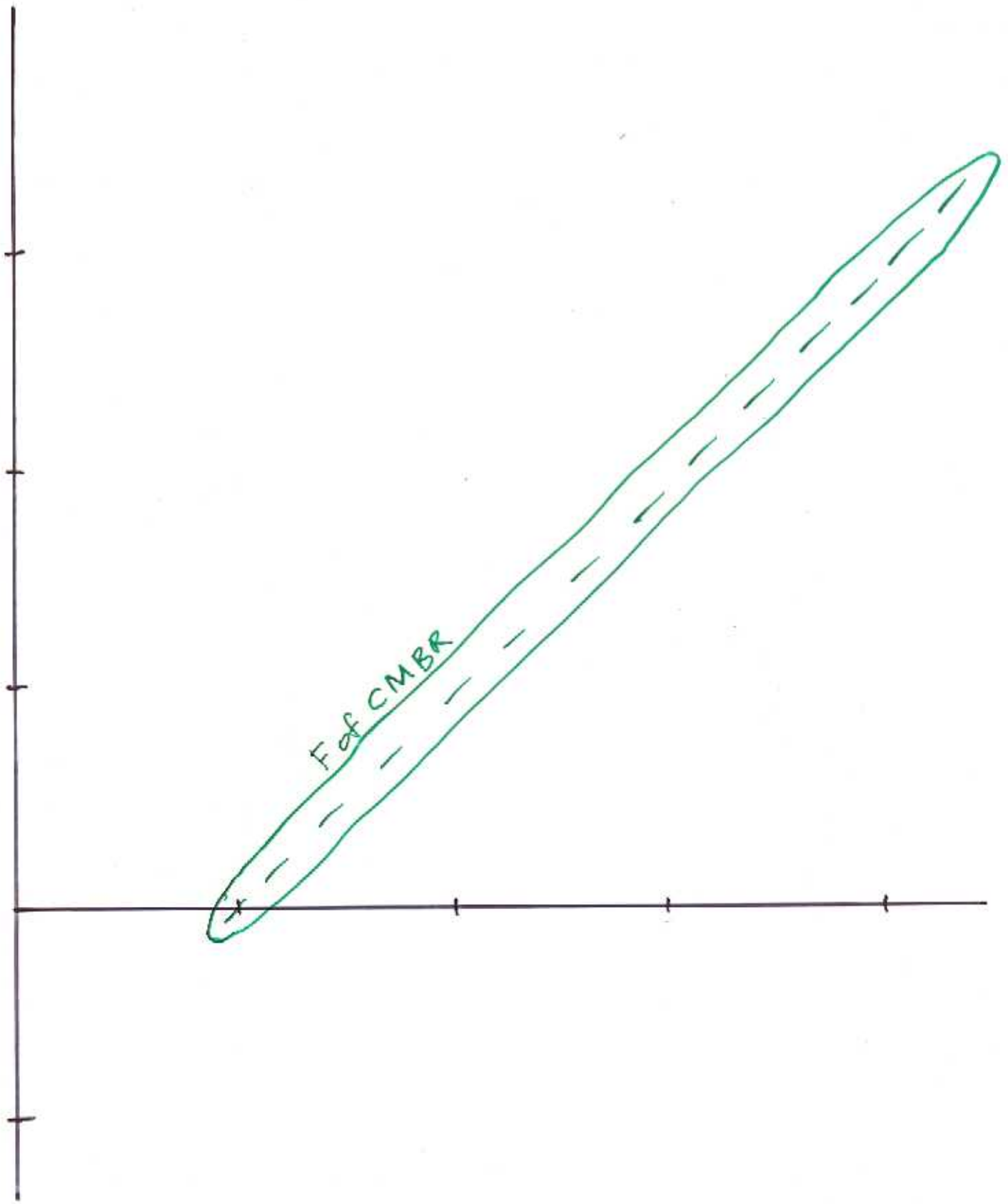


Angular correlation of relative temperature fluctuations

—— theory (fitted)

* I measurement of WMAP


$\lambda = \frac{\Lambda}{\alpha \epsilon_0}$
3
2
1
0
-1
1
2
3
4
$h = \frac{3H^2}{\alpha \epsilon_0}$


F of CMBR

② Supernovae (SN Ia) observations

Supernovae: exploding stars, very bright

SN Ia type supernovae: "standard candles"
fix total luminosity

We do not know
- when has it exploded
- what is its distance
- the scenario of the expansion

We can measure
- apparent luminosity (L)
- red shift (z)

Assumed parameter: time of explosion (t)

INPUT: scenario of cosmic expansion → functions $L(t)$, $z(t)$

eliminate t → function $L(z)$ for each scenario

Measuring $L(z)$ → fitting cosmological parameters

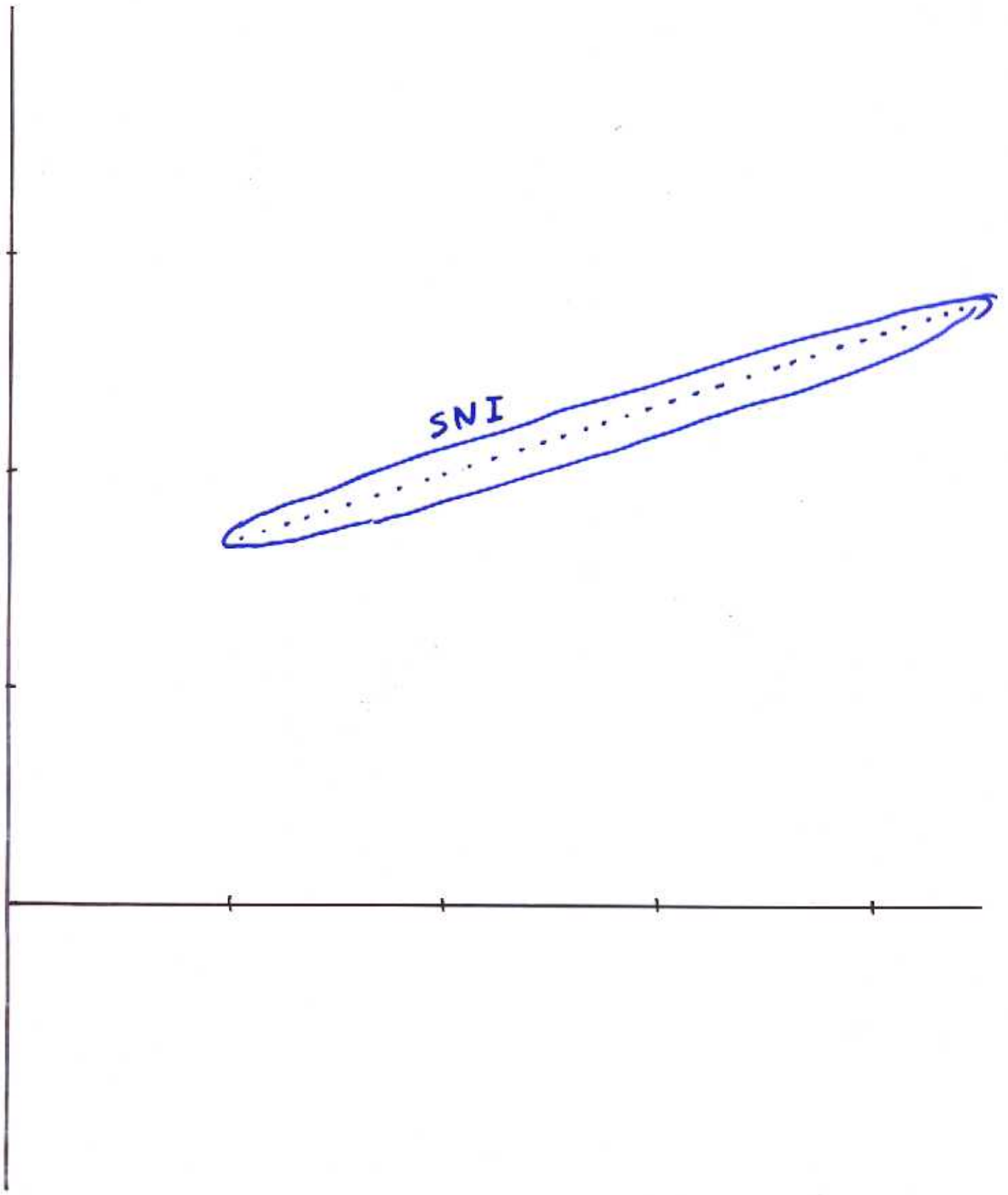
→ SUBSET OF PARAMETER SPACE

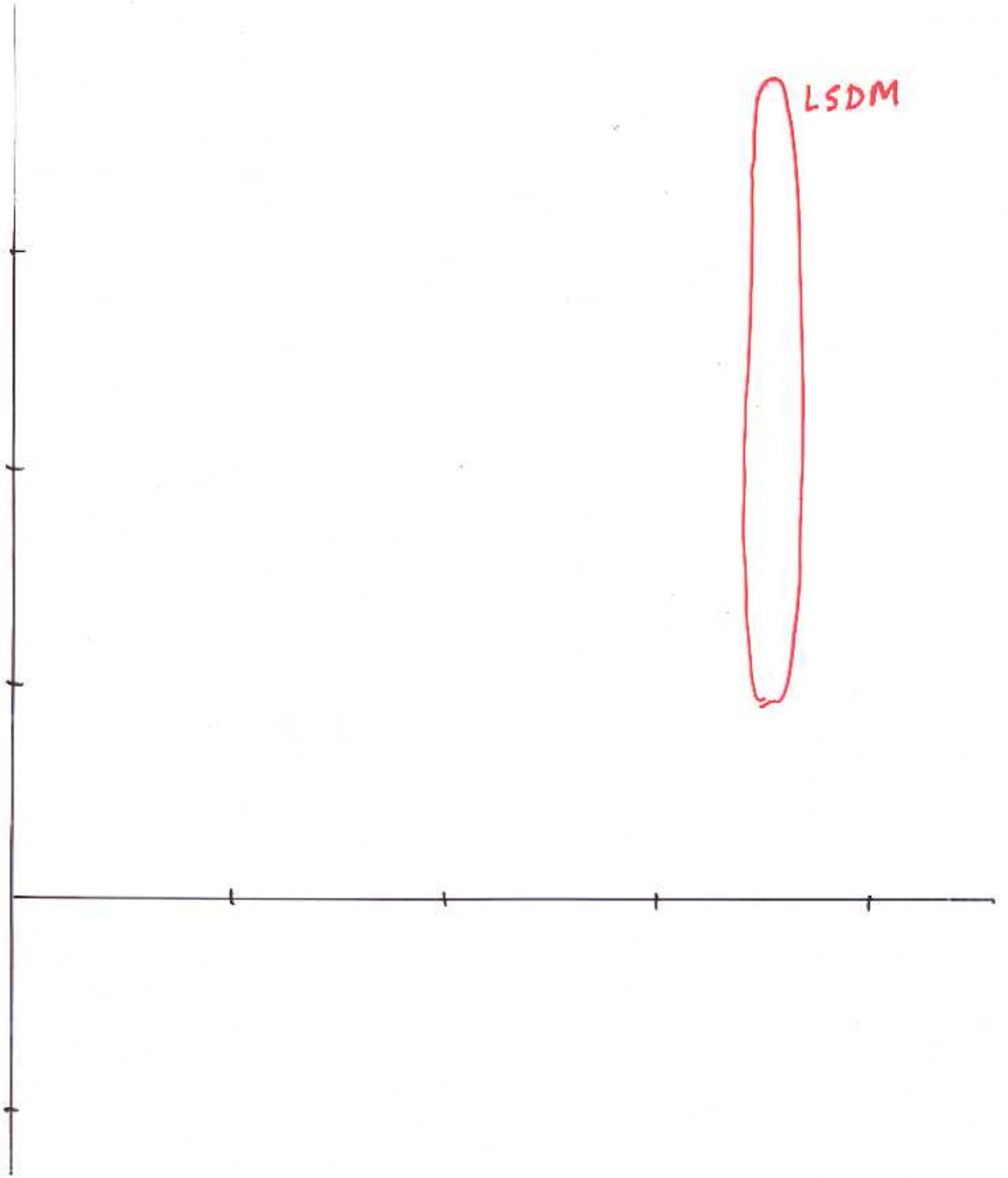
③ Large scale distribution of matter

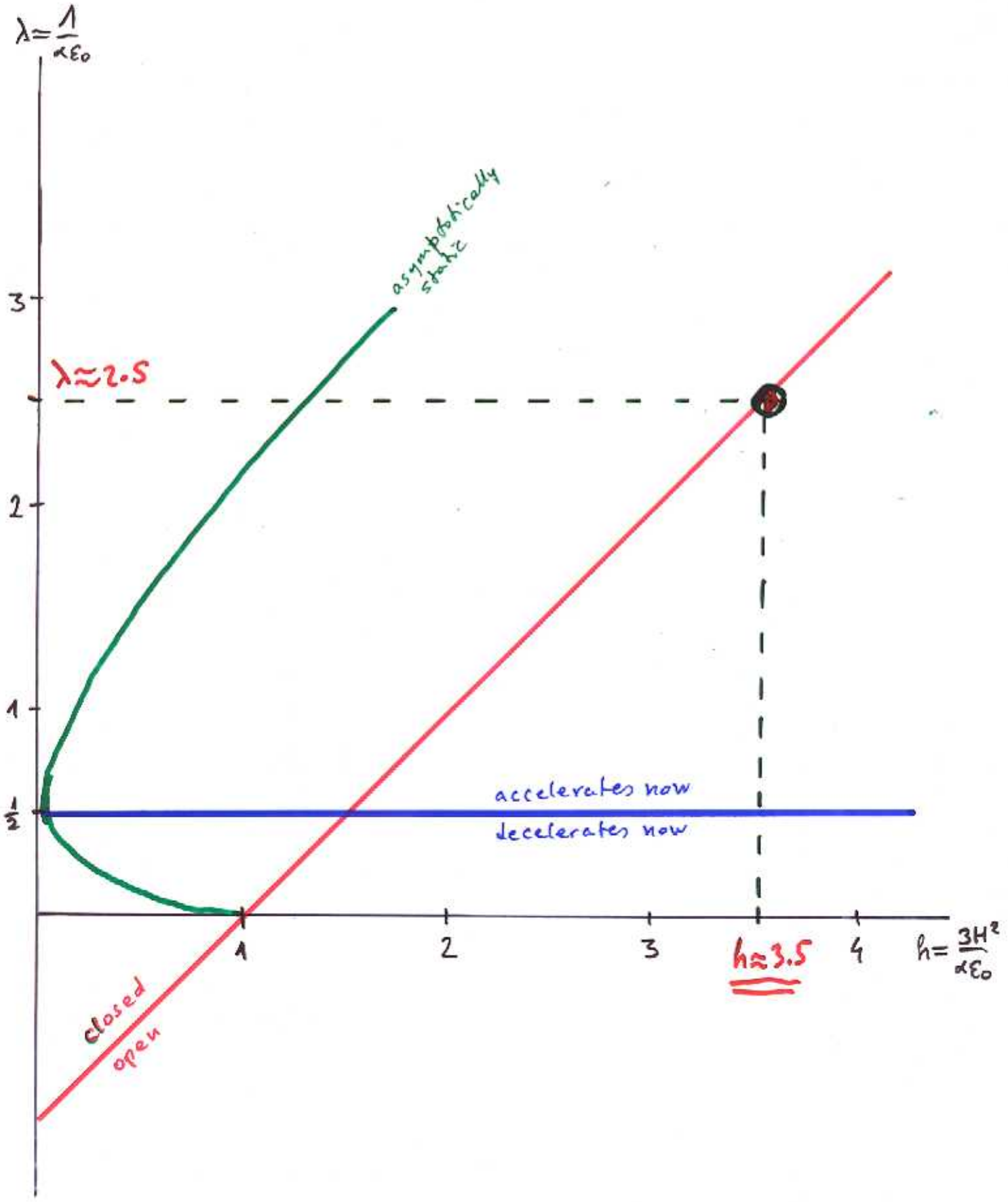
mapping the Universe (Sloan Digital Sky Survey)
10 Tbyte date (4 GB/s for 5 years)
real 3D-map of galaxy distribution

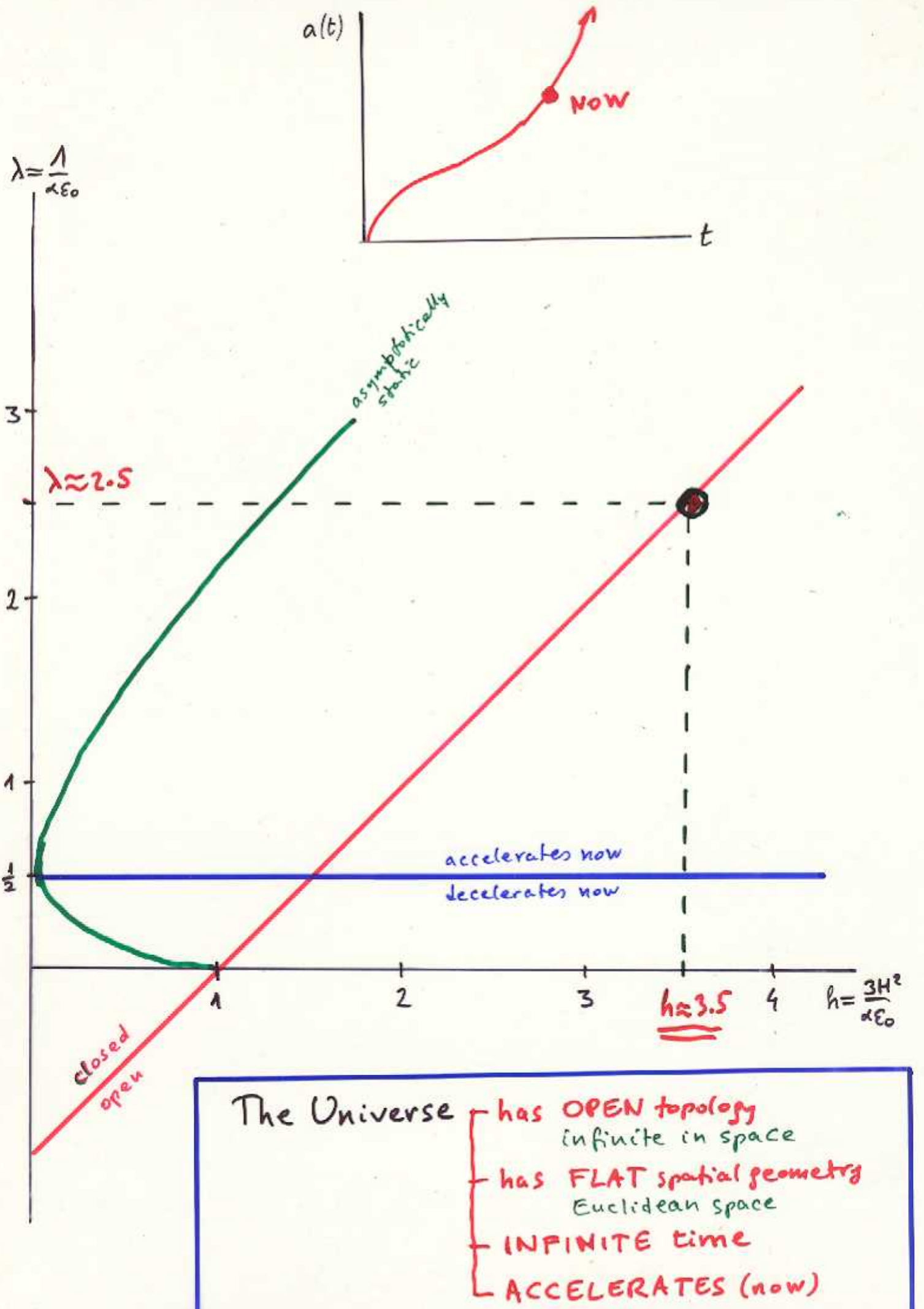
→ calculation of 2- and 3-point correlation functions
→ fitting cosmological parameters
→ PERMITTED SUBSET OF PARAMETER SPACE

1 + 2 + 3 : three different, INDEPENDENT methods


SNI


LSDM


$\lambda = \frac{\Lambda}{\alpha \varepsilon_0}$
asymptotically static
3
$\lambda \approx 2.5$
2
1
$\frac{1}{2}$
accelerates now
decelerates now
1
2
3
$h \approx 3.5$
4
$h = \frac{3H^2}{\alpha \varepsilon_0}$
closed
open



The Universe
- has OPEN topology
  infinite in space
- has FLAT spatial geometry
  Euclidean space
- INFINITE time
- ACCELERATES (now)

INTERPRETATION OF RESULTS

Precision cosmological measurements → $h \approx 3.5$, $\lambda \approx 2.5$ parameters of EFRW family of models

accuracy ~ 5% ↓ 1%

2006-2007

Supplementary result:
the age of the Universe : 13.7 billion years

EF equation: $\left(\frac{\dot{a}}{a}\right)^2 = H^2 = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\alpha\varepsilon}{3}$

$$\frac{k}{a^2} = \frac{\alpha\varepsilon}{3} + \frac{\Lambda}{3} - H^2 = \frac{\alpha}{3}\left(\varepsilon + \frac{\Lambda}{\alpha} - \frac{3H^2}{\alpha}\right)$$

$\frac{\Lambda}{\alpha} = \varepsilon_\Lambda$; $\frac{3H^2}{\alpha} = \varepsilon_{critical}$

Scalar field: $\frac{\Lambda}{\alpha} = V(\phi_0)$

$k > 0$	$\varepsilon + \varepsilon_\Lambda > \varepsilon_{crit}$	too much matter → closed space
$k = 0$ ⟺	$\varepsilon + \varepsilon_\Lambda = \varepsilon_{crit}$	just enough matter
$k < 0$	$\varepsilon + \varepsilon_\Lambda < \varepsilon_{crit}$	too few matter → open space

$\frac{\varepsilon}{\varepsilon_{crit}} = \frac{\alpha\varepsilon}{3H^2} = \frac{1}{h} \sim 0.28$

$\frac{\varepsilon_\Lambda}{\varepsilon_{crit}} = \frac{\Lambda}{3H^2} = \frac{\lambda}{h} \sim 0.72$

$\frac{\varepsilon + \varepsilon_\Lambda}{\varepsilon_{crit}} \sim 1$ just enough matter to flat the space

28% of them ~ ordinary matter
72% of them ~ cosmological constant or DARK ENERGY ($\sim V(\phi_0)$) or INFLATON or QUINTESSENCE

only 4% is the light emitting matter (stars..)

REMARK

curious:
- fluctuations of CMBW → universe is flat
- supernovae → acceleration
- large scale structure → value of h

together: accurate values of parameters

CONSEQUENCES OF MEASUREMENTS

(1) Einstein-Friedman family of cosmological models is ENOUGH to describe the PRESENT (and PAST) Universe

- using only 2 parameters of family ALL measured data (including complete functions) can be fitted successfully
- Occam's razor: there is NO NECESSITY to introduce and use various exotic cosmological models
  until — one of them can reproduce the upper results
  — contradictions occur in the standard model

(2) **Successful fit gives the parameters and properties of the present Universe**

IDENTITY CARD

photo: see below

NAME: OUR UNIVERSE

AGE: 13.7 BILLION YEARS

TOPOLOGY: OPEN (INFINITE IN SPACE)

SPATIAL GEOMETRY: FLAT, EUCLIDEAN

EXPANSION RATE: NOW ACCELERATING

EXPECTATION OF LIFE: INFINITE

MATERIAL COMPOSITION:

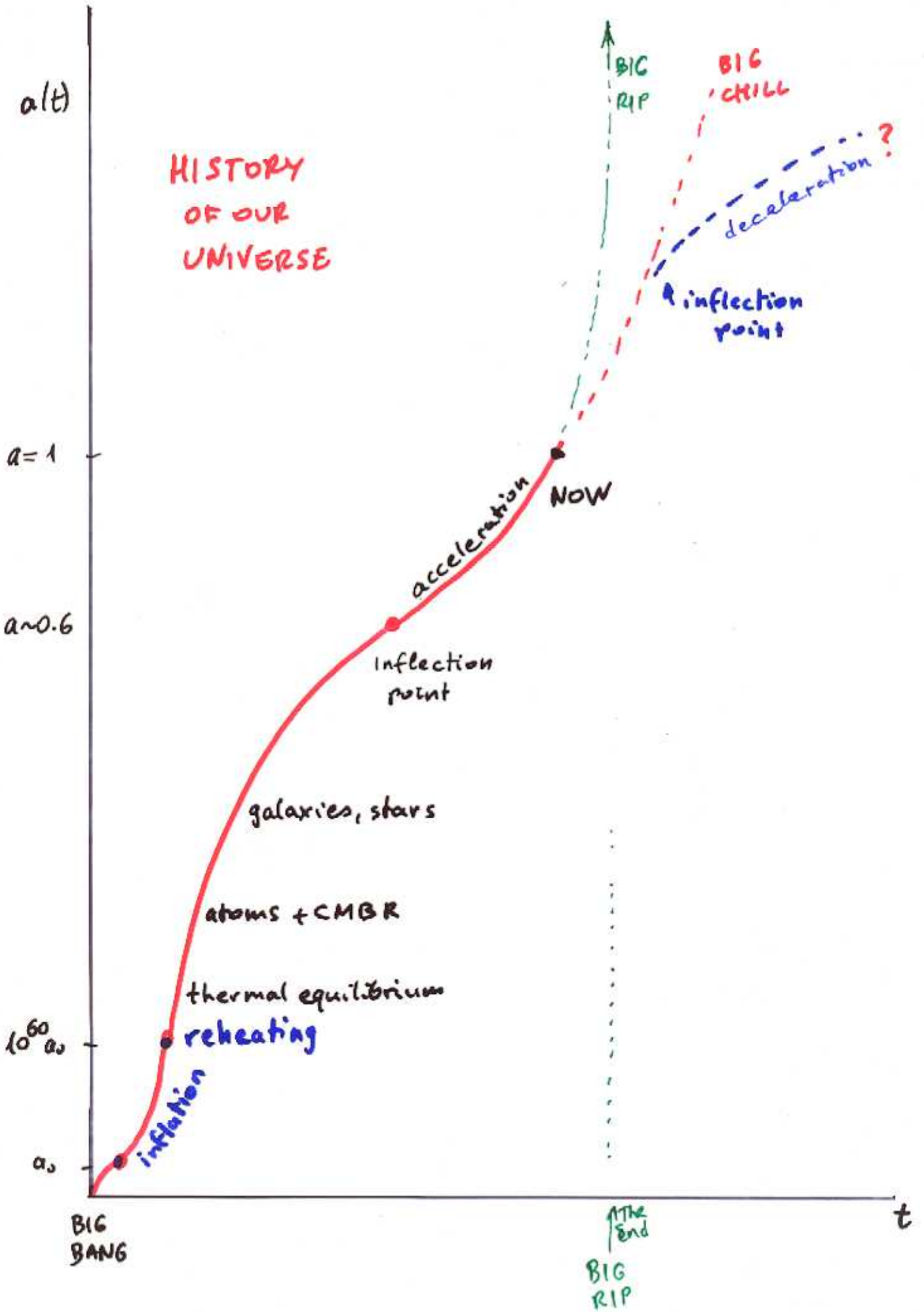
4% ordinary matter (atoms, plasma, gas, stars...)

24% "dark matter" (non emitting gas, black stars, black holes, unknown elementary particles)

72% "dark energy" (cosmological constant, scalar field: inflaton, quintessence, phantom matter...)

100%: just enough to flat the space

COMPETENCE TO CARRY LIFE AND INTELLIGENCE: CAPABLE


a(t)
HISTORY OF OUR UNIVERSE
BIG RIP
BIG CHILL
deceleration ?
inflection point
a = 1
NOW
acceleration
a~0.6
inflection point
galaxies, stars
atoms + CMBR
thermal equilibrium
reheating
$10^{60}\, a_0$
inflation
a_0
BIG BANG
THE End
BIG RIP
t

HISTORY OF THE UNIVERSE

according to the recent data and Standard Model of Cosmology

0 BIG BANG (13.7 billion years ago)

10^{-45} s Epoch of Quantum Gravity
quantum fluctuations of fields → energy density → metrics

10^{-34} s **INFLATION**: dominance of the scaler field energy
exponential blow up of size by a factor $\sim 10^{60}$
blow up of fluctuations

10^{-32} s end of inflation — reheating of the Universe
- birth of heavy particles and antiparticles
- generating small asymmetry between partides and antipadicles

THERMAL EQUILIBRIUM, "thermal death"
with small (10^{-5}) fluctuations of temperature

decelerating expansion, cooling
gradual annihilation of heavy particle-antipartile pairs

hadronic era

10^{-1} s annihilation of antiprotons and antineutrons

10^{2} s nuclear fusion: birth of light nuclei (^{2}H, ^{4}He...)

10^{4} s leptonic era, annihilation of positrons

radiation era

10^{5} years BIRTH OF ATOMS, decoupling of radiation from atoms
origin of CMBR

10^{8} years birth of first galexies and stars

10^{9} years birth of our Galexy and Sun .... and Earth

10^{10} years INFLECTION POINT: bepinning of acceleration
($a \sim 0.6$)

1.4×10^{10} years **TODAY**

? α) Λ is constant → acceleration forever (BIG CHILL)
β) Λ is an artefact of a scaler field, $V(\phi) \to 0$
acceleration ends:
γ) Λ is an artifact of phantom material: BIG RIP in finite time

THEORETICAL STATUS OF MODERN (PRECISION) COSMOLOGY

① What have we done?

We measured the main parameters of the Universe

Similar events

- Magellan's journey around the Earth
  - Columbus believed: the Earth is spherical.
    BUT he did not know the correct radius.
    Magellan "measured" it.
    ANY later maps must use Magellan's result.
- Mandelbrot discovers fractals
  - Everybody had known: the coast of Britain is more winding than that of South Africa but could not prove it numerically.
    The idea and definition of fractal dimension gave the quantitative description
    ANY later theory of sea coast erosion MUST reproduce the measured value of fractal dimension
- Cosmologists measured the parameters of the Universe
  - ANY later theory of cosmology (including present competitor theories, new theories based on new developments in relativity or particle physics) MUST reproduce these measured data (and function)

② The history of the Cosmological Principle

Originated by Giordano Bruno ~1600
"aesthetical" - philosophycal idea

→ Observed ISOTROPY + non-central status of the Earth
→ HOMOGENEITY of the Universe in a given instant

Einstein equations: very complicated mathematical problem (10 nonlinear coupled second order partial differential equations for ~~ten~~ 10 unknowns of 4 variables)

↓ cosmological principle makes it MORE and MORE simple: only 2 ordinary differential equations: they are exactly solvable

BUT is the cosmological principle TRUE?
Or only an oversimplifying assumption?

Precision cosmology MEASURED the correctness of the cosmological principle.

It is now an experimental FACT of finite accuracy, which works as a first approximation to describe the real distribution of matter.

The second approximation of perturbation theory is not a simple correction: it is a positive and fruitful theory of fluctuations of matter and space-time
→ which led to the correct interpretation of observed fluctuations and led to fit successfully the parameters.

The distribution of matter is isotropic an homogeneous
- not because it is beautiful or simple, but now we have a dynamical explanation for the origin of these properties.

② Have we THE correct theory of the Universe?

NO, because

- there are uncertainities in observations, calculations and fits:
  see e.g. Scientific American Aug 2005:
  Is our Universe out of Tune?
- there are yet competitor OLD theories (e.g. steady state)

  BUT: the good agreement between calculations and observations, the very possibility of data fitting to one of EFRW models has strengthened the authitencity of the "standard model", the "mainstream" cosmology and background theories (e.g. particle physics)
- there are competitor NEW theories
  - e.g. multidimensional models of particle physics (10 or 26 dimension of space-time)
    - we must recalculate cosmology
    - we must explain why 4 dimensions are of cosmic size and others microscopic
  - e.g. multiverse models
    - parallel Universes with different cosmological parameters and/or different physical constants
- the background theories (QFT, standard model of particle physics) are not so subtle and consolidated than general relativity

HOPE: following astrophysical observations of increasing accuracy may help to choose the correct version of models in particle physics

POSSIBILITY OF LOGICAL AND AXIOMATIC FOUNDATION OF COSMOLOGY

There are approaches to establish physical theories of space-time (special & general relativity) via logical and axiomatic methods.

Are they applicable to cosmology, to the physics of the ONE and ONLY REAL SPACE-TIME we know?

NO – in present state of cosmology

Nobody knows that the measured parameters of the Universe are "necessary" or "accidental" i.e. is their value a consequence of a deeper physical law or logical necessity – or only a pure measured number without any deeper meaning.

POSSIBLE DEVELOPMENTS

a) Data are "meaningless" – we can have the logical foundation of the space-time, the "container" but not that of the "contained" material.

b) Future developments and interactions between cosmology and quantum field theory leads to the Theory of Everything (TOE) – from which the proper values of the cosmological parameters can be DERIVED.
Such complete and "worldwide" theory NEEDS the logical foundation and axiomatic build-up. This will be the task of later generations of physicists, mathematicians and logicians.

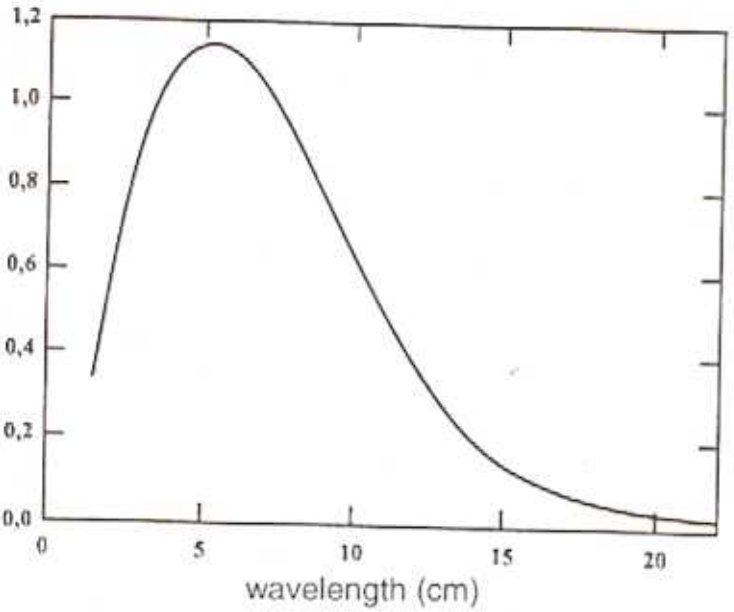
c) Cosmological parameters are "accidental" because of the existence of a MULTIVERSE in which our Universe is only one sample among the billions of parallel worlds.

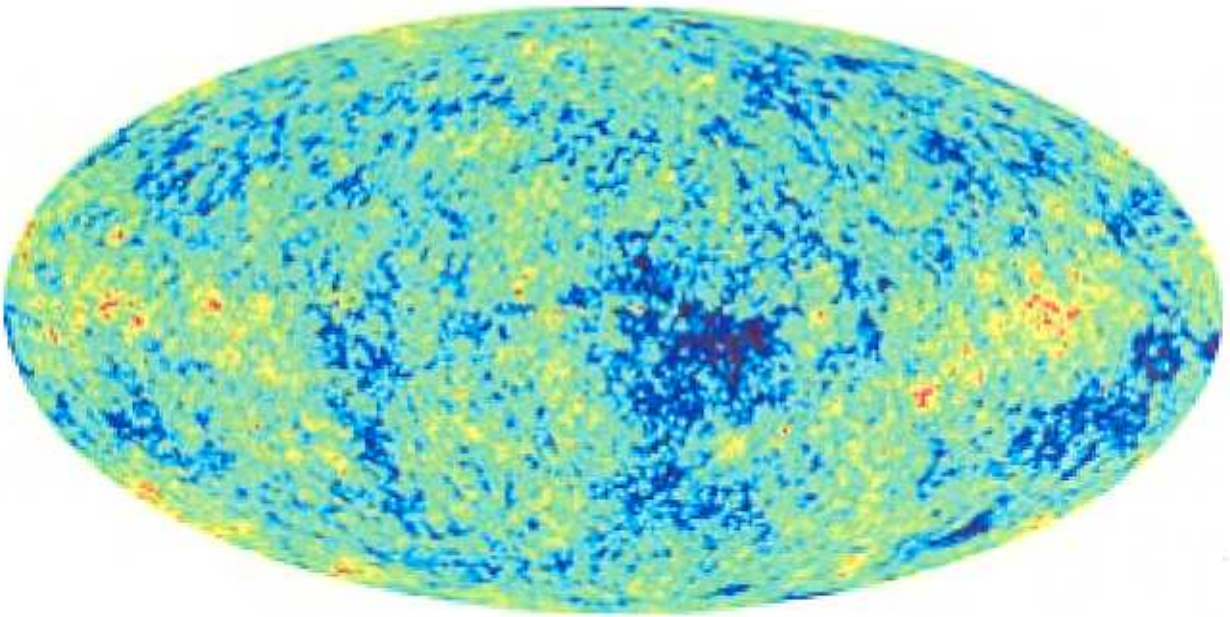
We live in this Universe because of the possibility of our very existence with these parameters (see the problems and idea of "anthropic principle").

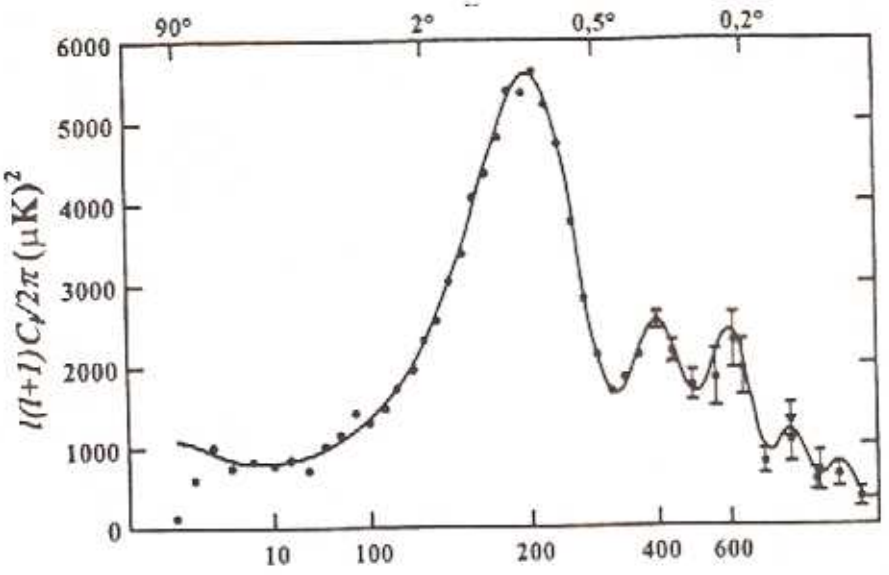
The task for logicians is then to explain and axiomatize the COMMON background laws of ALL of those Universes of the Multiverse.

To start in on dealing with this problem —
— this will be the birth of the real

COSMOLOGIC


1,2
1,0
0,8
0,6
0,4
0,2
0,0
0
5
10
15
20
wavelength (cm)




90°
2°
0,5°
0,2°
6000
5000
4000
3000
2000
1000
0
l(l+1)Cl/2π (μK)²
10
100
200
400
600